LOGARITHMS AND EXPONENTIAL EQUATION

EXPONENTIAL FORM $\quad 2^{-3}, \quad 5^{-1}, \quad a^2, \quad a^3$

$$\log_b N = L \Longrightarrow N = b^L$$

EX WRITE EXPONENTIAL FORM

$$\log_3 9 = 2 \longrightarrow 9 = 3^2$$

EX WRITE THE LOGARITHMIC FORM

$$5^2 = 25$$

$$2 = \log_5 25$$

EVALUATE $\log_3 81 = x$

$$\begin{array}{r|l} 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$3^4 = 81$

$\log_3 3^4 = 4 \log_3 3$

$= 4 \times 1$

$= 4$

$\boxed{\log_A A = 1}$

EVALUATE

$\log_9 \frac{1}{3}$

$\log_9 \frac{1}{3} = x$

$\frac{1}{3} = 9^x$

$3^{-1} = (3^2)^x$

$3^{-1} = 3^{2x}$

$2x = -1$

$x = -1/2$

Ex

$\log_5 \sqrt[3]{5} = ?$

$\log_5 \sqrt[3]{5} = x$

$\sqrt[3]{5} = 5^x$

$5^{1/3} = 5^x$

$x = 1/3$

DECIBEL

RATIO OF POWER	RATIO EXPRESSED IN BEL	DECIBEL
10:1	1 BEL (1B) =	10 dB
100:1	2 BEL (2B) =	20 dB
1000:1	3 BEL (3B) =	30 dB
10^4:1	4 BEL (4B)	= 40 dB
10^5:1	5 BEL (5B)	= 50 dB

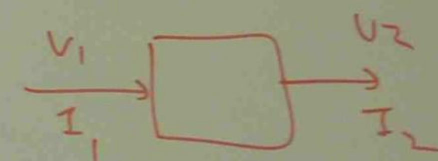
$$\text{NO. OF BEL} = \log_{10} \frac{P_2}{P_1}$$

P_1 = INPUT POWER

P_2 = OUTPUT POWER

1 BEL = 10 DECIBEL

$$\text{NO. OF DECIBEL} = 10 \log_{10} \frac{P_2}{P_1}$$



$$dB = 20 \log_{10} \frac{V_2}{V_1}$$

$$dB = 20 \log_{10} \frac{I_2}{I_1}$$

Ex IF AN AMPLIFIER HAS AN INPUT POWER OF 0.0013 WATT OUTPUT OF 6.5 WATT. CALCULATE POWER GAIN IN dB

$$dB = 10 \log_{10} \frac{P2}{P1}$$

$$= 10 \log_{10} \frac{6.5}{0.0013}$$

$$= 10 \log_{10} 5000$$

$$= 10 \times 3.7$$

$$= 37 \text{ dB}$$

Ex AN AMPLIFIER HAS INPUT POWER 1.7 mW,

(a) OUTPUT POWER 5.8 WATT. FIND POWER GAIN.

(b) AN ATTENUATOR HAS INPUT POWER 3.6 WATT, OUTPUT POWER 4.5 mW. FIND ATTENUATION.

