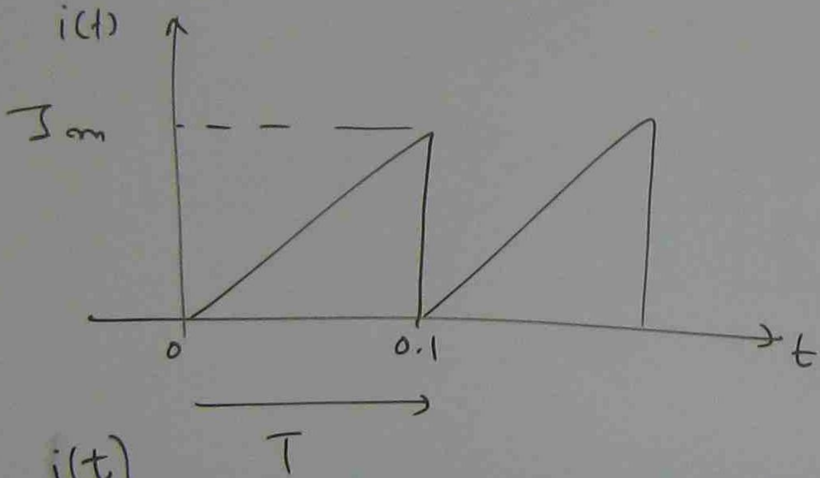
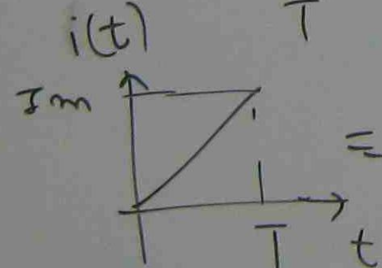
INTEGRATION OF ELECTRICAL WAVE FORMS

AVERAGE VALUE





$$= i(t) = \frac{I_m}{T} t$$

$$I_{AVE} = \frac{1}{T} \int_0^T i(t)\,dt$$

$$I_{AVE} = \frac{1}{T} \int_0^T \frac{I_m t}{T}\,dt$$

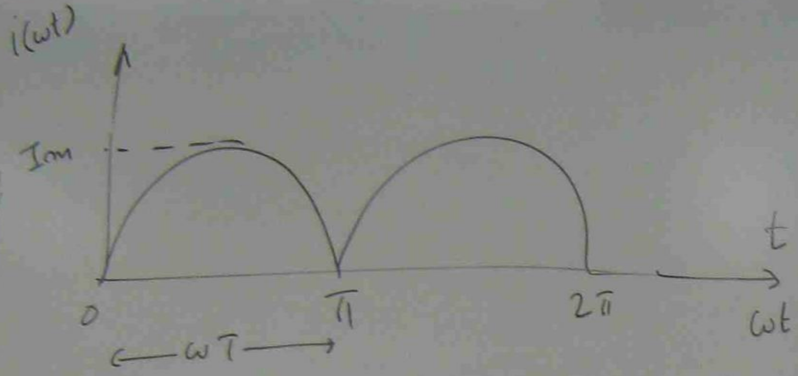
$$= \frac{I_m}{T^2} \int_0^T t\,dt$$

$$= \frac{I_m}{T^2} \left[\frac{t^2}{2}\right]_0^T$$

$$= \frac{I_m}{2T^2} \left[T^2 - 0\right]$$

$$= \frac{I_m}{2T^2} \times T^2$$

$$= \frac{I_m}{2}$$



$$i(\omega t) = I_m \sin \omega t \qquad 0 \leq \omega t \leq \pi$$

$$I_{AVE} = ?$$

$$I_{AVE} = \frac{1}{T}\int_0^T i(\omega t)\,dt \qquad \boxed{T = \frac{\pi}{\omega}}$$

$$T = \pi$$

$$i(\omega t) = I_m \sin \omega t$$

$$I_{AVE} = \frac{1}{\pi/\omega}\int_0^{\pi} I_m \sin \omega t \, dt$$

$$I_{AVE} = \frac{\omega}{\pi}\cdot\int_0^{\pi} I_m \sin \omega t \, dt$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin \omega t \, d\omega t$$

