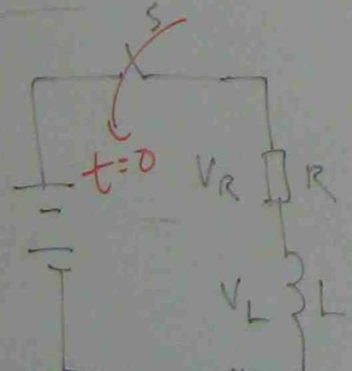
DIFFERENTIAL EQUATION & APPLICATION OF DIFFERENTIAL EQUATION IN ELECTRICAL CIRCUIT CALCULATION

$$\frac{dy}{dt} + 3y = 4 \quad \text{(FIRST ORDER)}$$

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 4 = 0 \quad \text{(SECOND ORDER)}$$

RL CIRCUIT



$V_R = IR$

$V_L = L\frac{di}{dt}$

$$E = V_R + V_L$$

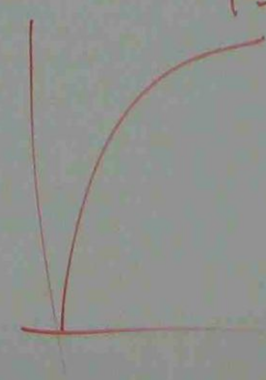
$$E = IR + L\frac{di}{dt}$$

$$i = I\left(1 - e^{-\frac{R}{L}t}\right)$$

VALUE OF CURRENT AT $t=0$

SWITCHING ON $t=0$

$$i = I\left(1 - e^{-\frac{Rt}{L}}\right)$$



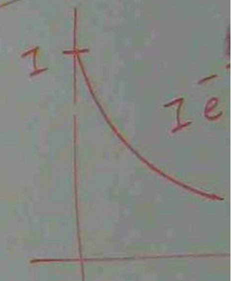
$$i = I\left(1 - e^{-\frac{Rt}{L}}\right)$$

$$= I - Ie^{-\frac{Rt}{L}}$$

CONSTANT

VARIABLE WITH e FUNCTION

SWITCHING OFF



GENERALIZ

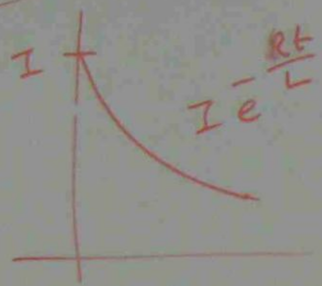
FOR ANS

THE EQU

L

$i(t$

SWITCHING OFF



GENERALIZED FORMAT
FOR ANSWER OF
THE EQUATION

$$L\frac{di}{dt} + IR = E$$

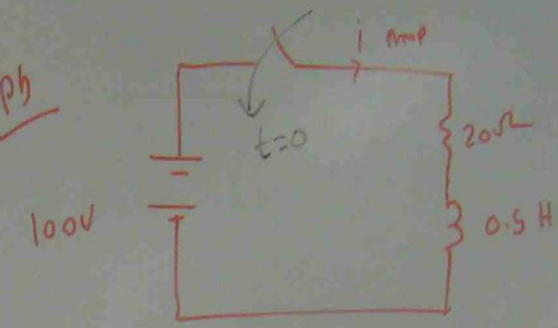
$$\boxed{i(t) = I_p + Ae^{-\frac{t}{\tau}} \qquad \tau = \frac{L}{R}}$$

$1 - e^{-\frac{Rt}{L}})$

$- Ie^{-\frac{Rt}{L}}$

VARIABLE
ANT WITH e FUNCTION

ph



FOR THE CIRCUIT SHOWN ABOVE, DETERMINE THE FOLLOWING VALUES AFTER THE SWITCH HAS BEEN CLOSED.

(a) THE FINAL VALUE OF CURRENT
(b) THE INITIAL VALUE OF CURRENT
(c) TIME CONSTANT OF THE CIRCUIT
(d) THE EQUATION OF THE CURRENT
(e) THE INITIAL RATE OF CHANGE OF CURRENT.

