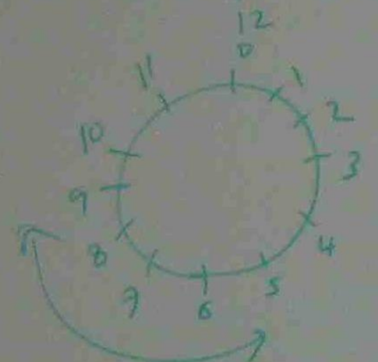
NORMAL PROBABILITY DISTRIBUTION

PROBABILITY – CHANCE (OR) LIKELIHOOD

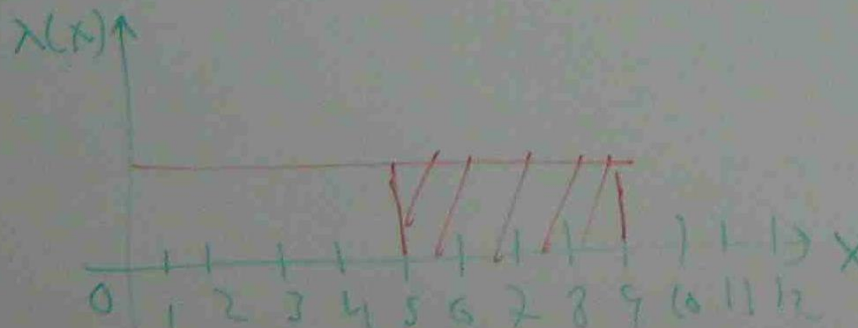


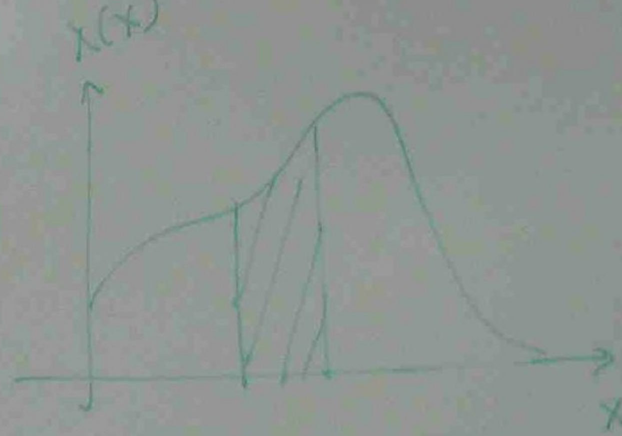
CHANCE TO STOP THE WATCH BETWEEN $5 \rightarrow 9$

TOTAL = 12 DIVISIONS

$5 \rightarrow 9$ = 4 DIVISIONS.

$$P(5 < x < 9) = \frac{4}{12} = \frac{1}{3}$$





PRACTICAL PROBABILITY GRAPH

NORMAL DISTRIBUTION

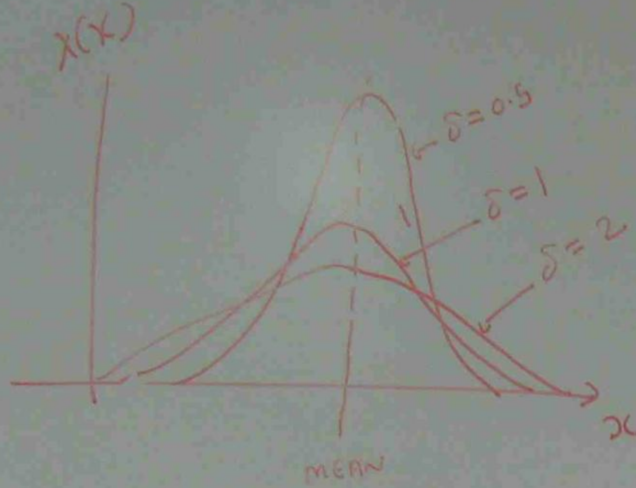
STANDARD DEVIATION $= \delta = \sqrt{\dfrac{\Sigma f x^2 - n(\bar{x})^2}{n-1}}$

HOW MUCH THE NUMBER DIFFERS FROM AVERAGE VALUE

	X		f
EVENT	1	→	5 TIMES
——	2	——	3 TIMES
——	3	——	4 TIMES.

