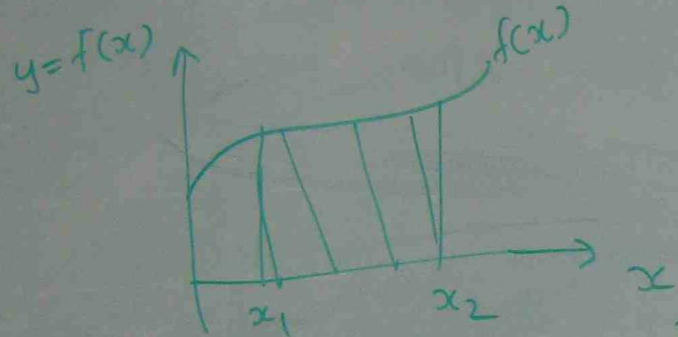
INTEGRATION



AREA UNDER CURVE = $\int_{x_1}^{x_2} f(x)\,dx$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

Ex

$$\int x^7\,dx = \frac{x^{7+1}}{7+1} + c$$

$$= \frac{x^8}{8} + c$$

Ex

$$\int x^{\frac{1}{5}}\,dx = \frac{x^{\frac{1}{5}+1}}{\frac{1}{5}+1} + c$$

$$\frac{x^{6/5}}{6/5} + c$$

$$\frac{5}{6}x^{6/5} + c$$

Ex $\int x^{-3}\,dx = \dfrac{x^{-3+1}}{-3+1} + c$

$= \dfrac{x^{-2}}{-2} + c$

$= -\frac{1}{2}x^{-2} + c$

Ex $\int x^{-1/2}\,dx = \dfrac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$

$= \dfrac{x^{1/2}}{1/2} + c$

$= 2x^{1/2} + c$

$\int (x^n + x^m)\,dx = \int x^n\,dx + \int x^m\,dx$

$= \dfrac{x^{n+1}}{n+1} + \dfrac{x^{m+1}}{m+1} + c$

Ex $\int (x^4 + 2x^3)\,dx$

$\int x^4\,dx + \int 2x^3\,dx$

$\dfrac{x^{4+1}}{4+1} + 2\int x^3\,dx$

$\dfrac{x^5}{5} + 2 \times \dfrac{x^{3+1}}{3+1} + c$

$\dfrac{x^5}{5} + 2\,\dfrac{x^4}{4} + c$

$$\frac{x^5}{5} + \frac{x^4}{2} + c \quad //$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

$$\int u^n \, du = \frac{u^{n+1}}{n+1} + c$$

$$\int (2x+3)^n \, d(2x+3) = \frac{(2x+3)^{n+1}}{n+1} + c$$

$$\int (2x+3)^n \, dx \neq \frac{(2x+3)^{n+1}}{n+1} + c$$

Ex $\int (2x+3)^3 \, dx = ?$

$$d(2x+3) = d2x + d3$$
$$= 2\,dx + 0$$
$$= 2\,dx$$

$$dx = \frac{d(2x+3)}{2}$$

$$\int (2x+3)^3 \, \frac{d(2x+3)}{2}$$

$$\frac{1}{2} \int (2x+3)^3 \, d(2x+3)$$

$$\frac{1}{2}\ \frac{(2x+3)^{3+1}}{3+1}+c$$

$$\frac{1}{2}\times\frac{(2x+3)^4}{4}+c$$

$$\frac{1}{8}(2x+3)^4+c$$

Ex $\int(-3x+2)^{-1/3}\,dx$

$$d(-3x+2)=d(-3x)+d2$$

$$d(-3x+2)=-3dx$$

$$dx=-\frac{1}{3}\,d(-3x+2)$$

$$\int(-3x+2)^{-\frac{1}{3}}\left(-\frac{1}{3}\,d(-3x+2)\right)$$

$$-\frac{1}{3}\int(-3x+2)^{-\frac{1}{3}}\,d(-3x+2)$$

$$-\frac{1}{3}\times\frac{(-3x+2)^{-\frac{1}{3}+1}}{-\frac{1}{3}+1}+c$$

$$-\frac{1}{3}\times\frac{(-3x+2)^{2/3}}{2/3}+c$$

$$-\frac{1}{3}\times\frac{3}{2}(-3x+2)^{2/3}+c$$

$$-\frac{1}{2}(-3x+2)^{2/3}+c$$

$$\frac{1}{2}\ \frac{(2x+3)^{3+1}}{3+1} + c$$

$$\frac{1}{2} \times \frac{(2x+3)^4}{4} + c$$

$$\frac{1}{8}(2x+3)^4 + c$$

Ex $\int (-3x+2)^{-1/3}\,dx$

$$d(-3x+2) = d(-3x) + d2$$

$$d(-3x+2) = -3dx$$

$$dx = -\frac{1}{3}.d(-3x+2)$$

$$\int (-3x+2)^{-\frac{1}{3}} \left(-\frac{1}{3}\,d(-3x+2)\right)$$

$$-\frac{1}{3}\int (-3x+2)^{-\frac{1}{3}}\,d(-3x+2)$$

$$-\frac{1}{3} \times \frac{(-3x+2)^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + c$$

