

Composite Highway Bridge Design: Worked Examples



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Composite highway bridge design: Worked Examples

In accordance with Eurocodes and the UK National Annexes

(including Corrigendum, March 2014)

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FOREWORD

This publication is the second of two SCI bridge design guides that reflect the rules in the Eurocodes. It gives two worked examples, one for a multi-girder bridge and one for a ladder deck bridge. It is a companion to a publication giving general guidance on composite highway bridge design.

The guidance in this publication has been developed from earlier well-established guidance in a number of SCI bridge design guides. The previous guides referred to BS 5400 for the basis of design.

The publication was prepared by David Iles, of The Steel Construction Institute. A technical review of the examples, to confirm compliance with the Eurocode rules, was carried out by Atkins. Thanks are expressed to Chris Hendy, Rachel Jones and Jessica Sandberg, all of Atkins, for their comments.

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[†] This publication includes references to Corus, which is a former name of Tata Steel in Europe

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SUMMARY

This publication presents worked examples of the detailed design of two composite highway bridges. Each bridge is formed by steel girders acting compositely with a reinforced concrete deck slab. The first example is of multi-girder form, the second is of ladder-deck form. The examples cover the principal steps in the verification of the designs in accordance with the Eurocodes, as implemented by the UK National Annexes.

The publication is complementary to SCI publication P356, *Composite highway bridge design*, which describes both forms of construction and presents general guidance and an introduction to the relevant detailed requirements of the Eurocodes.

INTRODUCTION

This publication presents two worked examples of the design of composite highway bridges using beam and slab construction. The evaluations of design values of actions (loads), action effects (bending moments, shears, etc.) resistances (of cross sections and of members in buckling) and limiting SLS criteria are carried out in accordance with the Eurocodes, as implemented by the UK National Annexes. Reference is made to selected documents providing non-contradictory complementary information.

References are made in the right-hand margins of the sheets to relevant clauses of the Eurocode Part, National Annex or other document. For brevity, the Eurocodes are designated as, for example, '3-1-5', meaning BS EN 1993-1-5 and its National Annex. National Annex clause numbers are all prefixed 'NA'.

The two examples are:

1. A two-span integral bridge, each span 28 m, carrying a two-lane roadway. The reinforced concrete deck acts compositely with four main girders of constant depth. The example shows the calculation of action effects (from the results of a computer global analysis) and the verification of the main girders in bending and shear. The adequacy of a bolted splice in the main girders is verified. Fatigue assessment is carried out for certain key details.
2. A three-span ladder deck bridge, spans 24.5 m, 42 m, 24.5 m, also carrying a two-lane roadway. The reinforced concrete deck acts compositely with a ladder-deck configuration of two main girders, at 11.7 m centres, and cross girders at 3.5 m centres. The main girders are of variable depth. The example shows the calculation of action effects (from the results of a computer global analysis) and the verification of the main girders and cross girders in bending and shear. The adequacy of the bolted connection between main girders and cross girders is verified. Fatigue assessment is carried out for certain key details.

The detailed design of the deck slab, for local loading, is not covered in either example.

WORKED EXAMPLE 1:

Multi-girder two-span bridge with integral abutments

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Subject Example 1: Multi-girder two-span bridge
Section 1: Structural arrangement