$\bar{X}$ = MEAN = AVERAGE

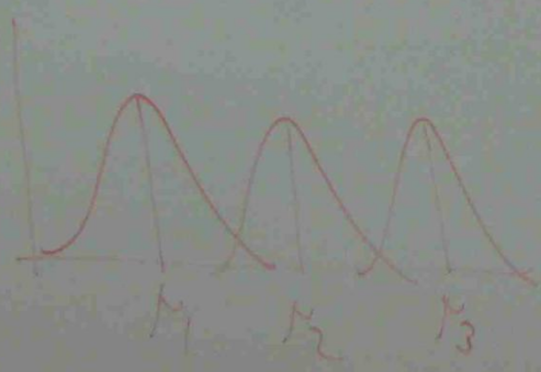
n = TOTAL NUMBER OF TIMES.



SAME MEN $= \mu = \frac{\sum x_1}{n_1} = \frac{\sum x_2}{n_2} = \frac{\sum x_3}{n_3}$

NORMAL DISTRIBUTION { SAME MEAN, DIFFERENT STANDARD DEVIATION

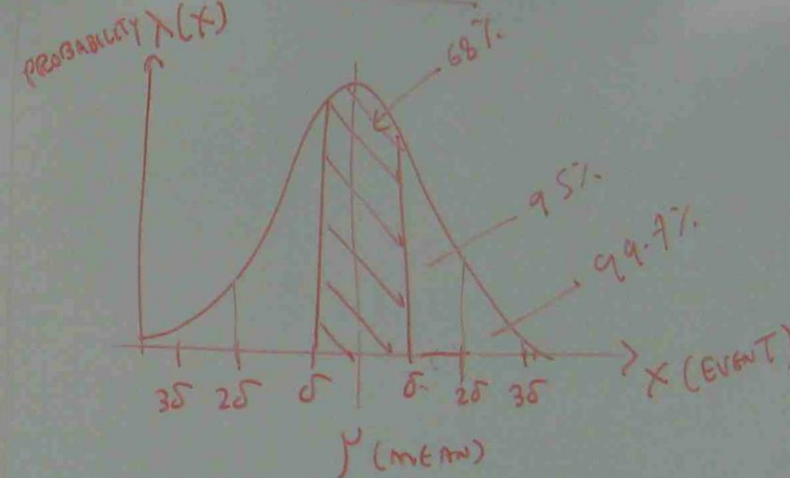
DIFFERENT MEAN, SAME STANDARD DEVIATION



s, δ

μ

NORMAL DISTRIBUTION



$P(\mu - \delta < x < \mu + \delta) = 68\% = 0.68$

$P(\mu - 2\delta < x < \mu + 2\delta) \rightarrow 0.95$

$P(\mu - 3\delta < x < \mu + 3\delta) \longrightarrow 0.997$

ERS

GE

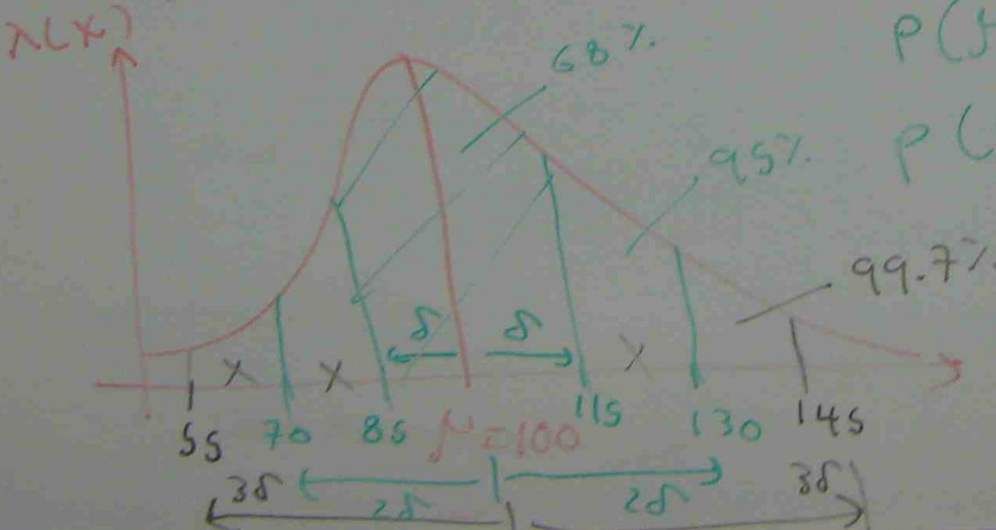
Ex (20) Suppose that IQ scores are normally distributed with a mean $\mu = 100$ and a standard deviation $\sigma = 15$.