(a) SWITCH CLOSED, INDUCTOR IS SHORT CIRCUITED

$$I_{FINAL} = \frac{V}{R} = \frac{100}{20} = 5A$$

(b) $I_{INITIAL} = 0$

(c) TIME CONSTANT $(\tau) = \dfrac{L}{R} = \dfrac{0.5}{20} = 0.025$ sec $= 25$ ms

(d) $E = IR + L\dfrac{dI}{dt}$

$100 = 20I + 0.5\dfrac{dI}{dt}$ CIRCUIT EQUATION

$i(t) = I\left(1 - e^{-t/\tau}\right)$

FINAL CURRENT

$= 5\left(1 - e^{-\frac{t}{0.025}}\right)$

$i(t) = 5\left(1 - e^{-40t}\right)$

(e) $\dfrac{di(t)}{dt} = \dfrac{d}{dt}\, 5\left(1 - e^{-40t}\right)$

$= \cancel{\dfrac{d}{dt}5} - \dfrac{d}{dt}5e^{-40t}$

$\to 0$

$= -5 \times (-40)\, e^{-40t}$

RATE OF CHANGE OF CURRENT $= 200\, e^{-40t}$ A/s

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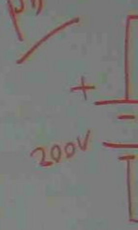

DE TC

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2

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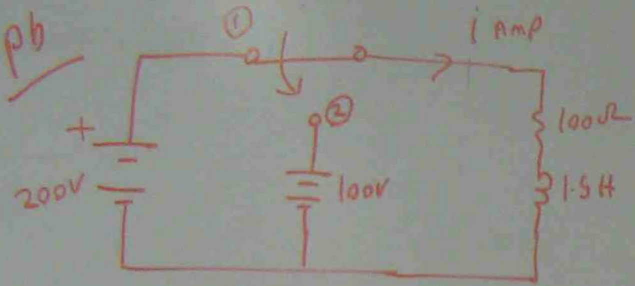
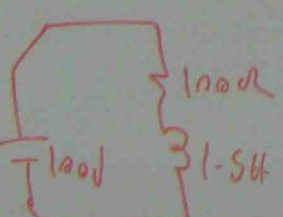


DETERMINE THE EQUATION OF THE CURRENT IN ABOVE FIGURE AFTER SWITCHING TO POSITION ②.
ASSUME THAT STEADY STATE CURRENT HAD BEEN ATTAINED IN POSITION ①

INITIAL CURRENT $I(0) = \frac{200V}{100\Omega} = 2$ Amp.

(OR) $I(t)\Big|_{t=0} = 2$ Amp (NOT STEADY STATE)

SWITCH AT ②



FINAL CURRENT $= \frac{100}{100} = 1$ Amp.

STEADY STATE CONSTANT VALUE

$$E = IR + L\frac{dI}{dt}$$

$$i(t) = I_p + A e^{-t/\tau}$$

FINAL

$$\tau = \frac{L}{R} = \frac{1.5}{100} = 0.015$$

$I_p = 1$ Amp ← FINAL

$$i(t) = 1 + A e^{-\frac{t}{0.015}}$$

WHEN TIME $(t=0)$, THE INITIAL CURRENT IS 2 Amp.

