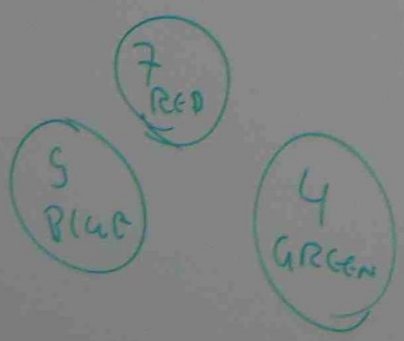
SET THEORY AND ELEMENTARY PROBABILITY

DEFINITION of PROBABILITY

$$P(A) = \frac{\text{NUMBER of TIMES A OCCUR}}{\text{TOTAL NUMBER of REPETITIONS OF EXPERIMENT}} = \frac{n}{N}$$

Pb (11) A BAG CONTAINS 7 RED BALLS, 5 BLUE AND 4 GREEN BALLS. FIND THE PROBABILITY THAT IF ONE BALL ONLY IS DRAWN RANDOMLY, IT WILL BE GREEN.



N = TOTAL NUMBER OF BALLS = $7+5+4 = 16$

n = NO. of GREEN BALL = 4

$$P(\text{GREEN}) = \frac{n}{N} = \frac{4}{16} = \frac{1}{4}$$

Pb (12)

pb (12) IF A SINGLE DRAW IS MADE FROM A STANDARD PACK OF CARDS. FIND THE PROBABILITY OF DRAWING A JACK (OR) A QUEEN (OR) A KING.

1 PACK = 52 CARDS.

JACK = 4 CARDS

QUEEN = 4 CARDS

KING = 4 CARDS.

$P(A) = P(\text{JACK}) = \frac{4}{52} = \frac{1}{13}$

$P(B) = P(\text{QUEEN}) = \frac{4}{52} = \frac{1}{13}$

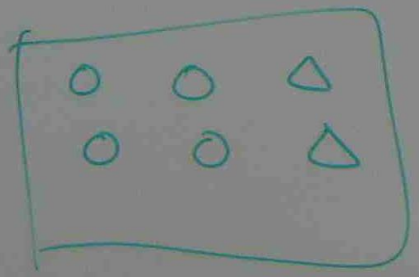
$P(C) = P(\text{KING}) = \frac{4}{52} = \frac{1}{13}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$= \frac{1}{13} + \frac{1}{13} + \frac{1}{13}$

$= \frac{3}{13}$

$4 = 16$



$\frac{2}{6}$ $P(\triangle)$

$1 - \frac{2}{6} = \frac{4}{6}$ $1 - P(\triangle)$

$$P(\overline{A}) = 1 - P(A)$$

Pb (14) THERE ARE 52 CARDS IN A STANDARD PACK IN WHICH 39 CARDS CONTAIN HEART.

CALCULATE THE PROBABILITY THAT THE CARD CONTAINING HEART IS DRAWN AND PROBABILITY THAT THE CARD NOT CONTAINING HEART IS DRAWN.

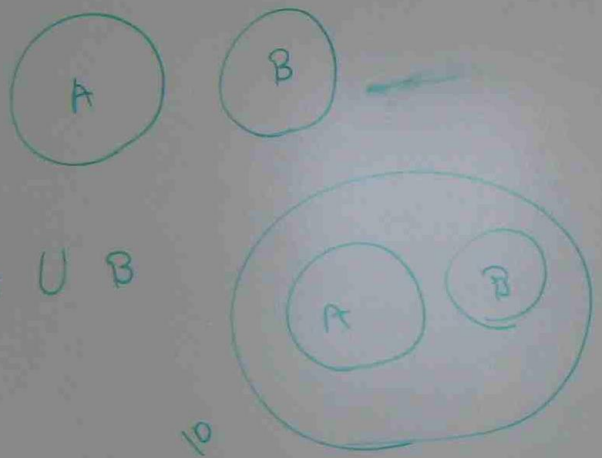
$$P(A) = P(\text{HEART}) = \frac{39}{52} = \frac{3}{4}$$

