DIFFERENTIATION OF QUOTIENT FUNCTIONS

$$y = \frac{u(x)}{v(x)}$$

$$\frac{dy}{dx} = \frac{v(x)\frac{du(x)}{dx} - u(x)\frac{dv(x)}{dx}}{(v(x))^2}$$

Ex DIFFERENTIATE

$$y = \frac{x}{x+1}$$

$$\frac{dy}{dx} = \frac{(x+1)\frac{d}{dx}x - x\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$\frac{dy}{dx} = \frac{(x+1)\times 1 - x\left[\frac{d}{dx}x + \frac{d}{dx}1\right]}{(x+1)^2}$$

$$= \frac{x+1 - x(1)}{(x+1)^2}$$

$$= \frac{x+1-x}{(x+1)^2}$$

$$= \frac{1}{(x+1)^2}$$

$$y = \frac{x^2+2x+1}{x^2-2x+1}$$

FIND $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{(x^2-2x+1)\frac{d}{dx}(x^2+2x+1) - (x^2+2x+1)\frac{d}{dx}(x^2-2x+1)}{[x^2-2x+1]^2}$$

$$= \frac{(x^2-2x+1)\left[\frac{d}{dx}x^2 + \frac{d}{dx}2x + \frac{d}{dx}1\right] - (x^2+2x+1)\left[\frac{d}{dx}x^2 - \frac{d}{dx}2x + \frac{d}{dx}1\right]}{[x^2-2x+1]^2}$$

$$= \frac{(x^2-2x+1)(2x^{2-1}+2+0) - (x^2+2x+1)\left[2x^{2-1} - 2 + 0\right]}{[x^2-2x+1]^2}$$

$$= \frac{(x^2-2x+1)(2x+2) - (x^2+2x+1)(2x-2)}{(x^2-2x+1)^2}$$

$$= \frac{(x^2-2x+1)\times 2(x+1) - (x^2+2x+1)2(x-1)}{(x^2-2x+1)^2}$$

$$\frac{2(x-1)^2(x+1) - 2(x+1)^2(x-1)}{[(x-1)^2]^2}$$

$$\frac{2(x-1)(x+1)[(x-1)-(x+1)]}{(x-1)^4}$$

$$\frac{2(x-1)(x+1)[\cancel{x}-1-\cancel{x}-1]}{(x-1)^4}$$

$$\frac{2(x-1)(x+1)[-2]}{(x-1)^4}$$

$$\frac{-4(x+1)}{(x-1)^3}$$ //

DIFFERENTIATING TRIGONOMETRIC FUNCTIONS

$$\frac{d}{dx}\sin(x) = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$$

DIFFERENTIATING EXPONENTIAL FUNCTION

$$y = e^x$$

$$\frac{dy}{dx} = \frac{de^x}{dx} = e^x$$

If $y = e^u$

$$\frac{dy}{dx} = \frac{d}{dx}e^u = e^u \frac{du}{dx}$$

Pb DIFFERENTIATE (i) $y = e^{ax}$

(ii) $y = e^{\frac{1}{2}bx^2 \ldots}$

(i)

$$y = e^{ax}$$

$$\frac{dy}{dx} = \frac{de^{ax}}{dx} = e^{ax}\frac{dax}{dx}$$

$$= e^{ax} \times a \ldots$$

$$= a e^{ax}$$

(ii) $y = e^{\frac{1}{2}bx^2 + x}$

$$\frac{dy}{dx} = \frac{d}{dx} e^{\frac{1}{2}bx^2 + \ldots}$$

$$= e^{(\frac{1}{2}bx^2 + x \ldots}$$

Pb DIFFERENTIATE (i) $y = e^{ax}$

(ii) $y = e^{\frac{1}{2}bx^2 + x}$

(i) $y = e^{ax}$

$$\frac{dy}{dx} = \frac{de^{ax}}{dx} = e^{ax}\frac{dax}{dx}$$

$$= e^{ax} \times a \frac{\cancel{dx}}{\cancel{dx}}$$

$$= a e^{ax}$$

(ii) $y = e^{\frac{1}{2}bx^2 + x}$

$$\frac{dy}{dx} = \frac{d}{dx} e^{\frac{1}{2}bx^2 + x}$$

$$= e^{(\frac{1}{2}bx^2 + x)} \frac{d}{dx}\left(\frac{1}{2}bx^2 + x\right)$$

$$= e^{\frac{1}{2}bx^2 + x}\left[\frac{d}{dx}\frac{1}{2}bx^2 + \frac{d}{dx}x\right]$$