$$= \frac{I_m}{\pi}\int_0^{\pi} \sin \omega t \, d\omega t$$

$$= \frac{I_m}{\pi}\Big[-\cos \omega t\Big]_0^{\pi}$$

$$= \frac{I_m}{\pi}(-)\Big[\cos \pi - \cos 0\Big]$$

$$= \frac{I_m}{\pi}\times(-)\Big[-1-1\Big]$$

$$= \frac{2 I_m}{\pi}$$

$$= 0.636\, I_m$$

RMS VALUE

$$I_{Rms} = \sqrt{\frac{1}{T}\int_0^T [i(t)]^2\,dt}$$

TRIANGULAR WAVE

$$i(t) = \frac{I_m}{T}\,t$$

$$I_{Rms} = \sqrt{\frac{1}{T}\int_0^T \left(\frac{I_m t}{T}\right)^2 dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T \frac{I_m^2\, t^2}{T^2}\,dt}$$

$$= \sqrt{\frac{1}{T} \times \frac{I_m^2}{T^2}\int_0^T t^2\,dt}$$

$$= \sqrt{\frac{I_m^2}{T^3}\left[\frac{t^3}{3}\right]_0^T}$$

$$= \sqrt{\frac{I_m^2}{T^3} \times \frac{T^3 - 0}{3}}$$

$$= \sqrt{\frac{I_m^2}{T^3} \times \frac{T^3}{3}}$$

$$= \sqrt{\frac{I_m^2}{3}}$$

$$I_{rms} = \frac{I_m}{\sqrt{3}}$$

SINUSOIDAL WAVE

$$i(t) = I_m \sin \omega t$$

$$T = \frac{\pi}{\omega}$$

$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T (I_m \sin \omega t)^2 \, dt}$$

$$= \sqrt{\frac{1}{\frac{\pi}{\omega}} \int_0^{\pi} I_m^2 \sin^2 \omega t \, dt}$$

$$= \sqrt{\frac{\omega}{\pi} \int_0^{\pi} I_m^2 \sin^2 \omega t \, dt}$$

$$= \sqrt{\frac{1}{\pi} \int_0^{\pi} I_m^2 \sin^2 \omega t \, d\omega t}$$

$$= \sqrt{\frac{I_m^2}{\pi} \int_0^{\pi} \sin^2 \omega t \, d\omega t}$$

$$\cos 2\omega t = 1 - 2 \sin^2 \omega t$$

$$\sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}$$

$$= \sqrt{\frac{I_m^2}{\pi} \int_0^{\pi} \frac{1 - \cos 2\omega t}{2} \, d\omega t}$$

$$= \sqrt{\frac{I_m^2}{\pi \times 2} \int_0^{\pi} (1 - \cos 2\omega t)\, d\omega t}$$

$$= \sqrt{\frac{I_m^2}{2\pi} \left[\int_0^{\pi} d\omega t - \int_0^{\pi} \cos 2\omega t \, d\omega t \right.}$$

$\pi = 180^\circ$

$\sin 180 = 0$

$$= \sqrt{\frac{I_m^2}{2\pi} \left[\omega t - \frac{1}{2} \sin \omega t \right]_0^{\pi}}$$

$$= \sqrt{\frac{I_m^2}{2\pi} \left[\left(\pi - \underbrace{\frac{1}{2} \sin \pi}_{0}\right) - \underbrace{\left(0 - \frac{1}{2} \sin 0\right)}_{0} \right.}$$

$$= \sqrt{\frac{I_m^2}{2\cancel{\pi}} \times \cancel{\pi}} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