Indicate the followings on the graph.

(a) Approximately 68% of people have IQ between 85 and 115

(b) Approximately 95% of people have IQ between 70 and 130

(c) Almost every one (approximately 99.7%) have IQ between 55 and 145.



$$P(\mu - \sigma < x < \mu + \sigma) = 0.68$$

$$P(\mu - 2\sigma < x < \mu + 2\sigma) = 0.95$$

$$P(\mu - 3\sigma < x < \mu + 3\sigma) = 0.997$$

STANDARD SCORE (Z SCORE)

STANDARD SCORE (Z-SCORE) $= \dfrac{\text{RAW DATA} - \text{MEAN}}{\text{STANDARD DEVIATION}}$

$$z = \frac{x - \mu}{\delta}$$

IF $x > \mu$ $z = +$

IF $x < \mu$ $z = -$

IF $x = \mu$ $z = 0$ NO ERROR

EX 21 THE DISTRIBUTION OF ADULT FEMALE HEIGHT HAS A MEAN $\mu = 165.5$ cm, STANDARD DEVIATION $\delta = 8$ cm.

FIND THE STANDARD SCORE (RELATIVE HEIGHT TO MEAN) CORRESPONDING TO A FEMALE HEIGHT OF (a) 164 cm (b) 178 cm

(a) $z = \dfrac{x - \mu}{\delta} = \dfrac{164 - 165.5}{8} = -0.3$

(b) $z = \dfrac{x - \mu}{\delta} = \dfrac{178 - 165.5}{8} = 1.44$

EX 22 THE WEIGHT OF A CERTAIN SPECIES OF ELEPHANT ARE DISTRIBUTED WITH A MEAN OF 20 TONNES AND A STANDARD DEVIATION OF 2 TONNES. SUPPOSE ALSO THAT WEIGHTS OF GARDEN SNAILS ARE DISTRIBUTED WITH A MEAN OF 30 GRAM AND STANDARD DEVIATION 3 GRAM.

WHICH IS HEAVIER, AN ELEPHANT WEIGHING 23 TON (OR) A SNAIL WEIGHING 34.5 GRAMS.

S

0.9997

(a) $z = \dfrac{x - \mu}{\delta} = \dfrac{164 - 165.5}{8} = -0.3$

(b) $z = \dfrac{x - \mu}{\delta} = \dfrac{178 - 165.5}{8} = 1.44$

Ex 22 THE WEIGHT OF A CERTAIN SPECIES OF ELEPHANT ARE DISTRIBUTED WITH A MEAN OF 20 TONNES AND A STANDARD DEVIATION OF 2 TONNES. SUPPOSE ALSO THAT WEIGHTS OF GARDEN SNAILS ARE DISTRIBUTED WITH A MEAN OF 30 GRAM AND STANDARD DEVIATION 3 GRAM.

WHICH IS HEAVIER, AN ELEPHANT WEIGHING 23 TON (OR) A SNAIL WEIGHING 34.5 GRAMS.