$$-\frac{1}{3} \times \frac{(-3x+2)^{2/3}}{2/3} + c$$

$$-\frac{1}{3} \times \frac{3}{2}(-3x+2)^{2/3} + c$$

$$-\frac{1}{2}(-3x+2)^{2/3} + c$$

EXERCISE

$$\int (5x+8)^{-2}\,dx$$

INTEGRATE THE FOLLOWINGS

(i) x^{5}

(ii) $\frac{1}{3} x^{-1/2}$

(iii) $6x^{-2}$

(iv) $3x^{1/5}$

(v) $\frac{1}{2} x^{-1/3} + x^{2}$

(vi) $(2x+3)^{3}$

(vii) $(1+x)^{-4}$

(viii) $(3-x)^{1/2}$

(ix) $(5x+6)^{-1/3}$

(x) $(3x-2)^{-3/2}$

INTEGRATION OF TRIGONOMETRIC FUNCTIONS

$\frac{d}{dx} \sin x = \cos x \longleftrightarrow \int \sin x \, dx = -\cos x + c$

$\frac{d}{dx} \cos x = -\sin x \longleftrightarrow \int \cos x \, dx = \sin x + c$

$\frac{d}{dx} \tan x = \sec^2 x \longleftrightarrow \int \sec^2 x \, dx = \tan x + c$

$\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x \longleftrightarrow \int \operatorname{cosec}^2 x \, dx = -\cot x + c$

$\frac{d}{dx} \sec x = \sec x \tan x \longleftrightarrow \int \sec x \tan x = \sec x + c$

$\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x \longleftrightarrow \int \operatorname{cosec} x \cot x = -\operatorname{cosec} x + c$

$$\frac{d}{dx}\sin^2 x = 2\sin^{2-1} x \frac{d}{dx}\sin x = 2\sin x\cos x$$

$$\int \sin^2 x \, dx$$

$$\boxed{\begin{aligned} \cos 2x &= 2\cos^2 x - 1 \\ \cos 2x &= 1 - 2\sin^2 x \end{aligned}}$$

$$\int \sin^2 x \, dx$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$2\sin^2 x = 1 - \cos 2x \longrightarrow \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\int \frac{1-\cos 2x}{2}\,dx = \int \frac{1}{2}\,dx - \int \frac{\cos 2x}{2}\,dx$$

$$= \frac{1}{2}\int dx - \frac{1}{2}\int \cos 2x\,dx$$

$$= \frac{x}{2} - \frac{1}{2}\int \cos 2x\,\frac{d2x}{2}$$

$$= \frac{x}{2} - \frac{1}{4}\int \cos 2x\,d2x$$

$$= \frac{x}{2} - \frac{1}{4}\sin 2x + C$$

pb

$$\int \cos^2 x\,dx = ?$$

$$\cos 2x = 2\cos^2 x - 1$$

$$2\cos^2 x = 1 + \cos 2x$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\int \frac{1+\cos 2x}{2}\,dx$$

$$\int \frac{1}{2}\,dx + \int \frac{\cos 2x\,dx}{2}$$

$$\frac{x}{2} + \int \frac{\cos 2x\,d2x}{2\times 2}$$

$\frac{x}{2} + \frac{1}{4}\int \cos 2x \, d2x$

$\frac{x}{2} + \frac{1}{4} \sin 2x + c$

$\frac{x}{2} + \frac{1}{4}$

pb

$\boxed{\int \tan^2 x \, dx = ?}$

$$\boxed{\begin{aligned} \sec^2 x &= \tan^2 x + 1 \\ \operatorname{cosec}^2 x &= \cot^2 x + 1 \end{aligned}}$$

$\sec^2 x = \tan^2 x + 1$

$\therefore \tan^2 x = \sec^2 x - 1$

$$\int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx$$

$$= \int \sec^2 x \, dx - \int dx$$

$$= \tan x - x + c$$

$\sin 2x = 2 \sin x \cos x$

pb

$\int \sin x \cos x \, dx = ?$

BuT

$\sin 2x = 2 \sin x \cos x$

$\therefore \sin x \cos x = \frac{\sin 2x}{2}$

$$\int \frac{\sin 2x}{2} dx = \int \frac{\sin 2x \, d2x}{2 \times 2}$$

$$= \frac{1}{4} \int \sin 2x \, d2x$$

$$= -\frac{\cos 2x}{4} + C$$

$\frac{d}{dx} \sin x \cos x = \sin x \frac{d}{dx} \cos x + \cos x \frac{d}{dx} \sin x$