(a) POWER GAIN (dB) $= 10 \log_{10} \frac{P2}{P1}$

$$= 10 \log_{10} \frac{5.8}{1.7 \times 10^{-3}}$$

$$= 10 \times 3.53 = 35.3 \text{ dB}$$

(b) POWER ATTENUATION (dB) $= 10 \log_{10} \frac{P2}{P1}$

$$= 10 \log_{10} \frac{4.5 \times 10^{-3}}{3.6}$$

$$= 10 \log_{10} 1.25 \times 10^{-3}$$

$$= -29 \text{ dB}$$

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THREE LAWS OF LOGARITHM

FIRST $\log(A \times B) = \log A + \log B$

SECOND $\log \frac{A}{B} = \log A - \log B$

THIRD $\log A^n = n \log A$

EVALUATE $\log_3 36 - 2 \log_3 2$

$\log_3 36 - \log_3 2^2$

$\log_3 36 - \log_3 4$

$\log_3 \frac{36}{4}$

$\log_3 9 = \log_3 3^2 = 2 \log_3 3$

$= 2 \times 1$

$= 2$

CHANGE OF BASE

$$\boxed{\log_a N = \frac{\log_b N}{\log_b a}}$$

Solution of Logarithmic Equation

Ex(1) $\log_{10} x = -2$ Find X

$$X = 10^{-2} = \frac{1}{100} = 0.01$$

Ex(2) $\log_2 (0.5V + 0.2) = -2$

Find V

$$\log_2 (0.5V + 0.2) = -2$$

$$0.5V + 0.2 = 2^{-2}$$

$$0.5V + 0.2 = \frac{1}{2^2}$$

$$0.5V + 0.2 = \frac{1}{4}$$

$$0.5V + 0.2 = 0.25$$

$$0.5V = 0.05$$

$$V = \frac{0.05}{0.5}$$

$$= 0.1$$

Ex(3) Solve $2^x + 5 = 13$

$$2^x = 13 - 5 = 8$$

$$2^x = 2^3$$

$$x = 3$$

Ex(4) $\log_{10} \frac{k}{k - x} = t$

Find X

$$\frac{k}{k-x} = 10^t$$

$$\frac{k}{10^t} = k - X$$

$$X = k - \frac{k}{10^t} = k\left(1 - \frac{1}{10^t}\right) = \frac{k(10^t - 1)}{10^t}$$

Ex Plot the

X vs

X = 0

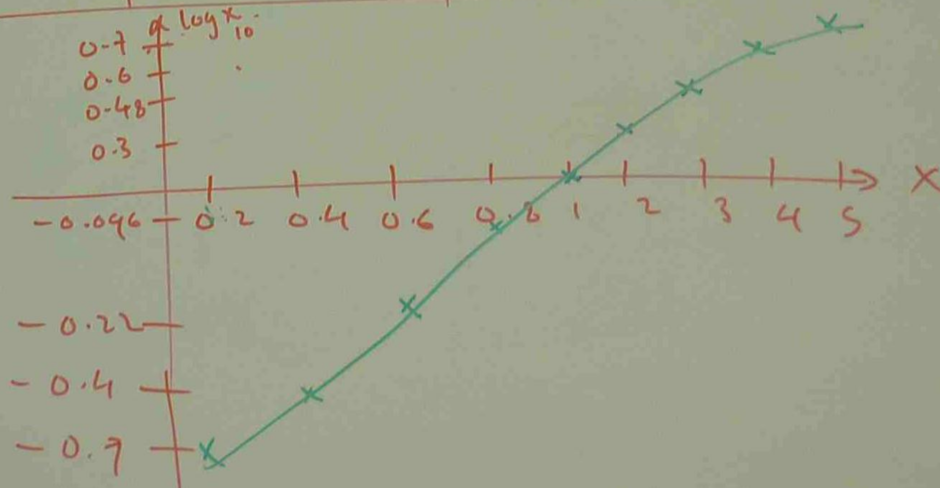
Y = lo

EX PLOT THE FOLLOWING GRAPH

X VS $Y = \log_{10} X$ FOR THE FOLLOWING X VALUES

$X = 0.2,\ 0.4,\ 0.6,\ 0.8,\ 1,\ 2,\ 3,\ 4,\ 5$

X	0.2	0.4	0.6	0.8	1	2	3	4	5
$Y = \log_{10} X$	−0.7	−0.4	−0.22	−0.096	0	0.3	0.48	0.6	0.7



$$1-\frac{1}{10^t}) = k(\frac{10^t-1}{10^t})$$

EXERCISE

FIND (a) $\log_3 \frac{1}{3^{15}}$

(b) USE CALCULATOR AND FIND THE VALUE

$$\log_{10}\sqrt{563} - \log_{10}\sqrt{107}$$

(c) AN AMPLIFIER HAS 10mW INPUT POWER AND 40W OUTPUT POWER

FIND GAIN IN dB

(d) FIND (k) $3^{k+1} = 5$

NON LINEAR EMPIRICAL EQUATIONS

CONVERSION TO LINEAR FORM

$$y = ax^2 + bx$$

TO CHANGE TO LINEAR FORM, DIVIDE BY X

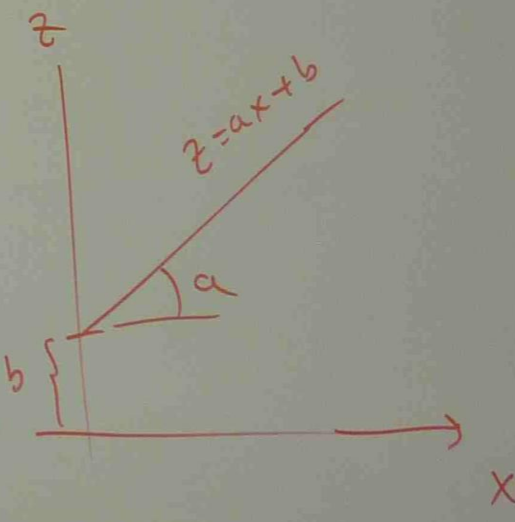
$$\frac{y}{x} = \frac{ax^2}{x} + \frac{bx}{x}$$

