DIFFERENTIATION OF A FUNCTION OF A FUNCTION

$y = \sin^2(x^2+1)$

Ex $y = \sin^3(2x^2-1)$ FIND y' $\left(\frac{dy}{dx} = ?\right)$

LET $u = 2x^2 - 1$

$\frac{du}{dx} = \frac{d}{dx}(2x^2-1)$

$= 2 \times 2\, x^{2-1}$

$= 4x$

$y = \sin^3 u$

$\frac{dy}{du} = 3 \sin^{3-1} u \frac{d \sin u}{du}$

$= 3 \sin^2 u \times \cos u$

$$\boxed{\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}}$$

$$= 4x \times 3\sin^2 u \cos u$$

But $u = 2x^2 - 1$

$$= 4x \; 3\sin^2(2x^2-1) \cos 2x^2 - 1$$

$$= 12x \sin^2 2x^2 - 1 \cos 2x^2 - 1$$

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DIFFERENTIATION OF IMPLICIT FUNCT...

pb

$x^2 + y^2 = 9$ Find $\frac{dy}{dx}$

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx} 9$$

$$\frac{d}{dx} x^2 + \frac{d}{dx} y^2 = 0$$

$$2x^{2-1} \cancel{\frac{dx}{dx}} + 2y^{2-1} \frac{dy}{dx} = 0$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$2y \times \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

Qb DIFFERENTIATE FOLLOWING FUNCTIONS WITH RESPECT TO X.

a) $x^2 + y^2 = 4$

b) $y \log_e x = 2$

(a) $\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx} 4$

$$\frac{d}{dx} x^2 + \frac{d}{dx} y^2 = 0$$

$$2x^{2-1} + 2y\frac{dy}{dx} = 0$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