Rms VALUE of complex WAVE forms

PERIOD $T = \frac{2\pi}{\omega}$

$$i(t) = I_0 + A_1 \cos \omega t + A_2 \cos 2\omega t + A_3 \cos 3\omega t + \cdots + B_1 \sin \omega t + B_2 \sin 2\omega t + B_3 \sin 3\omega t + \cdots$$

$$I_{Rms} = \sqrt{I_0^2 + \frac{1}{2}\left(A_1^2 + A_2^2 + A_3^2 + \cdots\right) + \frac{1}{2}\left(B_1^2 + B_2^2 + B_3^2 + \cdots\right)}$$

$$= \sqrt{I_{DC}^2 + A_{1\,Rms}^2 + A_{2\,Rms}^2 + \cdots + B_{1\,Rms}^2 + B_{2\,Rms}^2 + \cdots}$$

$$= \sqrt{DC^2 + (\text{sum of Rms})^2}$$

$\ldots n\, 3\omega t + - - -$

ph

$i(t) = 10 + 15 \sin 100t + 5 \sin 200t$ Amp, FIND I_{Rms} =?!

$$I_{Rms} = \sqrt{10^2 + \frac{1}{2}(15^2 + 5^2)} = 15 \text{ Amp}$$

FORM FACTOR

$$\text{FORM FACTOR} = \frac{\text{Rms VALUE}}{\text{AVERAGE VALUE}} = \frac{I_{Rms}}{I_{AVE}}$$

WAVE SHAPE	FORM FACTOR
RECTIFIED SQUARE WAVE	1
RECTIFIED SINE WAVE	1.11
TRIANGULAR WAVE	1.15

ph

A VOLTAGE WAVE FORM IS REPRESENTED BY THE EQUATION

$v(t) = 10 - 10 \sin \omega t$ VOLT

DC COMPONENT + AC COMPONENT

DETERMINE THE FOLLOWINGS

(a) AVERAGE VALUE

(b) Rms VALUE

(c) FORM FACTOR

$$V_{AVE} = \frac{1}{T}\int_0^T v(t)\,dt$$

(OR)

$$V_{AVE} = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)\,d\theta$$

$\boxed{\omega = 2\pi f}$ $\boxed{\theta = \omega t}$

$$V_{AVE} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$\boxed{\omega T = 2\pi}$$

$$= \frac{1}{T}\int_0^T (10 - 10\sin\omega t)\,dt$$

$$= \frac{1}{T}\left[\int_0^T 10\,dt\right] - \int_0^T 10\sin\omega t\,dt\Big]$$

$$= \frac{1}{T}\left[10(t)_0^T - \int_0^T \frac{10\sin\omega t\,d\omega t}{\omega}\right]$$

$$= \frac{1}{T}\left[10T - \frac{10}{\omega}\int_0^T \sin\omega t\,d\omega t\right]$$

$$= \frac{1}{T}\left[10T + \frac{10}{\omega}\left[\cos\omega t\right]_0^T\right.$$

$$= \frac{1}{T}\left[10T + \frac{10}{\omega}(\cos\omega T - \cos 0)\right)$$

$$= \frac{1}{T}\left[10T + \frac{10}{\omega}(\cos 2\pi - 1)\right)$$

$$\frac{1}{T}\left(10T + \cancel{\frac{10}{\omega}(1-1)}\right)$$

$\searrow 0$

$$= 10\ V$$

$$V_{AVE} = \frac{1}{2\pi}\int_0^{2\pi} V(\theta)\,d\theta$$

$$\frac{1}{2\pi}\int_0^{2\pi}(10 - 10\sin\theta)\,d\theta$$

$$\frac{1}{2\pi}\left[\int_0^{2\pi} 10\,d\theta - \int_0^{2\pi} 10\sin\theta\,d\theta\right]$$

$$\frac{1}{2\pi}\left[10(\theta)_0^{2\pi} + 10(\cos\theta)_0^{2\pi}\right]$$

$$\frac{1}{2\pi}\left[20\pi + 10\cancel{(\cos 2\pi - \cos 0)}\right]$$

$$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T (v(t))^2\,dt}$$

(OR)

$$V_{Rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} [v(\theta)]^2\,d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} [10 - 10\sin\theta]^2\,d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} 10^2(1-\sin\theta)^2\,d\theta}$$