$$P(\overline{A}) = 1 - P(A)$$

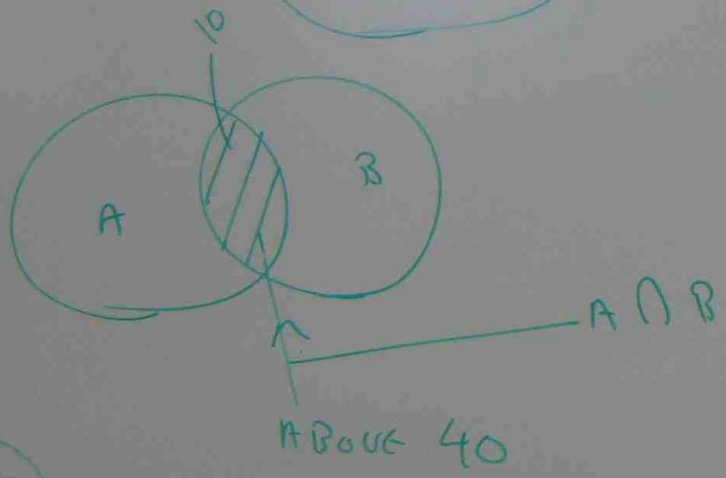
$$= 1 - \frac{3}{4}$$

$$= \frac{1}{4}$$

UNION, INTERSECT, SUBSET



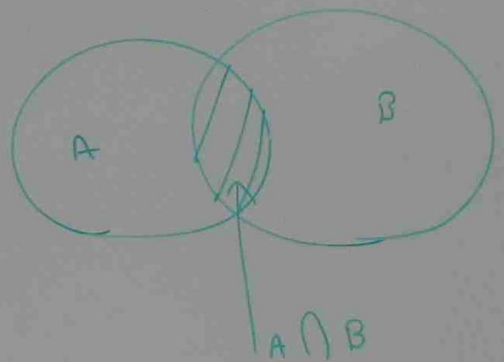
A ∪ B



(A) 60 STUDENTS, 10 ABOVE AGE 40

(B) 50 STUDENTS, 10 ABOVE AGE 40

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



$$P(A) = \frac{60}{110}$$

$$P(B) = \frac{50}{110}$$

$$P(A \cap B) = \frac{10}{110}$$

$$P(A \cup B) = \frac{60}{110} + \frac{50}{110} - \frac{10}{110}$$

$$= \frac{100}{110}$$

Pb (15) $P(S) = 0.4$, $P(T) = 0.5$, $P(S \cap T) = 0.15$

FIND $P(S \cup T)$

$$P(S \cup T) = P(S) + P(T) - P(S \cap T)$$

$$= 0.4 + 0.5 - 0.15$$

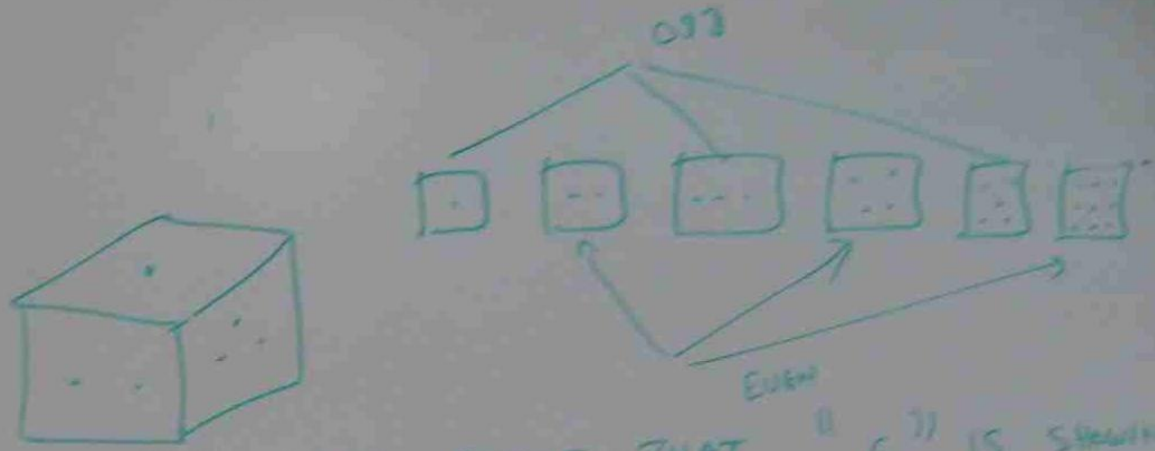
$$= 0.75$$

PROBABILITY OF SUBSET

$A \subset B$, A IS SUBSET OF B

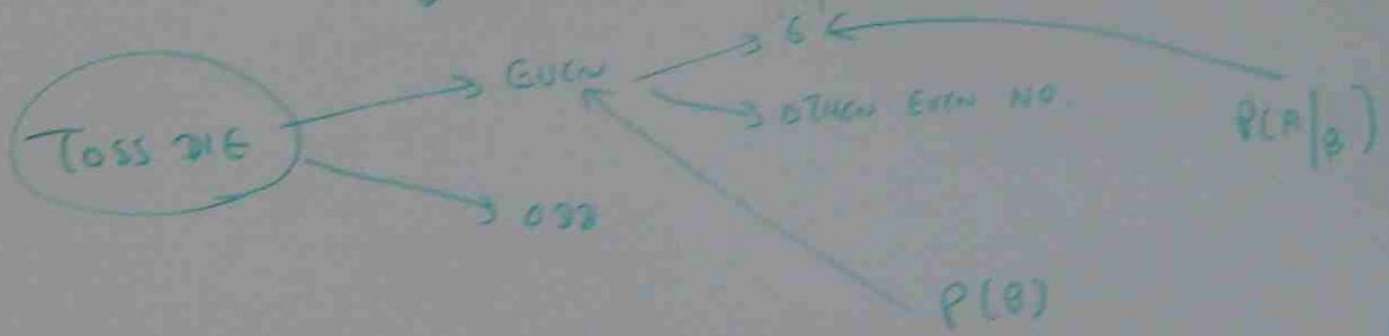
$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Pb (16) A SINGLE DIE IS TOSSED. WHAT IS THE PROBABILITY OF A 6 APPEARING ON THE UPPER MOST FACE GIVEN THAT NUMBER SHOWING IS EVEN.



A = EVENT THAT "6" IS SHOWING

B = EVENT THAT EVEN NUMBER IS SHOWING