(b) $y \log_e x = 2$

$$\frac{d}{dx}\left[y \log_e x\right] = \frac{d}{dx} 2$$

$$\boxed{\frac{d}{dx} u.v = u\frac{dv}{dx} + v\frac{du}{dx}}$$

$$\boxed{\frac{d}{dx} \log_e f(x) = \frac{f'(x)}{f(x)}}$$

$$y \frac{d}{dx} \log_e x + \log_e x \frac{dy}{dx} = 0$$

$$y \times \frac{\frac{dx}{dx}}{x} + \log_e x \times \frac{dy}{dx} = 0$$

$$\frac{y}{x} + \log_e x \times \frac{dy}{dx} = 0$$

$\log_e x \times \frac{dy}{dx}$

$\frac{dy}{dx} =$

$= \frac{d}{dx} 4$

0

$= 0$

(b) $y \log_e x = 2$

$$\frac{d}{dx}\left[y \log_e x\right] = \frac{d}{dx} 2$$

$$\boxed{\frac{d}{dx} u.v = u\frac{dv}{dx} + v\frac{du}{dx}}$$

$$\boxed{\frac{d}{dx} \log_e f(x) = \frac{f'(x)}{f(x)}}$$

$$y \frac{d}{dx} \log_e x + \log_e x \frac{dy}{dx} = 0$$

$$y \times \frac{\frac{dx}{dx}}{x} + \log_e x \times \frac{dy}{dx} = 0$$

$$\frac{y}{x} + \log_e x \times \frac{dy}{dx} = 0$$

$$\log_e x \times \frac{dy}{dx} = -\frac{y}{x}$$

$$\frac{dy}{dx} = \frac{-y/x}{\log_e x}$$

DIFFERENTIATE THE FOLLOWING TRIGONOMETRIC FUNCTIONS

(i) $\sin(10x+4) + \cos(7x+1)$

(ii) $\tan^2 3\theta$

(iii) $\sec x \tan x$

(iv) $\operatorname{cosec}^4(x^2+1)$

(v) $\cot 5x \sin 6x$

(vi) $1 - 10\sin 10t + 5\sin 20t + 2.5\sin 30t$

(i) $\frac{d}{dx}\left[\sin(10x+4) + \cos(7x+1)\right]$

$\frac{d}{dx}\sin(10x+4) + \frac{d}{dx}\cos(7x+1)$

$\cos(10x+4)\frac{d}{dx}(10x+4) + \left(-\sin(7x+1)\right)\frac{d}{dx}(7x+1)$

$\cos(10x+4) \times 10 - \sin(7x+1) \times 7$

$10\cos(10x+4) - 7\sin(7x+1)$

(ii) $u = \tan 3\theta$

$u^2 = \tan^2 3\theta$

$\frac{d}{d\theta}u^2 = 2u\frac{du}{d\theta}$

$= 2u\frac{d}{d\theta}\tan 3\theta$

$2u \quad \sec^2 3\theta$

$2\tan 3\theta \times \sec^2 3\theta$

$6\tan 3\theta \sec^2 3\theta$

METRIC FUNCTIONS

(i) $\frac{d}{dx}\left[\sin(10x+4) + \cos(7x+1)\right]$

$\frac{d}{dx}\sin(10x+4) + \frac{d}{dx}\cos(7x+1)$

$\cos(10x+4)\frac{d}{dx}(10x+4) + (-\sin(7x+1))\frac{d}{dx}(7x+1)$

$\cos(10x+4) \times 10 - \sin(7x+1) \times 7$

$10\cos(10x+4) - 7\sin(7x+1)$

(ii) $u = \tan 3\theta$

$u^2 = \tan^2 3\theta$

$\frac{d}{d\theta}u^2 = 2u\frac{du}{d\theta}$

$= 2u\frac{d}{d\theta}\tan 3\theta$

$2u \sec^2 3\theta \times \frac{d\,3\theta}{d\theta}$

$2\tan 3\theta \times \sec^2 3\theta \times 3$

$6\tan 3\theta \sec^2 3\theta$ //

(iii) $\frac{d}{dx} \sec x \tan x = \sec x \frac{d}{dx} \tan x + \tan x \frac{d}{dx} \sec x$

$$= \sec x \times \sec^2 x + \tan x \times \sec x \tan x$$

$$= \sec^3 x + \sec x \tan^2 x$$

$$= \sec x \left(\sec^2 x + \tan^2 x \right)$$

(iv) $\operatorname{cosec}^4 (x^2+1)$

$u = x^2+1 \longrightarrow \operatorname{cosec}^4 u$

$$\frac{d}{dx} \operatorname{cosec}^4 u = 4 \operatorname{cosec}^{4-1} u \frac{d}{dx} u$$

$$= 4 \operatorname{cosec}^3 u \times \frac{d}{dx}(x^2+1)$$

$$= 4 \operatorname{cosec}^3 (x^2+1) \times 2x$$

$$= 8x \operatorname{cosec}^3 (x^2+1)$$

(v) cot 5x sin 6x

$$\frac{d}{dx} \cot 5x \cdot \sin 6x = \cot 5x \frac{d}{dx} \sin 6x + \sin 6x \frac{d}{dx} \cot 5x$$

$$\cot 5x \frac{d}{dx} 6x + \sin 6x \times (-\operatorname{cosec}^2 5x) \frac{d}{dx} 5x$$

$$= \cot 5x \cos 6x \frac{d}{dx} 6x + \ldots$$

$$= \cot 5x \cos 6x - 5 \sin 6x \operatorname{cosec}^2 5x$$

$$= 6 \cot 5x \cos 6x$$

(vi) $\frac{d}{dt}\left[1 - 10 \sin 10t + 5 \sin 20t + 2.5 \sin 30t\right]$

$$\frac{d}{dt} 1 - \frac{d}{dt} 10 \sin 10t + \frac{d}{dt} 5 \sin 20t + \frac{d}{dt} 2.5 \sin 30t$$

$$0 - 10 \cos 10t \frac{d}{dt} 10t + 5 \cos 20t \frac{d}{dt} 20t + 2.5 \cos 30t \frac{d}{dt} 30t$$

$$-100 \cos 10t + 100 \cos 20t + 75 \cos 30t$$

pb THE POTENTIAL DIFFER...