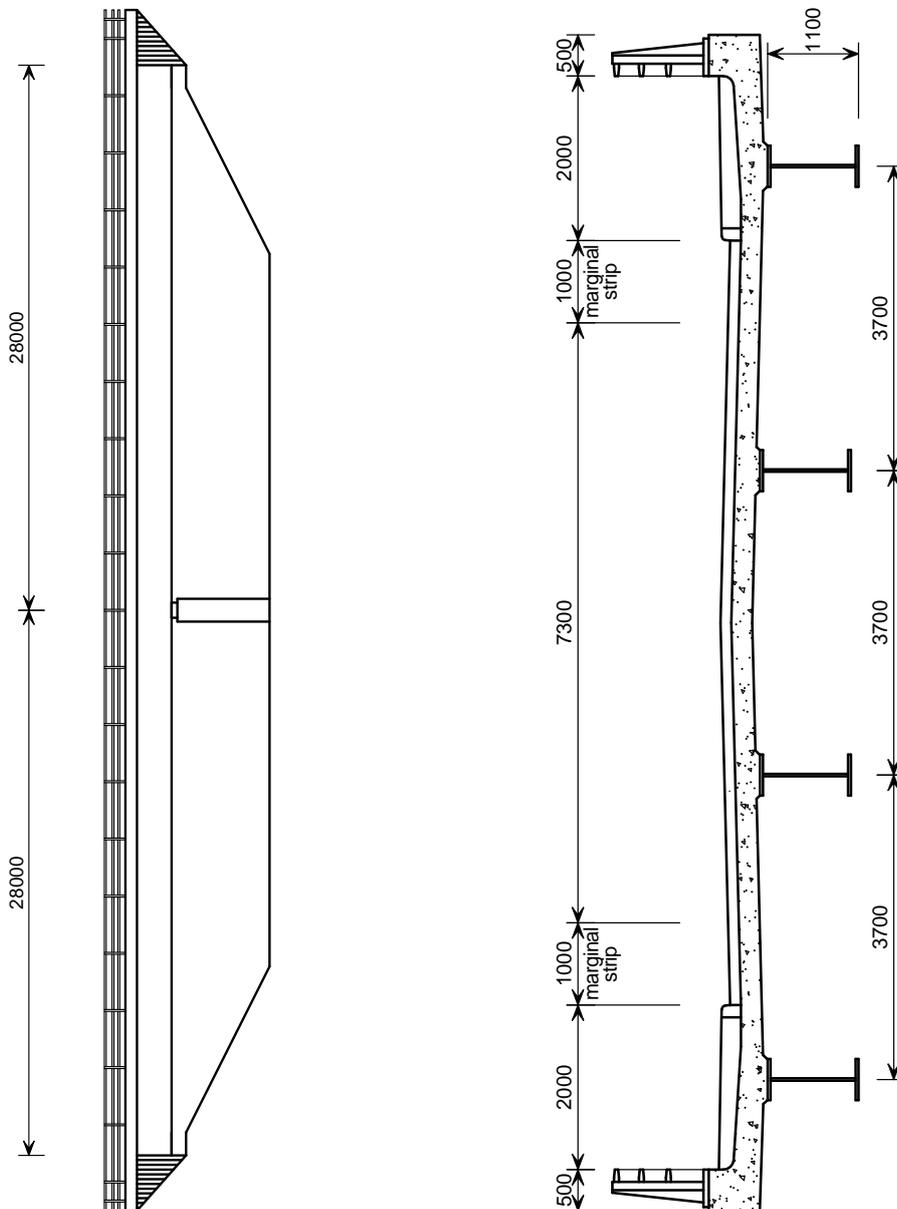
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1 Structural arrangement

The bridge carries a 2-lane single carriageway rural road over another road. The carriageway has 1.0 m wide marginal strips, in accordance with TD 27/05 and has a 2 m wide footway on either side (this width is slightly less than the width for footways given by TA 90/05). A four-girder arrangement has been chosen, and a deck slab thickness of 250 mm has been assumed. The deck cantilevers 1.6 m outside the centrelines of the outer girders; a 250 mm thick slab is likely to be adequate for this length, carrying footway loading or accidental traffic loading.

TD 27/05^[1]

TA 90/05^[2]





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2 Design basis

The bridge is to be designed in accordance with the Eurocodes, as modified by the UK National Annexes.

The basis of design set out in EN 1990 is verification by the partial factor method.

In this example the ultimate limit state STR/GEO is verified for persistent/transient design situations, using the combination of actions given by (6.10):

$$E \left(\sum \gamma_{G,j} G_{k,j} + \gamma_P P + \gamma_{Q,1} Q_{k,1} + \sum_{i>1} \gamma_{Q,i} \psi_{0,i} Q_{k,i} \right)$$

EN 1990
(6.10)

The fatigue limit state is verified for the reference stress range due to the application of the simplified fatigue load model (see below).

