TRIGONOMETRIC IDENTITIES

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sec^2\theta = 1 + \tan^2\theta$$

$$\operatorname{cosec}^2\theta = 1 + \cot^2\theta$$

Ex PROVE THAT $\operatorname{cosec}\theta - \sin\theta = \cos\theta \cot\theta$

$$\operatorname{cosec}\theta - \sin\theta$$

$$\frac{1}{\sin\theta} - \sin\theta$$

$$\frac{1 - \sin^2\theta}{\sin\theta}$$

$$\frac{\cos^2\theta}{\sin\theta}$$

$$\cos\theta \times \frac{\cos\theta}{\sin\theta}$$

$$\cos\theta \times \cot\theta$$

Ex SIMPLIFY $\dfrac{\operatorname{cosec}\theta}{\sec\theta}$

$$= \frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}} = \frac{\cos\theta}{\sin\theta} = \cot\theta$$

$\sin(90-\theta) = \cos\theta$

$\cos(90-\theta) = \sin\theta$

Ex Simplify $\dfrac{\cos(90-\theta)}{\tan\theta}$

$$\frac{\sin\theta}{\frac{\sin\theta}{\cos\theta}} = \sin\theta \times \frac{\cos\theta}{\sin\theta}$$

$$= \cos\theta$$

Ex Simplify

(a) $\tan^2\theta - \sec^2\theta$

(b) $\cot^2 B - \operatorname{cosec}^2 B$

(c) $\sqrt{\operatorname{cosec}^2\theta - \cot^2\theta}$

(a) $\tan^2\theta - \sec^2\theta$

$$\boxed{\begin{aligned} 1+\tan^2\theta &= \sec^2\theta \\ 1+\cot^2\theta &= \operatorname{cosec}^2\theta \end{aligned}}$$

$\tan^2\theta - \sec^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}$$

$$\frac{\sin^2\theta - 1}{\cos^2\theta} = \frac{-\cos^2\theta}{\cos^2\theta} = -1$$

$$\boxed{\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 \\ \sin^2\theta - 1 &= -\cos^2\theta \end{aligned}}$$

ANOTHER METHOD

$\tan^2\theta - [1+\tan^2\theta]$

$\tan^2\theta - 1 - \tan^2\theta$

$= -1$

(b) $\cot^2\beta - \operatorname{cosec}^2\beta$

$\cot^2 B - [1 + \cot^2 B]$

$\cot^2 B - 1 - \cot^2 B$

$= -1$

(c) $\sqrt{\operatorname{cosec}^2\theta - \cot^2\theta}$

$\sqrt{1 + \cot^2\theta - \cot^2\theta}$

$\sqrt{1} = 1$

Ex PROVE THAT

$\sin^2 x + \tan^2 x + \cos^2 x = \sec^2 x$

$\sin^2 x + \cos^2 x + \tan^2 x$

$1 + \tan^2 x = \sec^2 x$

PROVE

Ex $\sqrt{\dfrac{1 - \cos^2 A}{1 - \sin^2 A}} = \tan A$

$\sqrt{\dfrac{\sin^2 A}{\cos^2 A}}$

$\dfrac{\sin A}{\cos A} = \tan A$

EXERCISE

PROVE (a) $\cot^2\theta(1 - \cos^2\theta) = \cos^2\theta$

(b) $\dfrac{\sec^2 A - 1}{1 + \tan^2 A} = \sin^2 A$

(c) $\dfrac{\cos^2\theta - 1}{1 - \sec^2\theta} = \cos^2\theta$

Ex PROVE (a) $\cos^2 x \tan^2 x + \cos^2 x = 1$

(b) $(\cos\theta + \sin\theta)(\cos\theta - \sin\theta) = 2\cos^2\theta - 1$

(a) $\cos^2 x \tan^2 x + \cos^2 x$

$\cos^2 x\left(\tan^2 x + 1\right)$

$\cos^2 x \times \sec^2 x$

$\cos^2 x \times \dfrac{1}{\cos^2 x} = 1$

(b) LHS

$(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)$

$\cos^2\theta + \cancel{\sin\theta\cos\theta} - \cancel{\sin\theta\cos\theta} - \sin^2\theta$

$\cos^2\theta - \sin^2\theta$

$\cos^2\theta - \left(1 - \cos^2\theta\right)$

$\cos^2\theta - 1 + \cos^2\theta = 2\cos^2\theta - 1$

Ex PROVE

(a) $\dfrac{\sin x}{1-\cos x} - \dfrac{\sin x}{1+\cos x} = 2\cot x$

(b) $\dfrac{1+\cot\theta}{\operatorname{cosec}\theta} = \dfrac{\tan\theta + 1}{\sec\theta}$

(a) $\dfrac{\sin x}{1-\cos x} - \dfrac{\sin x}{1+\cos x}$

$\dfrac{\sin x(1+\cos x) - \sin x(1-\cos x)}{(1-\cos x)(1+\cos x)}$

$\dfrac{\sin x + \sin x\cos x - [\sin x - \sin x\cos x]}{1 - \cos x + \cos x - \cos^2 x}$

$\dfrac{\cancel{\sin x} + \sin x\cos x - \cancel{\sin x} + \sin x\cos x}{1 - \cos^2 x}$

$\dfrac{2\sin x\cos x}{\sin^2 x} = \dfrac{2\cos x}{\sin x} = 2\cot x$

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LHS

(b) $\dfrac{1+\cot\theta}{\operatorname{cosec}\theta}$

$\dfrac{1 + \frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$

$\dfrac{\frac{\cos\theta + \sin\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$

