

POLYNOMIALS

$$7x^{15} + 6x^8 - 4x^2$$

$$13x^2 - 6x - 8$$

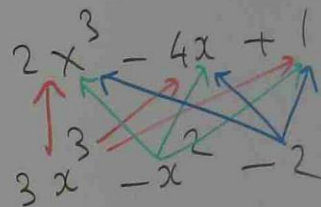
$$5x + 3 \quad \text{DEGREE } 1$$

$$4x^3 \quad \text{DEGREE } 3$$

$$x^m \times x^n = x^{m+n}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

Ex $(2x^3 - 4x + 1) \times (3x^3 - x^2 - 2)$



$$\begin{array}{r} 6x^6 - 12x^4 + 3x^3 \\ - 2x^5 + 4x^2 - 2 \\ \hline 6x^6 - 2x^5 - 12x^4 + 3x^3 - x^2 + 8x - 2 \end{array}$$

$$6x^6 - 2x^5 - 12x^4 + 3x^3 - x^2 + 8x - 2$$

Ex EXPAND

$$(2x-3)(x^2-4x+2)$$

FIND THE COEFFICIENT OF x^3 IN EXPANSION

$$\begin{array}{r} x^2 - 4x + 2 \\ 2x - 3 \end{array}$$

$$\begin{array}{r} 2x^3 - 8x^2 + 4x \\ - 3x^2 + 12x - 6 \end{array}$$

$$2x^3 - 11x^2 + 16x - 6$$

THE COEFFICIENT OF
 x^3 IS 2

Ex EVALUATE

$$2x^2 + 7x - 9 \text{ WHEN } x=4$$

$$2(4)^2 + 7 \times 4 - 9$$

$$2 \times 16 + 28 - 9$$

$$32 + 28 - 9$$

$$60 - 9 = 51$$

FACTORISATION

$$6 = 2 \times 3$$

$$12 = 2 \times 6$$

$$12 = 4 \times 3$$

$$8ab + 12ac$$

$$a(8b + 12c)$$

$$a(4 \times 2b + 4 \times 3c)$$

$$4a(2b + 3c)$$

EXERCISE

FACTORISE

(i) $4x + 6$

(ii) $9 - 12a$

(iii) $ab + a$

(iv) $6xy - 9x^2$

(v) $5x^2y - 10ax^2 + 5x^2$

(vi) $3a^2b^2 + 6ab^2$

Ex FACTORISE $e^x + e^{-x}$

$$e^{-2x} \times e^x = e^{-2x+x} = e^{-x}$$

$$e^x + e^{-2x} \times e^x$$

$$e^x (1 + e^{-2x})$$

Ex $6e^{2x} - 4e^x$

$$e^{2x} = e^{x+x} = e^x \times e^x$$

$$6e^x \times e^x - 4e^x$$

$$e^x (6e^x - 4)$$

$$2e^x (3e^x - 2)$$

Ex $\frac{e^x + 3e^{3x}}{e^{-x} + 3e^x}$

$$\frac{e^x + 3e^{2x+x}}{e^{-2x+x} + 3e^x}$$

$$\frac{e^x + 3e^{2x} \times e^x}{e^{-2x} \times e^x + 3e^x}$$

$$e^x (1 + 3e^{2x})$$

$$e^x \left(\frac{e^{-2x}}{e} + 3 \right)$$

$$\frac{1 + 3e^{2x}}{3 + e^{-2x}}$$

$$3 + e^{-2x}$$

$$(a+b)(a-b)$$

$$\begin{array}{r} \begin{array}{cc} a & + & b \\ \uparrow & \nearrow & \nearrow \\ a & - & b \end{array} \\ \hline a^2 + ab \\ - ab - b^2 \\ \hline a^2 - b^2 \end{array}$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$(x+y)(x+y)$$

$$\begin{array}{r} \begin{array}{cc} x & + & y \\ \uparrow & \nearrow & \nearrow \\ x & + & y \end{array} \\ \hline x^2 + xy \\ + xy + y^2 \\ \hline x^2 + 2xy + y^2 \end{array}$$