$= \sin(-\sin x) + \cos x * \cos x$

$= -\sin^2 x + \cos^2 x$

$$\boxed{\frac{d}{dx} uv = u\frac{dv}{dx} + v\frac{du}{dx}}$$

$$\frac{1}{10}\left[\int \frac{\cos 10x \, d10x}{10} + x\right]$$

$$\frac{1}{10}\left[\frac{\sin 10x}{10} + x\right] + C$$

$$\frac{\sin 10x}{100} + \frac{x}{10} + C$$

pb $\int \tan^2 3x \, dx = ?$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\sec^2 x = \tan^2 x + 1$$

$$\therefore \sec^2 3x = \tan^2 3x + 1$$

$$\tan^2 3x = \sec^2 3x - 1$$

$$\int (\sec^2 3x - 1) \, dx$$

$$\int \sec^2 3x \, dx - \int dx$$

$$\int \frac{\sec^2 3x \, d3x}{3} - x$$

$$\frac{\tan 3x}{3} - x + C$$

$\sin x$

x

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

pb $\int \sin 3x \cos 4x \, dx = ?$

$$\sin 3x \cos 4x = \frac{1}{2}\left[\sin(3x+4x) + \sin(3x-4x)\right]$$

$$= \frac{1}{2}\left[\sin 7x + \sin(-x)\right]$$

$$= \frac{1}{2}\left[\sin 7x - \sin x\right]$$

$$\int \frac{1}{2}\left[\sin 7x - \sin x\right] dx$$

$$\frac{1}{2}\left[\int \sin 7x \, dx - \int \sin x \, dx\right]$$

$$\frac{1}{2}\left[\int \frac{\sin 7x \, d7x}{7} - (-\cos x)\right]$$

$$\frac{1}{2}\left[\frac{(-\cos 7x)}{7} + \cos x\right] + c$$

$$\frac{1}{2}\left[-\frac{\cos 7x}{7} + \cos x\right] + c$$

$$-\frac{\cos 7x}{14} + \frac{\cos x}{2} + c$$

EXERCISE

INTEGRATE THE FOLLOWING TRIGONOMETRIC FUNCTIONS

(i) $\sec^2 x$

(ii) $\tan x$

(iii) $\cos 3x$

(iv) $\sin 6x$

(v) $\cot x$

(vi) $\operatorname{cosec} x \cot x$

(vii) $\sec x \tan x$

Pb $\int \frac{1}{5} \cos^2 5x \, dx = ?$

$\cos 2x = 2\cos^2 x - 1$

$\therefore \cos 10x = 2\cos^2 5x - 1$

$\cos^2 5x = \frac{\cos 10x + 1}{2}$

$\int \frac{1}{5} \left(\frac{\cos 10x + 1}{2} \right) dx$

$\frac{1}{10} \int (\cos 10x + 1) \, dx$

$\frac{1}{10} \left[\int \cos 10x \, dx + \int dx \right]$

pb $\int \sin x \sin 3x \, dx$

$$\sin x \sin 3x = \frac{1}{2}\left[\cos(x-3x) - \cos(x+3x)\right]$$

$$= \frac{1}{2}\left[\cos(-2x) - \cos 4x\right]$$

$$= \frac{1}{2}\left[\cos 2x - \cos 4x\right]$$

$$\int \frac{1}{2}\left[\cos 2x - \cos 4x\right] dx$$

$$\frac{1}{2}\left[\int \cos 2x \, dx - \int \cos 4x \, dx\right]$$

$$\frac{1}{2}\left[\int \frac{\cos 2x \, d2x}{2} - \int \frac{\cos 4x \, d4x}{4}\right]$$

$$\frac{\sin 2x}{4} - \frac{\sin 4x}{8} + c \quad //$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

pb $\int \sin 6x \cos x \, dx$

$\sin 6x \cos x = \frac{1}{2}\left[\sin(6x+x) + \sin(6x-x)\right]$

$= \frac{1}{2}\left[\sin 7x + \sin 5x\right]$

$\int \frac{1}{2}\left[\sin 7x + \sin 5x\right] dx$

$\frac{1}{2}\left[\int \sin 7x \, dx + \int \sin 5x \, dx\right]$

$\frac{1}{2}\left[\int \frac{\sin 7x \, d7x}{7} + \int \frac{\sin 5x \, d5x}{5}\right]$