$$i(t)\Big|_{t=0} = 2$$

$$2 = 1 + A e^{-\frac{0}{0.015}}$$

$$2 = 1 + A e^{0}$$

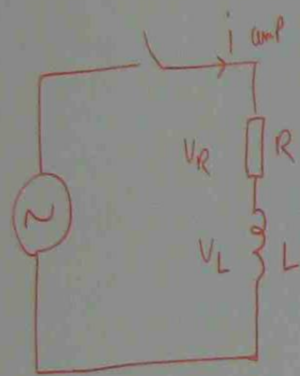
$2 = 1 + A$

$\therefore A = 2 - 1 = 1$

$$\therefore i(t) = 1 + 1 e^{-\frac{t}{0.15}}$$

$$= 1 + e^{-t/0.15} = 1 + e^{-6}$$

RESPONSE OF RL AND RC CIRCUITS TO AC VOLTAGE



$e = E_m \sin(\omega t + \theta)$

$V_R = iR$

$V_L = L\frac{di}{dt}$

$V_R + V_L = e$

$iR + L\frac{di}{dt} = E_m \sin(\omega t + \theta)$

$i = i_c + i_p$

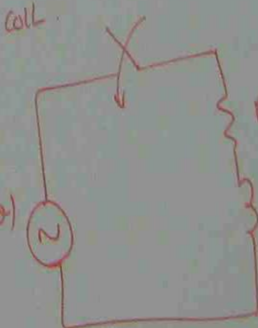
COMPLEMENTARY COMPONENT

PARTICULAR COMPONENT

GENERAL SOLUTION

$i = A e^{-\frac{t}{\tau}} + I_p \sin(\omega t + \theta - \phi)$

pb/ AN E.M.F $e = 100 \sin(314t + \theta)$ VOLT IS APPLIED TO A COIL OF RESISTANCE 200Ω AND INDUCTANCE 0.5 HENRY WHEN θ is 30 DETERMINE THE EQUATION OF THE RESULTING CURRENT IN THE COIL.



$X_L = \omega L$

$= 314 \times 0.$

$= 157$

$Z = R + jX_L = 200 + j157$

$\phi = \tan^{-1}\frac{157}{200} = 38.$

$i =$

$I =$

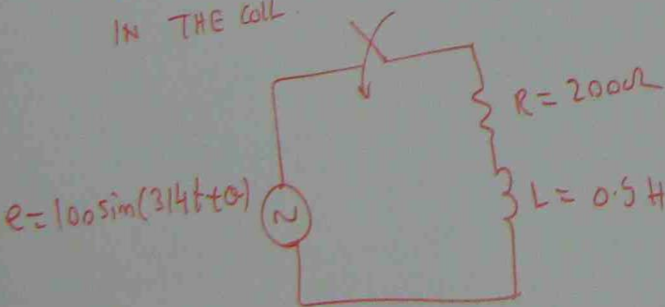
$\theta -$

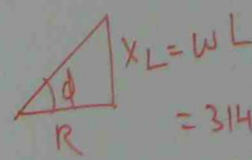
OLTAGE

GENERAL SOLUTION

$$i = A e^{-\frac{t}{\tau}} + I_p \sin(\omega t + \theta - \phi)$$

pb/ AN E.M.F $e = 100 \sin(314t + \theta)$ VOLT IS APPLIED TO A COIL OF RESISTANCE 200Ω AND INDUCTANCE 0.5 HENRY WHEN θ is 30° DETERMINE THE EQUATION OF THE RESULTING CURRENT IN THE COIL.





$$= 314 \times 0.5$$
$$= 157$$

$$Z = R + jX_L = 200 + j157$$

$$\phi = \tan^{-1}\frac{157}{200} = 38.13^\circ$$

$$i = A e^{-t/\tau} + I_p \sin(\omega t + \theta - \phi)$$

$$\tau = \frac{L}{R} = \frac{0.5}{200} = 0.0025 = 2.5\,ms$$

$$\theta - \phi = 30 - 38.13 = -8.13$$

$$i = A e^{-\frac{t}{2.5\times10^{-3}}} + I_p \sin(314t - 8.13)$$

$$I_p = \frac{E}{Z} = \frac{100}{\sqrt{R^2 + X_L^2}} = \frac{100}{\sqrt{200^2 + 157^2}} = 0.$$

$$i = A e^{-400t} + 0.392 \sin(314t - 8.13)$$

$$t = 0 \rightarrow i = 0$$

$$0 = Ae^{-400\times0} + 0.392\sin(314\times0 - 8.13)$$

$$0 = Ae^{0} + 0.392\sin(-8.13)$$

$$0 = A - 0.392\sin 8.13$$

$$A = 0.0556$$

$$i = 0.0556e^{-400t} + 0.392\sin(314t - 8.13)$$

0.3932

13)