$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(\bar{A}) = 1 - P(A)$$

pb (17) SUPPOSE WE HAVE 3 EVENTS X, Y, Z

$$P(X) = \frac{1}{2}, \quad P(Y) = \frac{1}{4}, \quad P(Z) = \frac{1}{8}$$

FIND (a) $P(Z \cap X)$, (b) $P(Z \cup X)$

(c) $P(X \cup Y)$, (d) $P(X \cup \bar{Y})$

(a) $P(Z \cap X) = P(Z) \times P(X)$

$= \frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$

(b) $P(Z \cup X) = P(Z) + P(X) - P(Z \cap X)$

$= \frac{1}{8} + \frac{1}{2} - \frac{1}{16}$

$= \frac{2+8}{16} - \frac{1}{16}$

$= \frac{10}{16} - \frac{1}{16}$

$= \frac{10-1}{16} = \frac{9}{16}$

(c) $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$= P(X) + P(Y) - P(X) \times P(Y)$

$= \frac{1}{2} + \frac{1}{4} - \frac{1}{2} \times \frac{1}{4}$

$= \frac{4+2}{8} - \frac{1}{8} = \frac{6}{8} - \frac{1}{8} = \frac{5}{8}$

(d) $P(X \cup \bar{Y}) = P(X) + P(\bar{Y}) - P(X \cap \bar{Y})$

$= P(X) + [1 - P(Y)] - P(X) \times P(\bar{Y})$

$= P(X) + [1 - P(Y)] - P(X) \times [1 - P(Y)]$

$= \frac{1}{2} + \left[1 - \frac{1}{4}\right] - \frac{1}{2} \times \left[1 - \frac{1}{4}\right]$