$= 2\cos^2\theta - 1$

EX PROVE

(a) $\dfrac{\sin x}{1-\cos x} - \dfrac{\sin x}{1+\cos x} = 2\cot x$

(b) $\dfrac{1+\cot\theta}{\operatorname{cosec}\theta} = \dfrac{\tan\theta + 1}{\sec\theta}$

(a) $\dfrac{\sin x}{1-\cos x} - \dfrac{\sin x}{1+\cos x}$

$\dfrac{\sin x(1+\cos x) - \sin x(1-\cos x)}{(1-\cos x)(1+\cos x)}$

$\dfrac{\sin x + \sin x\cos x - [\sin x - \sin x\cos x]}{1 - \cos x + \cos x - \cos^2 x}$

$\dfrac{\cancel{\sin x} + \sin x\cos x - \cancel{\sin x} + \sin x\cos x}{1 - \cos^2 x}$

$\dfrac{2\sin x\cos x}{\sin^2 x} = \dfrac{2\cos x}{\sin x} = 2\cot x$ #

LHS

(b) $\dfrac{1+\cot\theta}{\operatorname{cosec}\theta}$

$\dfrac{1 + \frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$

$\dfrac{\frac{\cos\theta + \sin\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$

$\dfrac{\cos\theta + \sin\theta}{\sin\theta} \times \sin\theta$

$\cos\theta + \sin\theta$

RHS

$\dfrac{\tan\theta + 1}{\sec\theta}$

$\dfrac{\frac{\sin\theta}{\cos\theta} + 1}{\frac{1}{\cos\theta}}$

$\dfrac{\frac{\sin\theta + \cos\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$

$\sin\theta + \cos\theta$

LHS = RHS.

PROVE $\cot A \cos A = \operatorname{cosec} A - \sin A$

LHS $\cot A \cos A$

$$\frac{\cos A}{\sin A} \times \cos A$$

$$\frac{\cos^2 A}{\sin A} = \frac{1-\sin^2 A}{\sin A} = \frac{1}{\sin A} - \frac{\sin^2 A}{\sin A}$$

$$= \frac{1}{\sin A} - \sin A$$

$$= \operatorname{cosec} A - \sin A$$

RHS

Ex SIMPLIFY

$$\sec^2\theta - \sin^2\theta - \cos^2\theta$$

$$\sec^2\theta - (\sin^2\theta + \cos^2\theta)$$

$$\sec^2\theta - 1 = \tan^2\theta$$

COMPOUND ANGLES

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

Ex SIMPLIFY $\cos x \cos 2x - \sin x \sin 2x$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\alpha = x, \quad \beta = 2x$$

$$\cos(\alpha+\beta) = \cos(x+2x)$$

$$= \cos 3x$$

Ex simplify $\sin^2 x \sin y + \cos^2 x \sin y$

$$\sin^2 x \sin y + \cos^2 x \sin y$$

$$\sin y \left(\sin^2 x + \cos^2 x \right)$$

$$\sin y \times 1 = \sin y$$

Ex simplify

$$\sin(\underset{\substack{\uparrow \\ \alpha}}{x+y}) \sin \underset{\substack{\uparrow \\ \beta}}{y} + \cos(x+y)\cos y$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \sin\alpha\sin\beta + \cos\alpha\cos\beta$$

$$\cos(x+y-y) = \sin(x+y)\sin y + \cos(x+y)\cos y$$

$$\downarrow$$

$$\cos x$$

Express $\dfrac{\sin(A+B)}{\cos(A-B)}$ in term of $\tan A$ and $\tan B$

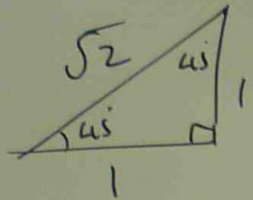
$$\frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B + \sin A\sin B}$$

$$\frac{\dfrac{\sin A\cos B + \cos A\sin B}{\cos A\cos B}}{\dfrac{\cos A\cos B + \sin A\sin B}{\cos A\cos B}}$$

$$\frac{\dfrac{\sin A\cancel{\cos B}}{\cos A\cancel{\cos B}} + \dfrac{\cancel{\cos A}\sin B}{\cancel{\cos A}\cos B}}{\dfrac{\cos A\cos B}{\cos A\cos B} + \dfrac{\sin A\sin B}{\cos A\cos B}}$$

$$\frac{\tan A + \tan B}{1 + \tan A\tan B}$$

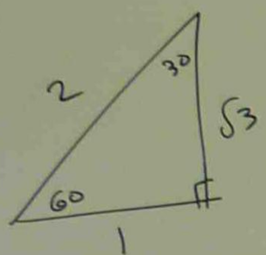
SPECIAL TRIANGLE



$\sin 45 = \frac{1}{\sqrt{2}}$

$\cos 45 = \frac{1}{\sqrt{2}}$

$\tan 45 = \frac{1}{1} = 1$



$\sin 60 = \frac{\sqrt{3}}{2}$

$\cos 60 = \frac{1}{2}$

$\tan 60 = \frac{\sqrt{3}}{1}$

$\sin 30 = \frac{1}{2}$

$\cos 30 = \frac{\sqrt{3}}{2}$

$\tan 30 = 1/\sqrt{3}$

EX FIND EXACT VALUE of $\sin 15$, $\sin 75$

$\sin 15 = \sin(45 - 30)$

$\sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$

$\sin(45 - 30) = \sin 45 \cos 30 - \cos 45 \sin 30$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3} - 1}{2\sqrt{2}}$

$\sin(75) = \sin(45 + 30)$

$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

$\sin(45 + 30) = \sin 45 \cos 30 + \cos 45 \sin 30$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$ ※