$\frac{1}{2}\left[-\frac{\cos 7x}{7} - \frac{\cos 5x}{5}\right] + C$

$-\frac{\cos 7x}{14} - \frac{\cos 5x}{10} + C$

pb $\int \cos 3x \cos 5x \, dx = ?$

$\cos 3x \cos 5x = \frac{1}{2}\left[\cos(3x+5x) + \cos(3x-5x)\right]$

$= \frac{1}{2}\left[\cos 8x + \cos(-2x)\right]$

$= \frac{1}{2}\left[\cos 8x + \cos 2x\right]$

$\int \frac{1}{2}\left[\cos 8x + \cos 2x\right] dx$

$\frac{1}{2}\left[\int \cos 8x \, dx + \int \cos 2x \, dx\right]$

$\frac{1}{2}\left[\int \frac{\cos 8x \, d8x}{8} + \int \frac{\cos 2x \, d2x}{2}\right]$

$\frac{1}{2}\left[\frac{\sin 8x}{8} + \frac{\sin 2x}{2}\right] + C$

$\frac{\sin 8x}{16} + \frac{\sin 2x}{4} + C$

pb

$\sin x$

$\frac{1}{2}$

eg

$$\int \sin x \sin 3x \, dx$$

$$\sin x \sin 3x = \frac{1}{2}\left[\cos(x-3x) - \cos(x+3x)\right]$$

$$= \frac{1}{2}\left[\cos(-2x) - \cos 4x\right]$$

$$= \frac{1}{2}\left[\cos 2x - \cos 4x\right]$$

$$\int \frac{1}{2}\left[\cos 2x - \cos 4x\right] dx$$

$$\frac{1}{2}\left[\int \cos 2x \, dx - \int \cos 4x \, dx\right]$$

$$\frac{1}{2}\left[\int \frac{\cos 2x \, d2x}{2} - \int \frac{\cos 4x \, d4x}{4}\right]$$

$$\frac{\sin 2x}{4} - \frac{\sin 4x}{8} + C$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

pb/ $\int \sin 6x \cos x \, dx$

$$\sin 6x \cos x = \frac{1}{2}\left[\sin(6x+x) + \sin(6x-x)\right]$$

$$= \frac{1}{2}\left[\sin 7x + \sin 5x\right]$$

$$\int \frac{1}{2}\left[\sin 7x + \sin 5x\right] dx$$

$$\frac{1}{2}\left[\int \sin 7x \, dx + \int \sin 5x \, dx\right.$$

$$\frac{1}{2}\left[\int \frac{\sin 7x \, d7x}{7} + \int \frac{\sin 5x \, d5x}{5}\right]$$

$$\frac{1}{2}\left[-\frac{\cos 7x}{7} - \frac{\cos 5x}{5}\right] + C$$

$$-\frac{\cos 7x}{14} - \frac{\cos 5x}{10} + C$$

pb/ $\int \cos 3x \cos 5x \, dx = ?$

$$\cos 3x \cos 5x = \frac{1}{2}\left[\cos(3x+5x) + \cos(3x-5x)\right]$$

$$= \frac{1}{2}\left[\cos 8x + \cos(-2x)\right]$$

$$= \frac{1}{2}\left[\cos 8x + \cos 2x\right]$$

$$\int \frac{1}{2}\left[\cos 8x + \cos 2x\right] dx$$

$$\frac{1}{2}\left[\int \cos 8x \, dx + \int \cos 2x \, dx\right]$$

$$\frac{1}{2}\left[\int \frac{\cos 8x \, d8x}{8} + \int \frac{\cos 2x \, d2x}{2}\right]$$

$$\frac{1}{2}\left[\frac{\sin 8x}{8} + \frac{\sin 2x}{2}\right] + C$$

$$\frac{\sin 8x}{16} + \frac{\sin 2x}{4} + C$$

pb/

sin

$\frac{1}{2}$

INTEGRATION of EXPONENTIAL FUNCTION

$$\int e^u\,du = e^u + c$$

ph

$$\int e^{3x}\,dx$$

$$\int \frac{e^{3x}\,d3x}{3}$$

$$\frac{e^{3x}}{3} + c$$

ph

$$\int x e^{x^2}\,dx \longrightarrow \int e^{x^2} x\,dx$$

$$dx^2 = 2x^{2-1}\,dx$$

$$= 2x\,dx$$

$$\therefore x\,dx = \frac{dx^2}{2}$$

$$\int e^{x^2} \frac{dx^2}{2}$$

$$\frac{1}{2}\int e^{x^2}\,dx^2$$

$$\frac{1}{2} e^{x^2} + c$$

$$\int e^{x^2} x\,dx$$

$$\int e^{x^2} \frac{dx^2}{2}$$

$$\frac{1}{2}\int e^{x^2} dx^2$$

$$\frac{1}{2} e^{x^2} + c$$

EXERCISE

INTEGRATE THE FOLLOWING EXPONENTIAL FUNCTIONS

(i) e^{-3x}

(ii) $2e^{4x}$

(iii) $5e^{-\frac{1}{3}x}$

(iv) $3xe^{x^2}$