$$= \sqrt{\frac{50}{\pi}\int_0^{2\pi} (1-\sin\theta)^2\,d\theta}$$

$$(1-x)^2 = 1 - 2x + x^2$$

$$= \sqrt{\frac{50}{\pi}\int_0^{2\pi} (1 - 2\sin\theta + \sin^2\theta)\,d\theta}$$

$$= \sqrt{\frac{50}{\pi}\left[\int_0^{2\pi} d\theta - \int_0^{2\pi} 2\sin\theta\,d\theta + \int_0^{2\pi}\sin^2\theta\,d\theta\right.}$$

$$= \sqrt{\frac{50}{\pi}\left[2\pi + 2(\cos\theta)_0^{2\pi} + \int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta\right.}$$

$$= \sqrt{\frac{50}{\pi}\left[2\pi + 2\,\underbrace{\cancel{[\cos 2\pi - \cos 0]}}_{0} + \int_0^{2\pi}\frac{1}{2}\,d\theta - \int_0^{2\pi}\frac{\cos 2\theta}{2}\,d\theta\right.}$$

$$= \sqrt{\frac{50}{\pi}\left[2\pi + \frac{2\pi}{2} - \left[\frac{\sin 2\theta}{4}\right]_0^{2\pi}\right.}$$

$$= \sqrt{\frac{50}{\pi}\left[3\pi - \underbrace{\cancel{\frac{\sin 4\pi}{4}}}_{0}\right.} = \sqrt{150} = 12.25\ V$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \sqrt{\frac{50}{\pi} \int_0^{2\pi} (1 - 2\sin\theta + \sin^2\theta)\, d\theta}$$

$$= \sqrt{\frac{50}{\pi} \left[\int_0^{2\pi} d\theta - \int_0^{2\pi} 2\sin\theta\, d\theta + \int_0^{2\pi} \sin^2\theta\, d\theta \right.}$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$= \sqrt{\frac{50}{\pi} \left[2\pi + 2 (\cos\theta)_0^{2\pi} + \int_0^{2\pi} \left(\frac{1 - \cos 2\theta}{2}\right) d\theta \right.}$$

$$\frac{\cos 2\theta\, d\, 2\theta}{4}$$

$$= \sqrt{\frac{50}{\pi} \left[2\pi + 2 \cancel{[\cos 2\pi - \cos 0]} + \int_0^{2\pi} \frac{1}{2} d\theta - \int_0^{2\pi} \frac{\cos 2\theta}{2} d\theta \right.}$$

$$0$$

$$= \sqrt{\frac{50}{\pi} \left[2\pi + \frac{2\pi}{2} - \left[\frac{\sin 2\theta}{4}\right]_0^{2\pi} \right.}$$

$$= \sqrt{\frac{50}{\pi} \left[3\pi - \cancel{\frac{\sin 4\pi}{4}} \right.} \to 0 \quad = \sqrt{150} = 12.25\ V$$

pb

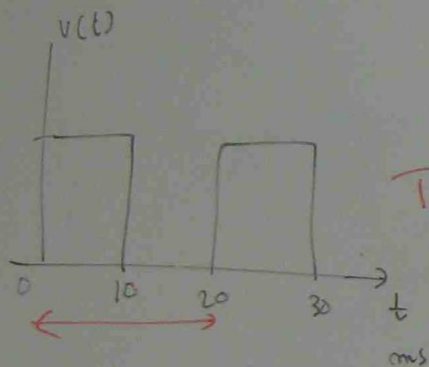
DETERMINE THE AVERAGE VALUE, RMS VALUE AND FORM FACTOR FOR EACH WAVE FORM SHOWN IN FIGURE

USE INTEGRAL CALCULUS

$$V_{AVE} = \frac{1}{T}\int_0^T v(t)\,dt$$

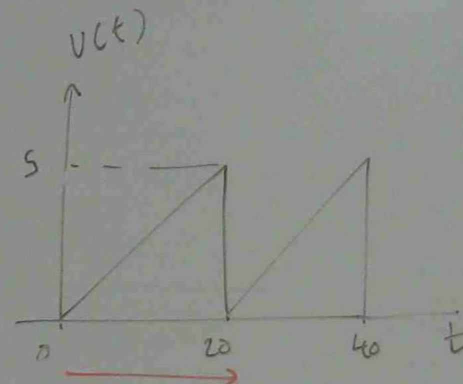
$$V_{rm} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$$