$$(x^2 + 2xy + y^2) = (x+y)(x+y)$$

Ex

Factorise

$$(a) \quad x^2 + 4x + 3$$

$$(b) \quad \theta^2 + 8\theta + 12$$

$$(c) \quad z^2 + 7z + 10$$

$$(d) \quad x^2 + 8x + 15$$

$$(e) \quad E^2 + 11E + 24$$

$$(a) \quad x^2 + 4x + 3$$

$$\begin{array}{r} \begin{array}{cc} x & & 3 \\ \uparrow & \nearrow & \nearrow \\ x & & 1 \end{array} \\ \hline 3x + x = 4x \end{array}$$

$$(x^2 + 4x + 3) = (x+3)(x+1)$$

$$(b) \theta^2 + 8\theta + 12$$

$$\begin{array}{r} \theta \quad \quad 6 \\ \swarrow \quad \searrow \\ \theta \quad \quad 2 \end{array}$$

$$2\theta + 6\theta = 8\theta$$

$$(\theta^2 + 8\theta + 12) = (\theta + 6)(\theta + 2)$$

$$(c) z^2 + 7z + 10$$

$$\begin{array}{r} z \quad \quad 5 \\ \swarrow \quad \searrow \\ z \quad \quad 2 \end{array}$$

$$2z + 5z = 7z$$

$$(z^2 + 7z + 10) = (z + 5)(z + 2)$$

$$(d) x^2 + 8x + 15$$

$$\begin{array}{r} x \quad \quad 5 \\ \swarrow \quad \searrow \\ x \quad \quad 3 \end{array}$$

$$3x + 5x = 8x$$

$$(x^2 + 8x + 15) = (x + 5)(x + 3)$$

$$(e) E^2 + 11E + 24$$

$$\begin{array}{r} E \quad \quad 8 \\ \swarrow \quad \searrow \\ E \quad \quad 3 \end{array}$$

$$3E + 8E = 11E$$

$$E^2 + 11E + 24 = (E + 8)(E + 3)$$

EXERCISE

FACTORISE

$$(a) f^2 + 17f + 16$$

$$(h) \theta^2 + 12\theta + 32$$

$$(a+b)(a-b)$$

$$\begin{array}{r} a+b \\ \uparrow \quad \nearrow \quad \searrow \quad \uparrow \\ a-b \\ \hline a^2+ab \\ -ab-b^2 \\ \hline a^2-b^2 \end{array}$$

$$a^2-b^2 = (a+b)(a-b)$$

$$(x+y)(x+y)$$

$$\begin{array}{r} x+y \\ \uparrow \quad \nearrow \quad \searrow \quad \uparrow \\ x+y \\ \hline x^2+xy \\ +xy+y^2 \\ \hline x^2+2xy+y^2 \end{array}$$

$$(x^2+2xy+y^2) = (x+y)(x+y)$$

Ex

Factorise

$$(a) \quad x^2+4x+3$$

$$(b) \quad x^2+8x+12$$

$$(c) \quad x^2+7x+10$$

$$(d) \quad x^2+8x+15$$

$$(e) \quad x^2+11x+24$$

$$(a) \quad x^2+4x+3$$

$$\begin{array}{r} x \quad \quad 3 \\ \nearrow \quad \searrow \\ x \quad \quad 1 \\ \hline 3x+x=4x \end{array}$$

$$(x^2+4x+3) = (x+3)(x+1)$$

$$(b) \theta^2 + 8\theta + 12$$

$$\begin{array}{r} \theta \quad \quad 6 \\ \swarrow \quad \searrow \\ \theta \quad \quad 2 \\ \hline 2\theta + 6\theta = 8\theta \end{array}$$

$$(\theta^2 + 8\theta + 12) = (\theta + 6)(\theta + 2)$$

$$(c) z^2 + 7z + 10$$

$$\begin{array}{r} z \quad \quad 5 \\ \swarrow \quad \searrow \\ z \quad \quad 2 \\ \hline 2z + 5z = 7z \end{array}$$