OF SELF INDUCTANCE L

$$V_L = L \frac{di}{dt}$$

IF $i = 10 \sin(314t +$

$$V_L = L \frac{d}{dt}\left[10 \sin(\right.$$

$$= L \times 10 \cos(31$$

$$= 10L \cos(31$$

$$= 3140L \cos($$

pb THE POTENTIAL DIFFERENCE ACROSS AN INDUCTOR OF SELF INDUCTANCE L IS

$$V_L = L \frac{di}{dt}$$

IF $i = 10 \sin(314t + 60^\circ)$, FIND THE POTENTIAL DIFFERENCE V_L

$$V_L = L \frac{d}{dt}\left[10 \sin(314t + 60^\circ)\right]$$

$$= L \times 10 \cos(314t + 60^\circ) \times \frac{d}{dt}(314t + 60^\circ)$$

$$= 10L \cos(314t + 60) \times 314$$

$$= 3140L \cos(314t + 60)$$

pb THE SELF INDUCTANCE OF A ROTOR WINDING OF A SALIENT POLE SYNCHRONOUS MACHINE IS

$$L = L_0 + L_2 \cos 2\theta$$

WHERE L_0 AND L_2 ARE CONSTANT AND θ IS THE ANGULAR POSITION OF THE ROTOR. FIND THE RATE OF CHANGE OF INDUCTANCE WITH ANGULAR POSITION.

$$\frac{dL}{d\theta} = \frac{d}{d\theta}\left(L_0 + L_2 \cos 2\theta\right)$$

$$= 0 + L_2 \frac{d}{d\theta} \cos 2\theta$$

$$= L_2(-\sin 2\theta) \times \frac{d}{d\theta} 2\theta$$

$$-2L_2 \sin 2\theta$$

(i) $\frac{d}{dx} e^{-ax} = e^{-ax} \frac{d}{dx}(-ax)$

$= -a e^{-ax}$

(ii) $\frac{d}{dx} e^{x^2+2x} = e^{x^2+2x} \frac{d}{dx}(x^2+2x)$

$= e^{x^2+2x}\left[\frac{d}{dx}x^2 + \frac{d}{dx} \cdot 2x\right]$

$= e^{x^2+2x}\left[2x+2\right]$

$= 2\, e^{x^2+2x}\left[x+1\right]$

(iii) $\frac{d}{dx} \log_e (x^2+2x+3) = \dfrac{\frac{d}{dx}(x^2+2x+3)}{x^2+2x+3}$

$= \dfrac{2x+2}{x^2+2x+3} = \dfrac{2(x+1)}{x^2+2x+3}$

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(iv) $$\frac{d}{dx} a^{x^2+1} = x^2+1 \quad a^{x^2+1-1} \quad \frac{d}{dx} x^2+1$$

$$= (x^2+1) \quad a^{x^2} \quad 2x$$

$$= 2x(x^2+1) \quad a^{x^2}$$

(v) $$\frac{d}{dx} x^{\sin x} = \sin x \quad x^{\sin x - 1} \quad \frac{d}{dx} \sin x$$

$$= \sin x \quad x^{\sin x - 1} \quad \cos x$$

$$= \sin x \cos x \quad x^{\sin x - 1}$$

pb FIND FIRST TWO DIFFERENTIAL COEFFICIENTS OF THE FOLLOWING FUNCTIONS

(i) $\log_e (x^2-1)$

(ii) $\sin^2 x$

(iii) $e^{\frac{x^2}{2}}$

(iv) a^x

(i) $y = \log_e (x^2-1)$

$$\frac{dy}{dx} = \frac{d}{dx} \log_e (x^2-1)$$

$$= \frac{\frac{d}{dx}(x^2-1)}{(x^2-1)}$$

$$= \frac{2x}{x^2-1}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \frac{2x}{x^2-1}$$