ELEPHANT

$\mu_1 = 20 \qquad \delta_1 = 2$

$x_1 = 23$

$z = \dfrac{x_1 - \mu_1}{\delta_1} = \dfrac{23 - 20}{2} = 1.5$

SNAIL

$\mu_2 = 30, \quad \delta_2 = 3$

$x_2 = 34.5$

$z = \dfrac{x_2 - \mu_2}{\delta_2} = \dfrac{34.5 - 30}{3}$

$z = 1.5$

TWO ANIMALS HAVE SAME RELATIVE WEIGHT.

Ex (23) IF A STUDENT SHOWS THE FOLLOWING RESULTS IN A SERIES OF TESTS. COMPARE THE OVERALL PERFORMANCE.

SUBJECT	STUDENT SCORE x	MEAN μ	STANDARD DEVIATION δ
MATHS	170	150	15
ENGLISH	120	120	10
HISTORY	188	200	12
SCIENCE	120	100	8

MATHS $z = \frac{x-\mu}{\delta} = \frac{170-150}{15} = 1.33$

ENGLISH $z = \frac{x-\mu}{\delta} = \frac{120-120}{10} = 0$

HISTORY $z = \frac{x-\mu}{\delta} = \frac{188-200}{12} = -1.0$

SCIENCE $z = \frac{x-\mu}{\delta} = \frac{120-100}{8} = 2.5$

PERFORMANCE

SCIENCE → BEST (2.5)

MATHS ⇒ 2nd BEST (1.33)

ENGLISH - NORMAL (0)

HISTORY = MOST POOR (−1.0)

Re

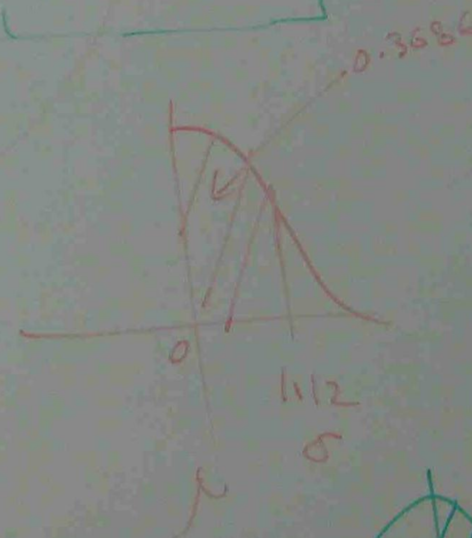
z

READING Z TABLE

Z TABLE

z_0	0.00	0.01	0.02	0.03 - - - 0.04
0.0				
0.1				
0.2				
⋮				
1.1			0.3686	
⋮				
3.0				

$1.12 = 1.1 + .02$



$0.3686 \longrightarrow 1.12$

$0.3690 \longrightarrow 1.12$

Ex (24) USE Z TABLE, CALCULATE THE AREA UNDER THE STANDARD NORMAL CURVE FOR THE FOLLOWING INTERVALS.

(a) BETWEEN $z=0$ AND $z=1.50$
(b) BETWEEN $z=0$ AND $z=-2.10$
(c) BETWEEN $z=-0.30$ AND $z=+2.25$
(d) TO THE RIGHT OF $z=1.95$
(e) TO THE LEFT OF $z=1.64$
(f) BETWEEN 0.60 AND 1.80