(a)



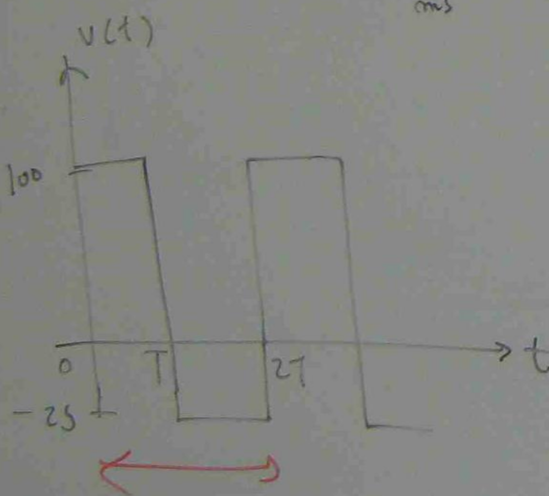
$T = 20$

(b)



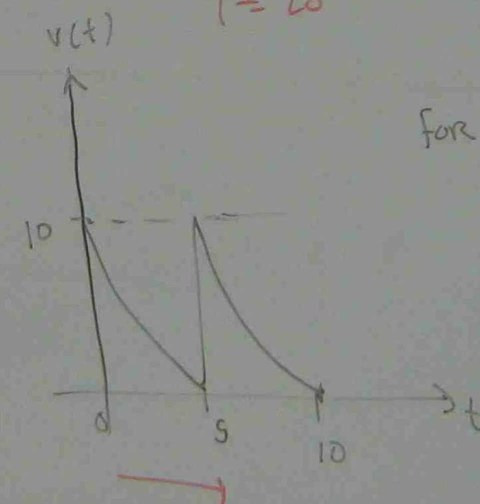
$T = 20$

(c)



$T = 2T$

(d)



$T = 5$

for $0 \le t \le 5$ ms

$v(t) = 10e^{-1000t}$

(a) $V_{AVE} = \frac{1}{T}\int_0^T v(t)\,dt$

$= \frac{1}{20}\left[\int_0^{10} 100\,dt + \int_{10}^{20} 0\,dt \to 0\right]$

$= \frac{1}{20}\left[100(t)_0^{10}\right]$

$= \frac{1}{20} \times 100 \times 10$

$= \frac{1000}{20} = 50\ V$

$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$

$= \sqrt{\frac{1}{20}\left[\int_0^{10} 100^2\,dt + \cancel{\int_{10}^{20} 0^2\,dt} \to 0\right]}$