$$\frac{y}{x} = ax + b$$

IF $\frac{y}{x} = z$

THEN $\boxed{z = ax + b}$

LINEAR FORM



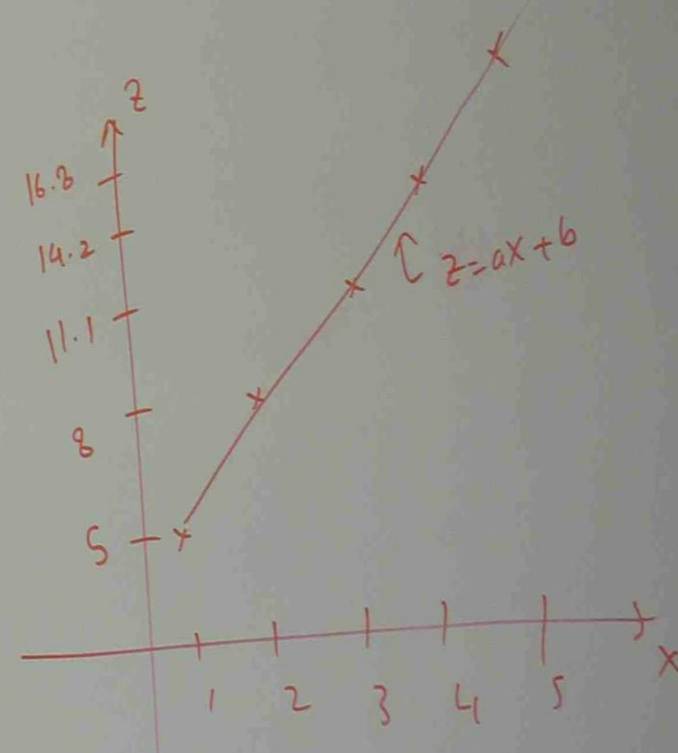
p) THE FOLLOWING VALUES OF x AND y ARE BELIEVED TO SATISFY THE EQUATION OF $y = ax^2 + bx$. FIND A LINEAR EQUATION THAT SUITS THIS INFORMATION AND SO EVALUATE

a & b.

x	1	2	3	4	5
y	5	16	34	57	84

$$z = \frac{y}{x} = \frac{ax^2 + bx}{x} = ax + b$$

x	1	2	3	4	5
$z = \frac{y}{x}$	$5/1 = 5$	$16/2 = 8$	$\frac{34}{3} = 11.1$	$\frac{57}{4} = 14.2$	$\frac{84}{5} = 16.8$



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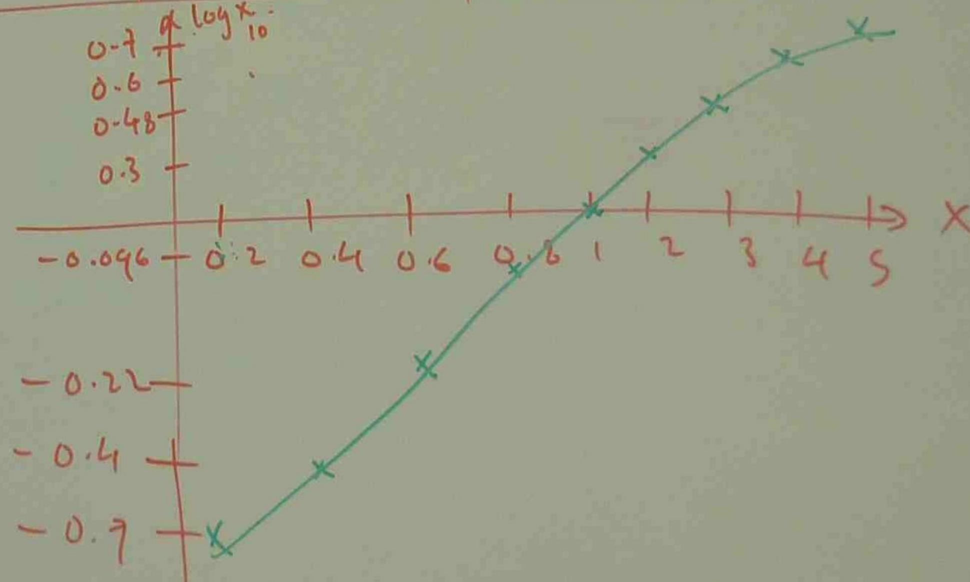

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