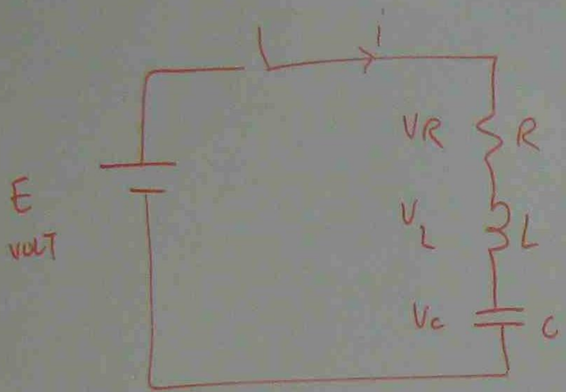
SOLVING THE SECOND ORDER DIFFERENTIAL EQUATION



$$V_R + V_L + V_c = E$$

$$iR + L\frac{di}{dt} + \frac{1}{c}\int i\,dt = E$$

$$\frac{d}{dt} iR + \frac{d}{dt} L\frac{di}{dt} + \frac{d}{dt}\frac{1}{c}\int i\,dt = \frac{d}{dt}E$$

$$R\frac{di}{dt} + L\frac{d^2 i}{dt} + \frac{1}{c} \times i = 0$$

$$\frac{R}{L}\frac{di}{dt} + \frac{d^2 i}{dt^2} + \frac{1}{Lc} i = 0$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{Lc} i = 0$$

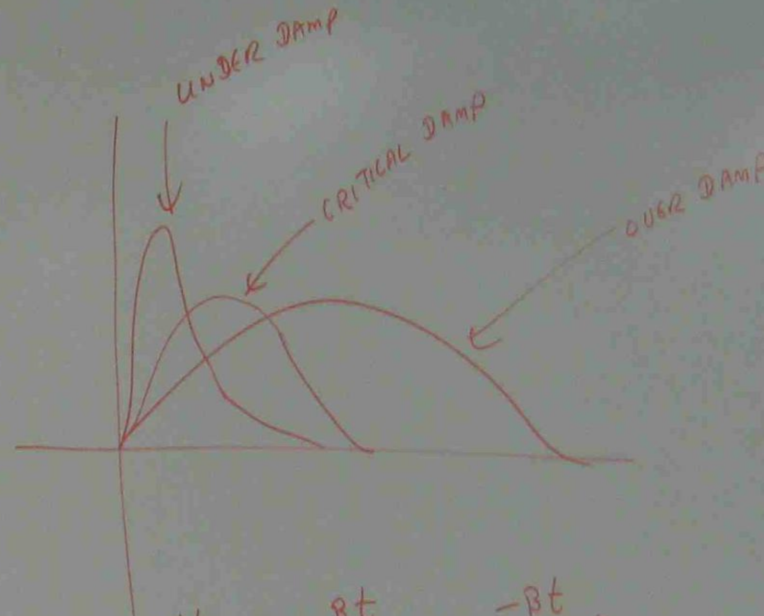
COMPARE WITH

$$a\frac{d^2 i}{dt^2} + b\frac{di}{dt} + ci = 0$$

$$am^2 + bm + c = 0$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$



LET

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\beta = \sqrt{\alpha^2 - \omega_0^2}$$

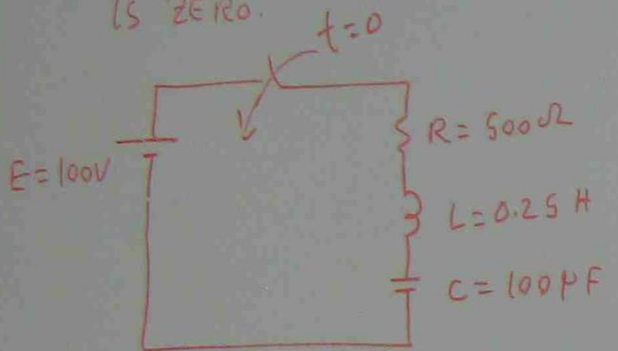
IF $\alpha > \omega_0 \Rightarrow$ OVER DAMP $\longrightarrow$ $i = e^{-\alpha t}\left(A_1 e^{\beta t} + A_2 e^{-\beta t}\right)$

IF $\alpha = \omega_0 \Rightarrow$ CRITICAL DAMP $\longrightarrow$ $i = e^{-\alpha t}\left(A_1 + A_2 t\right)$

IF $\alpha < \omega_0 \Rightarrow$ UNDER DAMP $\longrightarrow$ $i = e^{-\alpha t}\left(A_1 \sin \beta t + A_2 \cos \beta t\right)$

pb/ IN A GIVEN CIRCUIT, $E = 100V$ $R = 500\Omega$, $L = 0.25H$, $C = 100\mu F$.