$$(z^2 + 7z + 10) = (z + 5)(z + 2)$$

$$(d) x^2 + 8x + 15$$

$$\begin{array}{r} x \quad \quad 5 \\ \swarrow \quad \searrow \\ x \quad \quad 3 \\ \hline 3x + 5x = 8x \end{array}$$

$$(x^2 + 8x + 15) = (x + 5)(x + 3)$$

$$(e) E^2 + 11E + 24$$

$$\begin{array}{r} E \quad \quad 8 \\ \swarrow \quad \searrow \\ E \quad \quad 3 \\ \hline 3E + 8E = 11E \end{array}$$

$$E^2 + 11E + 24 = (E + 8)(E + 3)$$

EXERCISE

Factorise

$$(a) f^2 + 17f + 16$$

$$(h) \theta^2 + 12\theta + 32$$

Ex SOLVE $\frac{8x^2 + 40x + 48}{4x^2 - 36}$

$$\frac{8(x^2 + 5x + 6)}{4(x^2 - 9)}$$

$$\begin{array}{r} x \quad 3 \\ \swarrow \quad \searrow \\ x \quad 2 \end{array}$$

$$(x+3)(x+2)$$

$$2x + 3x = 5x$$

$$x^2 - 9 = (x-3)(x+3)$$

$$\frac{8(x+3)(x+2)}{4(x-3)(x+3)}$$

$$\frac{8(x+2)}{4(x-3)}$$

$$\frac{2(x+2)}{1}$$

Ex SOLVE (a) $\frac{6-2k}{3} = \frac{2(k-7)}{k}$

(b) $\frac{x-4}{3} = \frac{5-2x}{x-2}$

(c) $\frac{9}{R+3} = \frac{3+R}{9}$

(a) $\frac{6-2k}{3} = \frac{2(k-7)}{k}$

$$k(6-2k) = 3 \times 2(k-7)$$

$$6k - 2k^2 = 6(k-7)$$

$$\cancel{6k} - 2k^2 = \cancel{6k} - 42$$

$$-2k^2 = -42$$

$$k^2 = 21$$

$$k = \sqrt{21}$$

$$(b) \quad \frac{x-4}{3} = \frac{5-2x}{x-2}$$

$$(x-4)(x-2) = 3(5-2x)$$

$$x^2 - 4x - 2x + 8 = 15 - 6x$$

$$x^2 - 6x + 8 = 15 - 6x$$

$$x^2 - 6x + 6x = 15 - 8$$

$$x^2 = 7$$

$$x = \sqrt{7}$$

$$(c) \quad \frac{5}{R+3} = \frac{3+R}{5}$$

$$5 \times 5 = (R+3)(R+3)$$

$$25 = R^2 + 3R + 3R + 9$$

$$25 = R^2 + 6R + 9$$

$$R^2 + 6R - 16 = 0$$

$$\begin{array}{ccc} R & + & 8 \\ & \times & \\ R & - & 2 \end{array}$$

$$8R - 2R = 6R$$

$$(R+8)(R-2) = 0$$

$$R+8=0 \text{ (or) } R-2=0$$

$$\therefore R = -8 \text{ (or) } R = 2$$

APPLICATION OF QUADRATIC FORMULA

$$Ax^2 + Bx + C = 0$$

THEN
$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Ex Solve
 $(x-7)(x+2) = 4x-1$

$$x^2 - 7x + 2x - 14 = 4x - 1$$

$$x^2 - 5x - 14 = 4x - 1$$

$$x^2 - 5x - 14 - 4x + 1 = 0$$

$$x^2 - 9x - 13 = 0$$

$$Ax^2 + Bx + C = 0$$

$$A=1, B=-9, C=-13$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 1 \times (-13)}}{2 \times 1}$$