$$= e^{\frac{1}{2}bx^2 + x}\left[\frac{b}{2}\frac{d}{dx}x^2 + 1\right]$$

$$= e^{\frac{1}{2}bx^2 + x}\left[\frac{b}{2} \times 2x^{2-1} + 1\right]$$

$$= e^{\frac{1}{2}bx^2 + x}\left[bx + 1\right]$$ //

DIFFERENTIATING LOGARITHMIC FUNCTION

$$y = \log_e u(x) = \ln u(x)$$

$$\boxed{\frac{dy}{dx} = \frac{1}{u(x)} \frac{d}{dx} u(x)}$$

DIFFERENTIATE

$\log_e x^3$

$\log_e (x^2 - 1)$

$\log_e \sin x$

(i) $\frac{d}{dx} \log_e x^2 = \frac{1}{x^2} \frac{d}{dx} x^2$

$= \frac{1}{x^2} \times 2x^{2-1}$

$= \frac{2x}{x^2}$

$= \frac{2}{x}$

(ii) $\frac{d}{dx} \log_e (x^2-1) = \frac{1}{(x^2-1)} \frac{d}{dx}(x^2-1)$

$= \frac{1}{(x^2-1)} \left[\frac{d}{dx} x^2 - \frac{d}{dx} 1\right]$

$= \frac{1}{(x^2-1)} \left[2x\right] = \frac{2x}{x^2-1}$

(iii) $\frac{d}{dx} \log_e \sin x = \frac{1}{\sin x} \frac{d}{dx} \sin x$

$$= \frac{1}{\sin x} \cos x$$

$$= \frac{\cos x}{\sin x} = \cot x$$

$$\log_e u = \frac{\log_{10} u}{\log_{10} e} = \frac{\log_{10} u}{\log_{10} 2.718} = \frac{\log_{10} u}{0.434}$$

$$= 2.3 \log_{10} u$$

$$\log_{10} u = \frac{\log_e u}{2.3}$$

$$\frac{d}{dx} \log_{10} u = \frac{d}{dx}\left(\frac{\log_e u}{2.3}\right)$$

$$\frac{d}{dx} \log_{10} u = \frac{1}{2.3} \frac{d}{dx} \log_e u$$

$$= \frac{1}{2.3} \times \frac{1}{u} \frac{du}{dx}$$

SUCCESSIVE DIFFERENTIATION

$$y = x^3 + 3x^2 + 4$$

$$\frac{dy}{dx} = \frac{d}{dx}\left[x^3 + 3x^2 + 4\right]$$

$$= \frac{d}{dx}x^3 + \frac{d}{dx}3x^2 + \frac{d}{dx}4\,]$$

$$= 3x^{3-1} + 3\times 2x^{2-1} + 0$$

$$= 3x^2 + 6x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[3x^2 + 6x\right]$$

$$= \frac{d}{dx}3\left[x^2 + 2x\right]$$

$$\frac{d^2y}{dx^2} = 3\left[\frac{dx^2}{dx} + \frac{d2x}{dx}\right]$$

$$= 3\left[2x^{2-1} + 2\right]$$

$$= 3\left[2x + 2\right]$$

$$= 6(x+1)$$

pb) FIND THE FIRST THREE DIFFERENTIAL COEFFICIENTS OF THE FUNCTION

$$y = \frac{1}{2x+1} = (2x+1)^{-1}$$

$$\frac{dy}{dx} = \frac{d}{dx}(2x+1)^{-1} = -1(2x+1)^{-1-1}\frac{d}{dx}(2x+1)$$

$$= -1(2x+1)^{-2} \times 2 = -2(2x+1)^{-2}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[-2(2x+1)^{-2}\right] = -2\frac{d}{dx}(2x+1)^{-2}$$

$$= -2 \times (-2)(2x+1)^{-2-1}\frac{d}{dx}(2x+1)$$

$$= 4(2x+1)^{-3} \times 2$$

$$= 8(2x+1)^{-3}$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx} 8(2x+1)^{-3}$$