$= \sqrt{\frac{1}{20} \times 10000(t)_0^{10}}$

$= \sqrt{\frac{1}{20} \times 100{,}00 \times 10}$

$= \sqrt{5000}$

$= 70.7\ V$

Form factor $= \frac{V_{Rms}}{V_{AVG}}$

$= \frac{70.7}{50} = 1.4142$

(b) $v(t) = \frac{V_m}{T} \times t$

$= \frac{5}{20}t = 0.25t$

$V_{AVG} = \frac{1}{T}\int_0^T v(t)\,dt$

$V_{AVE} = \frac{1}{20}\int_0^{20} 0.25t\,dt$

$= \frac{1}{20} \times 0.25\left[\frac{t^2}{2}\right]_0^{20}$

$= \frac{1}{20} \times 0.25 \times \frac{20^2}{2}$

$= \frac{0.25 \times 20}{2} = 2.5V$

$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$

$= \sqrt{\frac{1}{20}\int_0^{20} (0.25t)^2\,dt}$

$= \sqrt{\frac{1}{20} \times 0.0625 \times \left[\frac{t^3}{3}\right]_0^{20}}$

$$= \sqrt{\frac{1}{20} \times 0.0625 \times \frac{20^3}{3}}$$

$$= \sqrt{\frac{20^2 \times 0.0625}{3}}$$

$$= 0.25 \times 20 \times \frac{1}{\sqrt{3}}$$

$$= \frac{5}{1.7321} = 2.887\ V$$

$$\text{Form Factor} = \frac{V_{Rms}}{V_{AVG}} = \frac{2.887}{2.5} = 1.155$$

(c) $$V_{AVE} = \frac{1}{T} \int_0^T v(t)\,dt$$

$$= \frac{1}{2T} \left[\int_0^T 100\,dt + \int_T^{2T} (-25)\,dt \right.$$

$$\frac{1}{2T} \left[\left[100\,(t)\Big|_0^T \right] - 25\,(t)\Big|_T^{2T} \right]$$

$$\frac{1}{2T} \left[100T - 25\,[2T - T] \right]$$

$$\frac{1}{2T} \left[100T - 25T \right]$$

$$\frac{75T}{2T} = 37.5\ V$$

$$V_{Rms} = \sqrt{\frac{1}{T} \int_0^T v(t)^2\,dt}$$

$$= \sqrt{\frac{1}{2T} \left(\int_0^T 100^2\,dt + \int_T^{2T} (-25)^2\,dt \right.}$$

$$= \sqrt{\frac{1}{2T} \left[100{,}00T + 625\,(2T - T) \right]}$$

$$= \sqrt{\frac{1}{2T} \left[10000T + \ldots \right.}$$

$$= \sqrt{5312}$$

$$= 72.89\ V$$

$$\text{Form Factor} = \frac{72.89}{37.5}$$

(d) $$V_{AVE} = \frac{1}{T}$$

$$= \frac{1}{5/1000} \int_0$$

$$= 200 \int_0$$

$[10000T + 625T]$

2

$89V$

$= \frac{72.89}{37.5} = 1.94$

$= \frac{1}{T}\int_0^T v(t)\,dt$

$\frac{1}{5/1000}\int_0^{5\times10^{-3}} 10\times e^{-1000t}\,dt$

$200\int_0^{5\times10^{-3}} \frac{10\times e^{-1000t}\,d{-1000}}{-1000}$

$= \frac{2000}{-1000}\left[e^{-1000t}\right]_0^{5\times10^{-3}}$

$-2\times\left[e^{-1000\times5\times10^{-3}} - e^{0}\right]$

$= -2\times\left[e^{-5}-1\right]$

$= -2\times\left[\frac{1}{148}-1\right]$

$= -2\times -0.993$

$= 1.98V.$

$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$

$V_{Rms} = \sqrt{\frac{1}{\frac{5}{1000}}\int_0^{5\times10^{-3}} (10\times e^{-1000t})^2\,dt}$

$= \sqrt{\frac{1000}{5}\times10\int_0^{5\times10^{-3}} e^{-2000t}\,dt}$

$= \sqrt{2000\times\int_0^{5\times10^{-3}} \frac{e^{-2000t}\,d(-2000t)}{-2000}}$

$= \sqrt{-\int_0^{5\times10^{-3}} e^{-2000t}\,d(-2000t)}$

$= \sqrt{-\left(e^{-2000t}\right)_0^{5\times10^{-3}}}$

$\sqrt{-[e^{-5}-1]}$

$\sqrt{1-e^{-5}}$

$0.996V$

$= \sqrt{\frac{1}{2T}\left[10000T + 625T\right]}$

$= \sqrt{5312}$

$= 72.89V$

$\text{Form Factor} = \frac{72.89}{37.5} = 1.94$

(d) $V_{AVE} = \frac{1}{T}\int_0^T v(t)\,dt$

$= \frac{1}{5/1000}\int_0^{5\times10^{-3}} 10\times e^{-1000t}\,dt$

$= 200\int_0^{5\times10^{-3}} \frac{10\times e^{-1000t}\,d(-1000t)}{-1000}$

$= \frac{2000}{-1000}\left[e^{-1000t}\right]_0^{5\times10^{-3}}$

$= -2\times\left[e^{-1000\times5\times10^{-3}} - e^{0}\right]$

$= -2\times\left[e^{-5} - 1\right]$

$= -2\times\left[\frac{1}{148} - 1\right]$

$= -2\times -0.993$

$= 1.98V.$

$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\,dt}$

$V_{Rms} = \sqrt{\frac{1}{\frac{5}{1000}}\int_0^{5\times10^{-3}}\left(10\times e^{-1000t}\right)^2 dt}$