$$= \frac{9 \pm \sqrt{81 + 52}}{2}$$

$$= \frac{9 \pm \sqrt{133}}{2}$$

$$= \frac{9 \pm 11.53}{2}$$

$$= \frac{9+11.53}{2} \text{ (or) } \frac{9-11.53}{2}$$

$$= \frac{20.53}{2} \text{ (or) } \frac{-2.53}{2}$$

$$= 10.265 \text{ (or) } -1.265$$

EXERCISE

(1) solve

$$50E^2 + 150E - 100 = 0$$

(2) solve

$$x^2 + 2x - 5 = 0$$

FUNCTIONS AND THEIR GRAPHS

$$f(x) = x - 3$$

$$\text{If } x=1 \rightarrow f(1) = 1 - 3 = -2$$

$$x=2 \rightarrow f(2) = 2 - 3 = -1$$

$$x=3 \rightarrow f(3) = 3 - 3 = 0$$

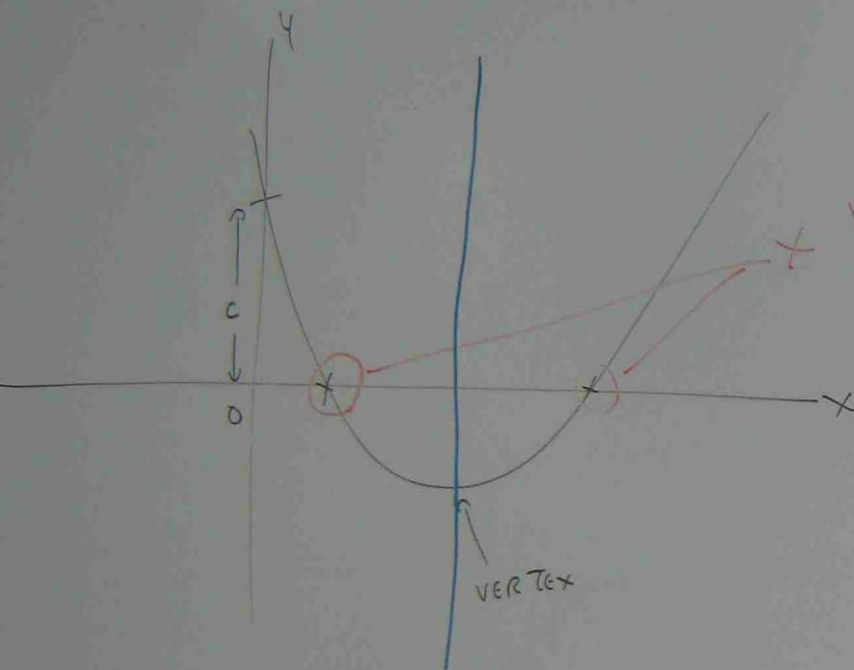
$$g(x) = 4 + x^2$$

$$\text{If } x=1 \rightarrow g(1) = 4 + 1^2 = 5$$

$$x=2 \rightarrow g(2) = 4 + 2^2 = 8$$

$$x=3 \rightarrow g(3) = 4 + 3^2 = 13$$

PLOTTING THE GRAPH FOR EQUATION $y = ax^2 + bx + c$



$$x = -\frac{b}{2a}$$

(Axis of symmetry)

STEP(1)