(a) $P(0<z<1.5) = 0.4332 - 0.0000$
$= 0.4332$

(b) $P(-2.1<z<0) = 0 - (-0.4821)$
$= 0.4821$

(c) $P(-0.30<z<2.25) = 0.4878 - (-0.1179)$
$= 0.4878 + 0.1179$
$= 0.6057$

(e) $P(z>1.95) =$ THE WHOLE AREA $- P(0<z<1.95)$
$= 0.5 - [0.4744 - 0]$
$= 0.5 - 0.4744 = 0.0256$

(e) $P(z<1.64)$
$=$ THE WHOLE AREA $+ (P(0<z<1.64))$
$= 0.5 + (0.4495 - 0)$
$= 0.5 + 0.4495$
$= 0.9495$

(f) $P(0.6<z<1.8)$
$= P(1.8) - P(0.6)$
$= 0.4641 - 0.2257$
$= 0.2384$

EX 25

(a) IF 37.7% OF POPULATION HAS A STANDARD SCORE BETWEEN THE MEAN (Z=1) AND SOME POSITIVE Z VALUE FIND THAT Z VALUE

(b) FIND THE Z VALUE CUTTING OFF TOP 5% OF POPULATION

(c) FIND THE Z SCORE CUTTING OFF BOTTOM 10% OF POPULATION

(d) FIND THE VALUE OF Z SO THAT 95% OF POPULATION HAS A SCORE BETWEEN $-Z$ AND $+Z$

(a) AREA = 37.7% $\Rightarrow$ 0.377 $\longrightarrow$ $Z = ?$

	0.06
1.1	0.3770

$Z = 1.16$

(b) CUTTING OFF TOP 5% = THE WHOLE ARE − 5%

$= 0.5 - 0.05 = 0.45$

	0.04	0.05
1.6	0.4495	0.4505

$0.4495 = 1.64$

$0.4505 = 1.65$

$0.4500 = \frac{1.64 + 1.65}{2}$

$= 1.645$

(c) CUTTING OFF BOTTOM 10% = THE WHOLE AREA − 10%

$= 0.5 - 0.1 = 0.4$

	0.08
1.2	0.3997

$0.3997 \simeq 0.4 \Rightarrow 1.28$

(d)

2 AREA = 0.95

1 AREA = $\frac{0.95}{2}$ = 0.475

0.475 ⟶ z = ?

0.06

1.9 ——— 0.475 ⟹ 1.96

EX (26) THE HEIGHT OF ADULT MALES IN AUSTRALIA IS APPROXIMATELY NORMALLY DISTRIBUTED WITH MEAN μ = 170 cm AND STANDARD DEVIATION σ = 7 cm.

(a) WHAT IS THE PROBABILITY THAT A MALE RANDOMLY SELECTED HAS HEIGHT (i) BETWEEN 160 cm & 175 cm

(ii) GREATER

(b) WHAT IS

OF HEA

(a) P (16

x

z_1 =

z

0

1.4

(ii) GREATER THAN 188CM

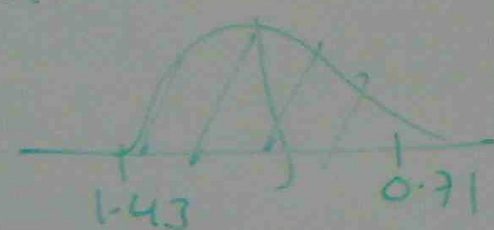
(b) WHAT IS THE GREATEST HEIGHT EXCEEDED BY 22% OF POPULATION.

(a) $P(\underset{x_1}{160} < z < \underset{x_2}{175})$ = THE AREA BETWEEN

$$z_1 = \frac{x_1 - \mu}{\sigma} \quad \text{AND} \quad z_2 = \frac{x_2 - \mu}{\sigma}$$

$$z_1 = \frac{160 - 170}{7} \quad \text{AND} \quad z_2 = \frac{175 - 170}{7}$$

$$z_1 = -1.43 \quad \text{AND} \quad z_2 = 0.71$$



TOTAL = AREA (1.43) + AREA (0.71)

0.03
|
1.4 — 0.4236

0.01
|
0.7 — 0.2611

= 0.4236 + 0.2611

= 0.6847

(ii)

$P(X > 188)$

$= \text{TOTAL AREA} - \text{THE AREA} \qquad z = \frac{x-\mu}{\sigma}$

$= 0.5 - \left(z = \frac{188-170}{7} = 2.57\right)$

0.07

2.5 — 0.4949

$= 0.5 - 0.4949 = 0.0051$

(b) GREATEST HEIGHT EXCEED BY 22% OF POPULATION

$= \text{THE WHOLE AREA} - 0.22$

$= 0.5 - 0.22 = 0.28$

FIND (Z)

0.07

0.7 — 0.2794 $\simeq$ 0.28

$\therefore z = 0.77$