$= \sqrt{\frac{1000}{5}\times100\int_0^{5\times10^{-3}} e^{-2000t}\,dt}$

$= \sqrt{20000\times\int_0^{5\times10^{-3}} \frac{e^{-2000t}\,d(-2000t)}{-2000}}$

$= \sqrt{-10\int_0^{5\times10^{-3}} e^{-2000t}\,d(-2000t)}$

$= \sqrt{-10\left(e^{-2000t}\right)_0^{5\times10^{-3}}}$

$\sqrt{\left[e^{-5} - 1\right]}\times3.16$

$\sqrt{1 - e^{-5}}\times3.16$

3.1

$FF = \frac{3.1}{1.98} = 1.59$

ph IF A $100\,\Omega$ RESISTOR DISSIPATES AN AVERAGE POWER OF 1000 WATT, DETERMINE

(a) RMS VALUE OF CURRENT

(b) MAXIMUM VALUE OF CURRENT IF THE WAVE FORM IS SINUSOIDAL

(c) MAXIMUM VALUE OF CURRENT IF THE WAVE FORM IS TRIANGULAR.

(a) $POWER = I^2 R$

$$I_{rms}^2 R = POWER$$

$$I_{rms}^2 \times 100 = 1000$$

$$I_{rms}^2 = 10$$

$$I_{rms} = \sqrt{10} = 3.16 \text{ Amp.}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I(t)^2\,dt}$$

SINUSOIDAL => $I_{rms} = \sqrt{\frac{1}{T}\int_0^T I(t)^2\,dt}$

(OR)

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} I(\theta)^2\,d\theta}$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(I_{max}\sin\theta)^2\,d\theta}$$

$$= \sqrt{\frac{1}{2\pi} I_m^2 \int_0^{2\pi}\sin^2\theta\,d\theta}$$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$$\sqrt{\frac{1}{2\pi} I_m^2 \int_0^{2\pi}\left[\frac{1-\cos 2\theta}{2}\right]d\theta}$$

$$I_m\sqrt{\frac{1}{2\pi}\left\{\left[\frac{1}{2}\theta\right]_0^{2\pi} - \left[\int_0^{2\pi}\frac{\cos 2\theta\, d2\theta}{4}\right]\right.}$$

$$\sqrt{\frac{1}{2\pi}\left[\frac{2\pi}{2} - \left[\frac{\sin 2\theta}{4}\right]_0^{2\pi}\right.}$$

$$I_m\sqrt{\frac{1}{2\pi}\left[\pi - \cancel{\left(\frac{\sin 4\pi - \sin 0}{4}\right)}\right]}$$

$$I_m \times \frac{1}{\sqrt{2}} = 3.16$$

$$I_m = \sqrt{2}\times 3.16 = 4.46\text{ Amp.}$$

ZCR

TRIANGULAR

$$i(t) = \frac{I_m t}{T}$$

$$I_{rms} = \sqrt{\frac{1}{T} \int_0^T \left(\frac{I_m t}{T}\right)^2 dt}$$

$$= \sqrt{\frac{1}{T} \times \frac{I_m^2}{T^2} \int_0^T t^2 dt}$$

$$= \sqrt{\frac{I_m^2}{T^3} \left[\frac{t^3}{3}\right]_0^T}$$

$$= \sqrt{\frac{I_m^2}{T^3} \times \frac{T^3}{3}}$$

$$I_{rms} = \frac{I_m}{\sqrt{3}}$$

$$3.16 = \frac{I_m}{1.7321}$$

$$I_m = 3.16 \times 1.7321$$

$$= 5.47 \text{ Amp.}$$