$= \frac{1}{2} + \frac{3}{4} - \frac{1}{2} \times \frac{3}{4}$

$= \frac{4+6}{8} - \frac{3}{8}$

$= \frac{10}{8} - \frac{3}{8} = \frac{7}{8}$ //

Pb(18) SUPPOSE THAT WE KNOW FROM EXPERIMENT THAT 28% OF PEOPLE READ THE WOMAN'S DAY ARTICLE AND 45% OF PEOPLE READ THE NEW IDEA.

READING WOMEN'S DAY AND NEW IDEA ARE INDEPENDENT OF EACH OTHER.

TWO PEOPLE ARE SELECTED AT RANDOM.

FIND THE PROBABILITY THAT

(a) NEITHER READ NEW IDEA

(b) BOTH READ WOMEN'S DAY

(c) BOTH DO NOT READ WOMEN'S DAY

$P(W) = P(\text{WOMEN'S DAY}) = 0.28$

$P(\overline{W}) = P(\text{DO NOT READ WOMAN DAY}) = 1 - P(W)$

$= 1 - 0.28$

$= 0.72$

$P(N) = 0.45$

$P(\overline{N}) = 1 - P(N) = 1 - 0.45 = 0.55$

(a) NEITHER READS NEW IDEA

(FROM TWO PEOPLE)

$P(\overline{N} \cap \overline{N}) = P(\overline{N}) \times P(\overline{N})$

$= 0.55 \times 0.55 = 0.3025$

(b) $P(W \cap W) = P(W) \times P(W)$

$= 0.28 \times 0.28 = 0.0784$

(c) $P(\bar{W} \cap \bar{W}) = P(\bar{W}) \times P(\bar{W})$

$= 0.72 \times 0.72 = 0.5184$

Pb (19) THERE ARE 10 PEOPLE TO BE CLASSIFIED ACCORDING TO SEX & AGE. 6 ARE FEMALES AND 4 ARE MALES. OF 6 FEMALES, 4 ARE UNDER 40, 2 ARE 40 (OR) MORE OF 4 MALES. 3 ARE UNDER 40, 1 IS 40 (OR) MORE FIND THE PROBABILITY THAT PERSON IS (i) FEMALE (ii) UNDER 40, (iii) FEMALE UNDER 40 (iv) UNDER 40 GIVEN THAT SHE IS FEMALE (v) MALE AND 40 (OR) MORE, (vi) MALE

GIVEN THAT HE IS UNDER 40.

(i) $P(F) = \frac{6}{10}$.

$P(A)$ UNDER 40

(ii) $P(A) = \frac{4+3}{10} = \frac{7}{10}$

$P(F)$, $P(A)$
FEMALE , UNDER 40

$P(A \cap F) = P(F \cap A)$

$P(F \cap A) =$ FEMALE UNDER 40 $= \frac{4}{10}$

(iv) $P(A/F) = \frac{P(A \cap F)}{P(F)} = \frac{P(F \cap A)}{P(F)} = \frac{\frac{4}{10}}{\frac{6}{10}}$

$= \frac{4}{10} \times \frac{10}{6}$

$= \frac{4}{6}$

⟷ AMONG THE FEMALES, FIND UNDER 40

(v) $P(\bar{A})$ = UNDER 40

40 OR MORE = $P(\bar{A})$

$P(m)$ = MALE

MALE AND 40 (OR) MORE = $P(m \cap \bar{A}) = P(m) \times P(\bar{A})$

$= P(m) \times (1 - P(A))$

$= \frac{4}{10} \times \left(1 - \frac{7}{10}\right)$

$= \frac{4}{10} \times \frac{3}{10} = \frac{12}{100} \simeq \frac{1}{10}$

(vi) $P(\bar{A}/m) = \frac{P(m \cap A)}{P(m)}$

$= \frac{3/10}{4/10} = \frac{3}{4}$