$$= 8\frac{d}{dx}(2x+1)^{-3}$$

$$= 8(-3)(2x+1)^{-3-1}\frac{d}{dx}(2x+1)$$

$$= -24(2x+1)^{-4} \times 2$$

$$= -48(2x+1)^{-4}$$

$$= \frac{-48}{(2x+1)^4}$$ //

EXERCISES

① DIFFERENTIATE

(i) $y = x^{10}$

(ii) $y = x^{3}$

(iii) $y = m x^{k}$

(iv) $y = (x-5)^{3}$

(v) $y = 15(x+4)^{7}$

(vi) $y = 4x^{-3.2}$

(vii) $y = 3.2(2x+3)^{1/5}$

② DIFFERENTIATE

(a) $y = 2x^{3/2} + x^{-3/2}$

(b) $y = 7x^{6} + 6x^{5} + 4x^{4} + 3x^{2} + 2x + 1$

(c) $y = \dfrac{1}{\sqrt{x}} + \dfrac{1}{\sqrt[3]{x^{2}}}$

(d) $y = -x^{-2} + \dfrac{1}{x} + x^{2}$

(3) THE LOSS IN ELECTRICAL MACHINE IS GIVEN BY

$$P = af + bf^2$$

WHERE P = POWER $\qquad a, b$ = CONSTANT

f = FREQUENCY

FIND $\frac{dP}{df}$

(4) THE INDUCED EMF OF A DC MACHINE IS GIVEN BY

$$E = 0.58 + 681.5\, I_f - 461.8\, I_f^2 + 46.3\, I_f^3$$

I_f = FIELD EXCITATION CURRENT

FIND $\frac{dE}{df}$

(5) DIFFERENTIATE

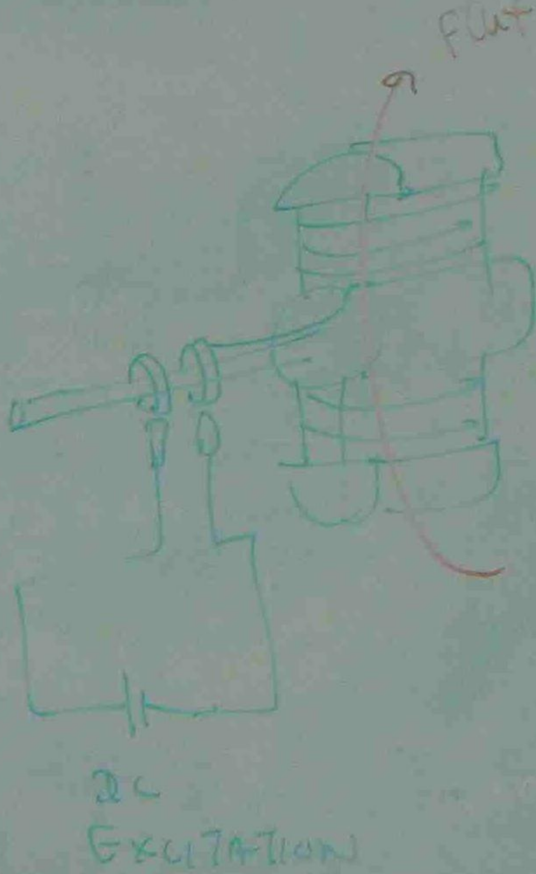
(i) $(x+1)^{-\frac{1}{2}}(x-5)$

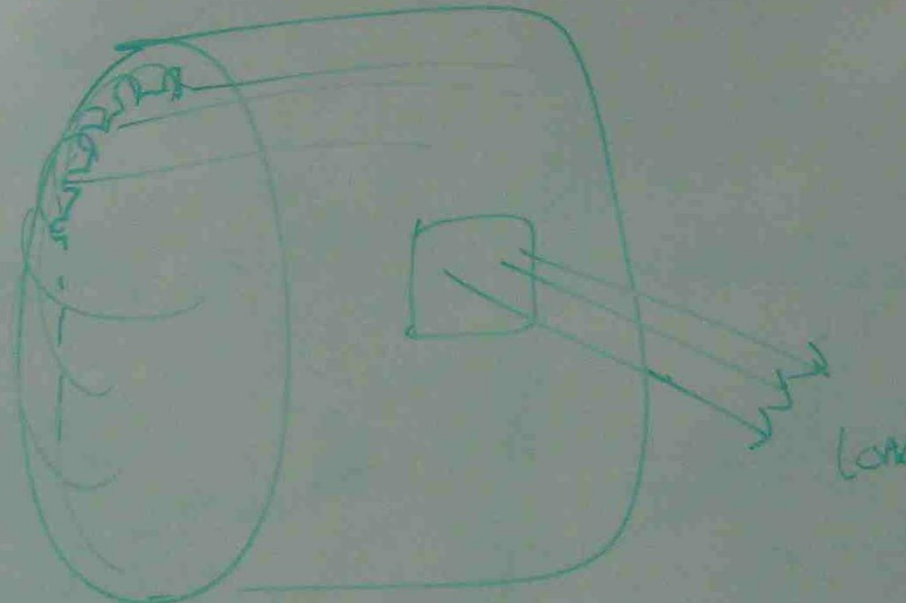
(ii) $(2x+7)^3(4x^2-5)^2$

(iii) $(5x^2+6x+3)(5x-1)$

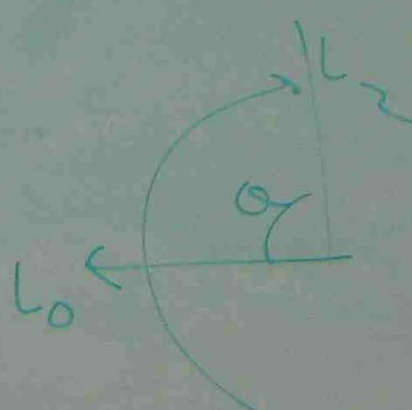
(iv) $\frac{4-x}{x-x^2}$

(v) $\frac{x^{1/4}}{x^{1/2}-1}$





$$N = \frac{120 f}{P}$$



pb

DIFFERENTIATE THE FOLLOWING EXPONENTIAL, LOGARITHMIC AND POWER FUNCTIONS:

(i) e^{-ax}

(ii) $e^{(x^2+2x)}$

(iii) $\log_e (x^2+2x+3)$

(iv) a^{x^2+1}

(v) $x^{\sin x}$

$$\boxed{\frac{d}{dx}\frac{u}{v} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}}$$

$$= \frac{(x^2-1)\frac{d}{dx}2x - 2x\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$= \frac{2(x^2-1) - 2x \times 2x}{(x^2-1)^2}$$

$$= \frac{2x^2 - 2 - 4x^2}{(x^2-1)^2}$$

$$= \frac{-2(x^2+1)}{(x^2-1)^2}$$

(ii) $y = \sin^2 x$

$$\frac{dy}{dx} = \frac{d}{dx}\sin^2 x = 2\sin^{2-1} x \frac{d\sin x}{dx} = 2\sin x\cos x$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \times 2\sin x\cos x$$