DETERMINE THE EQUATION OF THE CURRENT IF THE INITIAL CHARGE ON CAPACITOR IS ZERO.



$$\alpha = \frac{R}{2L} = \frac{500}{2\times 0.25} = 1000$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25\times 100\times 10^{-6}}} = 200$$

$$\beta = \sqrt{\alpha^2 - \omega_0^2}$$

$$= \sqrt{1000^2 - 200^2}$$

$$= 979.8$$

$\alpha > \omega_0 \Rightarrow$ OVER DAMP

$$i = e^{-\alpha t}\left[A_1 e^{\beta t} + A_2 e^{-\beta t}\right]$$

$$= A_1 e^{(-\alpha+\beta)t} + A_2 e^{(-\alpha-\beta)t}$$

$$= A_1 e^{(-1000+979.8)t} + A_2 e^{(-1000-979.8)t}$$

$$i = A_1 e^{-20.2t} + A_2 e^{-1979.8t}$$

I $t = 0 \rightarrow i = 0$

$$0 = A_1 e^{-20.2\times 0} + A_2 e^{-1979.8\times 0}$$

$$A_1 + A_2 = 0 \quad \text{—— (1)} \Rightarrow A_1 = -A_2$$

II

$$i = A_1 e^{-20.2t} + A_2 e^{-1979.8t}$$

$$\frac{di}{dt} = A_1 \frac{d}{dt} e^{-20.2t} + A_2 \frac{d}{dt} e^{-1979.8t}$$

$$= -20.2 A_1 e^{-20.2t} + (-1979.8) A_2 e^{-1979.8t}$$

$$\frac{di}{dt}$$

II

$L\frac{di}{dt}$ = INDUCTOR VOLTAGE

$$t=0 \rightarrow L\frac{di}{dt} = E$$

$$\frac{di}{dt} = \frac{E}{L} = \frac{100}{0.25} = 400$$

$$400 = -20.2 A_1 e^{-20.2\times 0} - 1979.8 A_2 e^{-1979.8\times 0}$$

$$-20.2 A_1 - 1979.8 A_2 = 400$$

$$-20.2(-A_2) - 1979.8 A_2 = 400$$

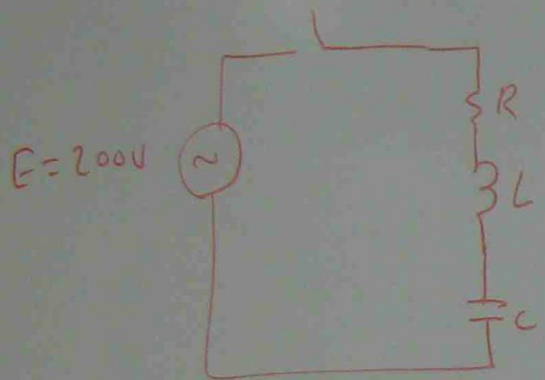
$$A_2 = -0.2041$$

$$\therefore A_1 = 0.02041$$

$$i = 0.02041 e^{-20.2t} - 0.204 e^{-1979.8t}$$

$-A_2$

EXERCISE



E = 200V, R = 100Ω L = 0.5 HENRY

C = 200 μF

FIND THE EQUATION OF THE CURRENT

IF THE INITIAL CHARGE ON CAPACITOR IS

20 MILLICOULOMBS

$\alpha = \frac{R}{2L}$ $\omega_0 = \frac{1}{\sqrt{LC}}$

COMPARE α & ω_0 SELECT THE EQUATION

CONDITION (1) $t = 0 \longrightarrow i = 0$

CONDITION (2)

$q = 20\ mC$

$V_c = \frac{q}{C} = \frac{20 \times 10^{-3}}{200 \times 10^{-6}}$

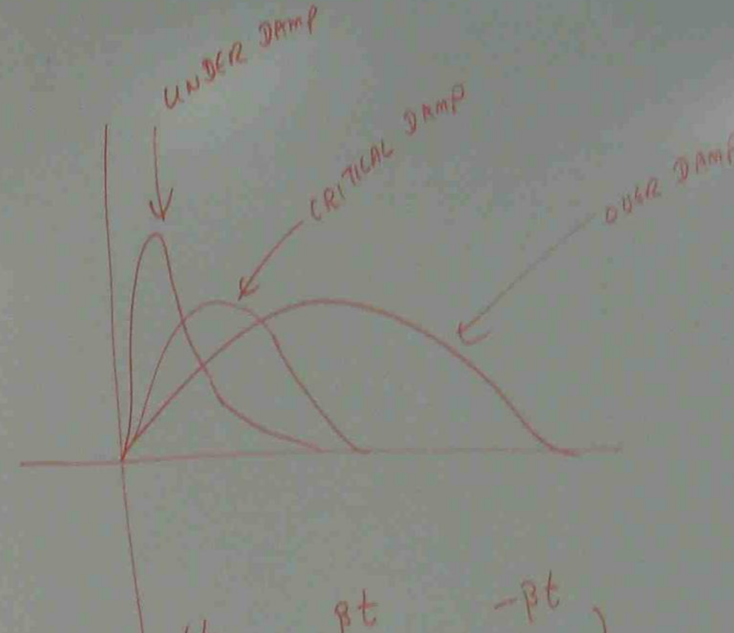
$= 100V$

$V_c = \frac{1}{C} \int i\,dt$

$t = 0 \left| \frac{di}{dt} = \frac{E}{L} = \frac{100V}{0.5} \right.$

$= 200$

$$m = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$



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