RC CIRCUIT

$$e = E_m\sin(\omega t + \theta)$$

$$v_R = iR$$

$$v_C = \frac{1}{C}\int i\,dt$$

$$v_R + v_C = e$$

$$iR + \frac{1}{C}\int i\,dt = e = E_m\sin(\omega t+\theta)$$

$$\frac{d}{dt}(iR) + \frac{d}{dt}\left[\frac{1}{C}\int i\,dt\right] = \frac{d}{dt}\left[E_m\sin(\omega t+\theta)\right]$$

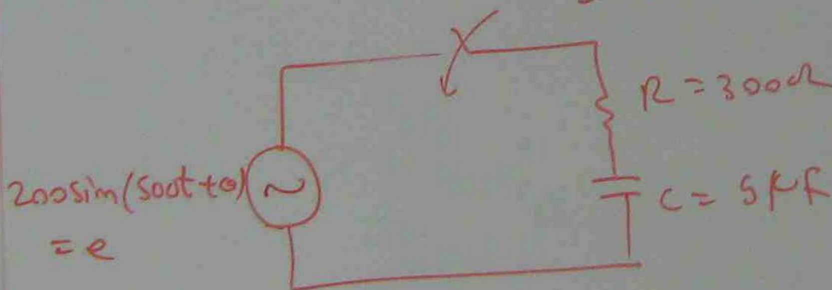
$$R\frac{di}{dt} + \frac{1}{C}\frac{d}{dt}\int i\,dt = E_m\frac{d}{dt}\sin(\omega t+\theta)$$

$$R\frac{di}{dt} + \frac{1}{C}\times i = E_m\omega\cos(\omega t+\theta)$$

THE GENERAL SOLUTION

$$i = A e^{-t/\tau} + I_p \,\mathrm{sim}\,(\omega t + \theta - \phi)$$

pb AN E.m.F OF $e = 200 \,\mathrm{sim}\,(500t + \theta)$ VOLT IS APPLIED TO AN RC CIRCUIT WHERE $R = 300\,\Omega$ AND $C = 5\,\mu F$ WHEN θ IS 60°. DETERMINE THE EQUATION OF THE CIRCUIT IF THE INITIAL CHARGE ON THE CAPACITOR IS 250 MICRO COULOMBS.



$$i = A e^{-t/\tau} + I_p \,\mathrm{sim}(\omega t + \theta - \phi)$$

$$\tau = R.C = 300 \times 5 \times 10^{-6} = 1.5 \times 10^{-3}$$

$$\theta = 60^\circ$$

$$Z = R - jX_c = 300 - j\frac{1}{2\pi f c}$$

$$= 300 - j\frac{1}{500 \times 5\times10^{-6}}$$

$$= 300 - j400\ \Omega$$

$$= \sqrt{300^2+400^2}\ \angle -\tan^{-1}\frac{400}{300}$$

$$Z = 500\ \angle -53.1\ \Omega$$

$$\phi = -53.1$$

$$I_p = \frac{E}{Z} = \frac{200}{500} = 0.4$$

$$i = A\,e^{-\frac{t}{1.5\times10^{-3}}} + 0.4\sin(500t + 60 - (-53.1))$$

$$i = A\,e^{-666.7t} + 0.4\sin(500t + 113.1)\ \underline{\underline{\text{Amp}}}$$

$$t=0 \rightarrow i(0) = A\,e^{-666.7\times0} + 0.4\sin(500\times0 + 113.1)$$

$$i(0) = A + 0.4\sin 113.1 \text{ ——— } 1$$

$$q = \int i\,dt = 250\ \mu C$$

INITIAL

$$iR + \frac{1}{C}\int i\,dt = e$$

$$iR + \frac{1}{C}\times q = e$$

$$i\times300 + \frac{250\times10^{-6}}{5\times10^{-6}} = 200\sin60$$

$$i\times300 + 50 = 200\sin60$$

$$i(0) = \frac{200\sin60 - 50}{300} = 0.4107$$

1

$$0.4107 = A + 0.4\sin113.1$$

$$A = 0.0433$$

$$i(t) = 0.0433\,e^{-666.7t} + 0.4(500t + 113.1)$$