$$= 2\left[\sin x\frac{d}{dx}\cos x + \cos x\frac{d}{dx}\sin x\right]$$

$$= 2\left[\sin x(-\sin x) + \cos x \times \cos x\right]$$

$$= -2\left[\sin^2 x - \cos^2 x\right]$$

$$= 2\left[-\sin^2 x + \cos^2 x\right]$$

$$2\left[1 - 2\sin^2 x\right]$$

$$= 2\left[-\sin^2 x + 1 - \sin^2 x\right]$$

(iii) $y = e^{\frac{x^2}{2}}$

$$\frac{dy}{dx} = \frac{d}{dx} e^{\frac{x^2}{2}} = e^{\frac{x^2}{2}} \frac{d}{dx} x^2/2$$

$$= e^{\frac{x^2}{2}} \times \frac{1}{2} \times 2x$$

$$= x e^{x^2/2}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} x e^{\frac{x^2}{2}}$$

$$= x \frac{d}{dx} e^{\frac{x^2}{2}} + e^{\frac{x^2}{2}} \frac{d}{dx} x$$

$$= x \times e^{\frac{x^2}{2}} \frac{d}{dx} x^2/2 + e^{\frac{x^2}{2}}$$

$$= x\, e^{x^2/2} \times \frac{\cancel{2}x}{\cancel{2}} + e^{x^2/2}$$

$$= e^{x^2/2} [x^2+1] \;//$$

(iv) $y = a^x$

$$\frac{dy}{dx} = \frac{d}{dx} a^x = x\, a^{x-1}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} [x \cdot a^{x-1}]$$

$$= x \frac{d}{dx} a^{x-1} + a^{x-1} \frac{dx}{dx}$$

$$= x \times (x-1)\, a^{x-1-1} + a^{x-1}$$

$$= x(x-1)\, a^{x-2} + a^{x-1}$$

pb THE CURRENT GROWTH IN A RESISTIVE - INDUCTIVE CIRCUIT FROM A SUDDENLY APPLIED BATTERY EMF IS

$$i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

WHERE E, R, L ARE CONSTANTS, t IS TIME

FIND THE RATE OF CHANGE OF CURRENT WITH RESPECT TO TIME.

$$\frac{di}{dt} = \frac{d}{dt}\left[\frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right)\right]$$

$$= \frac{E}{R}\left[-\frac{d}{dt}e^{-\frac{Rt}{L}}\right]$$

$$= -\frac{E}{R} \times e^{-Rt/L} \times \frac{d}{dt}\left(-\frac{Rt}{L}\right)$$

$$= -\frac{E}{R} \times e^{-\frac{Rt}{L}} \times \left(-\frac{R}{L}\right)$$

$$\left\{ \frac{di}{dt} = \frac{E}{L} \times e^{-\frac{Rt}{L}} \right.$$