Plot $y = 2x^2 - 12x + 13$

compare $ax^2 + bx + c$

$$\therefore a = 2, b = -12, c = 13$$

FIND AXIS OF SYMMETRY

$$\text{Axis of symmetry} = \frac{-b}{2a} = \frac{-(-12)}{2 \times 2} = 3$$

Y VALUE AT AXIS OF SYMMETRY

$$y = 2x^2 - 12x + 13$$

$$y = 2(3)^2 - 12 \times 3 + 13$$

$$= -5$$

$$x = 3 \text{ (Axis of symmetry)}$$

Vertex is located at $y = -5$

STEP ②

FIND X AXIS CROSSINGS

$$y = 0$$

FIND X VALUE

$$y = 2x^2 - 12x + 13$$

$$2x^2 - 12x + 13 = 0$$

$$Ax^2 + Bx + C = 0$$

$$A = 2, B = -12, C = 13$$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

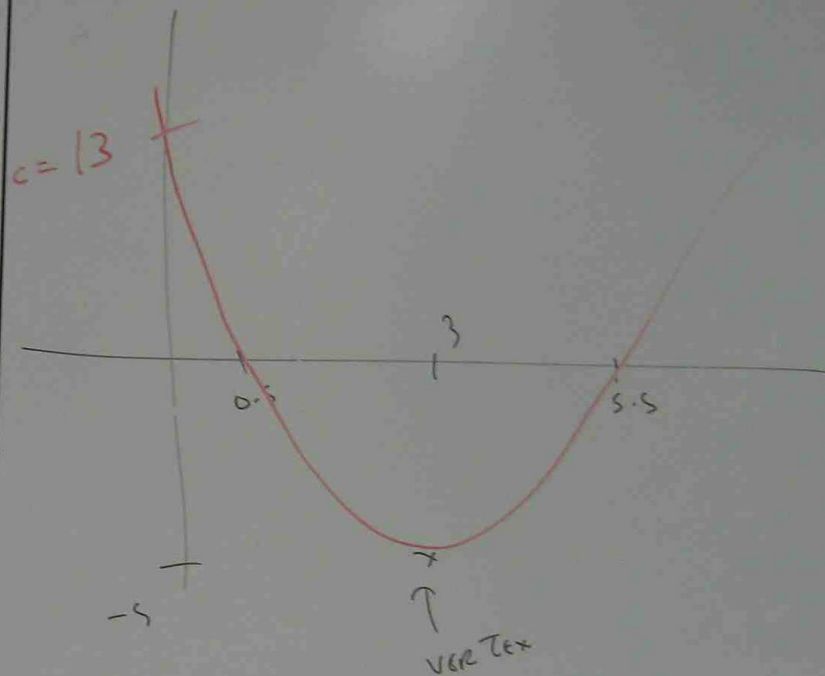
$$= \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \times 2 \times 13}}{2 \times 2}$$

$$= \frac{12 \pm \sqrt{100}}{4}$$

$$= \frac{12 \pm 10}{4}$$

$$= \frac{12+10}{4} \text{ (or) } \frac{12-10}{4}$$

$$= 5.5 \text{ (or) } 0.5$$



$$(b) \quad \frac{x-4}{3} = \frac{5-2x}{x-2}$$

$$(x-4)(x-2) = 3(5-2x)$$

$$x^2 - 4x - 2x + 8 = 15 - 6x$$

$$x^2 - 6x + 8 = 15 - 6x$$

$$x^2 - 6x + 6x = 15 - 8$$

$$x^2 = 7$$

$$x = \sqrt{7}$$

$$(c) \quad \frac{5}{R+3} = \frac{3+R}{5}$$

$$5 \times 5 = (R+3)(R+3)$$

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$$25 = R^2 + 6R + 9$$

$$R^2 + 6R - 16 = 0$$

$$\begin{array}{r} R \quad + \quad 8 \\ R \quad - \quad 2 \end{array}$$

$$8R - 2R = 6R$$

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$$R+8=0 \text{ (or) } R-2=0$$

$$\therefore R = -8 \text{ (or) } R = 2$$

pb THE FOLLOWING VALUES OF X AND Y ARE BELIEVED TO SATISFY THE EQUATION OF $y = ax^2 + bx$. FIND A LINEAR EQUATION THAT SUITS THIS INFORMATION AND SO EVALUATE a & b .

X	1	2	3	4	5
Y	5	16	34	57	84

$$z = \frac{y}{x} = \frac{ax^2 + bx}{x} = ax + b$$

X	1	2	3	4	5
$z = \frac{Y}{X}$	$5/1 = 5$	$16/2 = 8$	$34/3 = 11.1$	$57/4 = 14.2$	$84/5 = 16.8$