Section 3.3

Stresses in the structural steel, concrete and reinforcement are verified at the serviceability limit state for the characteristic combination of actions given by (6.14b)

$$E \left(\sum G_{k,j} + P + Q_{k,1} + \sum_{i>1} \psi_{0,i} Q_{k,i} \right)$$

EN 1990
(6.14b)

Crack widths in the deck slab are verified at the serviceability limit state for the quasi-permanent combination of actions given by (6.16b)

$$E \left(\sum G_{k,j} + P + \sum_{i \geq 1} \psi_{2,i} Q_{k,i} \right)$$

EN 1990
(6.16b)

2.1 Partial factors on actions

For persistent design situations, partial factors on actions at ULS are given by the NA:

<i>Permanent actions</i>		Unfavourable actions	Each of these actions is represented by a single characteristic value; factors for favourable actions would only be needed if the total effect of the action were favourable.
Concrete self weight	γ_G	1.35	
Steel self weight	γ_G	1.20	
Super-imposed dead	γ_G	1.20	
Road surfacing	γ_G	1.20	
Weight of soil	γ_G	1.35	
Self weight of other materials	γ_G	1.35	
Variable actions			
Road traffic actions	γ_Q	1.35	
Pedestrian actions (gr3, gr4)	γ_Q	1.35	
Wind actions	γ_Q	1.70	
Thermal actions	γ_Q	1.55	

BS EN 1990
NA.A2.4(B)

1-1-1/5.1(1)

No values are given for transient situations (such as during construction) but it is assumed that the above factors for permanent actions may be used.

The partial factor on shrinkage γ_{sh} is set at unity for both ULS and SLS by EN 1992-1-1.

2-1-1/2.4.2.1



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2.2 Factors for combination values

Factors for combination values of actions are given by the NA to BS 1990 as:

	ψ_0	ψ_1	ψ_2
LM1 - TS	0.75	0.75	0
LM1 - UDL	0.75	0.75	0
Footway loads with LM1	0.4	0.4	0
LM2 Single Axle	0	0.75	0
Horizontal Forces	0	0	0
gr5 vertical forces from SV vehicles	0	0	0
Wind - persistent design situations	0.5	0.2	0
Wind during execution	0.8	-	0
Wind during execution (Fw*)	1	-	0
Thermal actions	0.6	0.6	0.5

BS EN 1990
NA.A2.1

2.3 Factors on strength

The values of the various γ_M partial factors are given by the NA to BS EN 1993-2 as:

	ULS	SLS
γ_{M0}	1.00	
γ_{M1}	1.10	
γ_{M2}	1.25	
γ_{M3}	1.25	1.10

3-2/6.1,
NA.2.17

The values of the partial factors for strength of concrete and reinforcement at ULS are given by the NA to BS EN 1992-1-1 as $\gamma_c = 1.5$ and $\gamma_s = 1.15$.

2-1-1/2.4.2.4,
Table NA.1

2.4 Structural material properties

It is assumed that the following structural material grades will be used:

- Structural steel: S355 to EN 10025-2
- Concrete: C40/50 to EN 206-1
- Reinforcement: B500 to EN 10080 and BS 4449

For structural steel, the value of f_y depends on the product standard.

(Use 355 N/mm² for $t \leq 16$ mm; 345 N/mm² for $16 \text{ mm} > t \leq 40$ mm; and 335 N/mm² for $t > 40$ mm)

For concrete, $f_{ck} = 40$ MPa

For reinforcement $f_{yk} = \text{N/mm}^2$

The modulus of elasticity of both structural steel and reinforcing steel is taken as 210 GPa (as permitted by EN 1994-2).

NA to 3-1-1
2-1-1,
Table 3.1

4-2/3.2



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The modulus of elasticity of the concrete is given by EN 1992-1-1 as:

$E_{cm} = 35$ GPa. This 28-day value will be used for determination of all short-term effects and resistances and the modular ratio is thus

$$n_0 = 210/35 = 6.0$$

For long-term effects, the modular ratio is given by 4-2/5.4.2.2 as:

$$n_L = n_0(1 + \psi_L \varphi_t)$$

For the evaluation of the creep coefficient $\varphi_t (= \varphi(t, t_0)$ in 2-1-1/B.1) it is assumed that the first loading is applied at an average age of $t_0 = 21$ days and that the relative humidity is 70%.

$$\text{For } t \rightarrow \infty \quad \varphi(t, t_0) = \varphi_0$$

$$\text{Where } \varphi_0 = \varphi_{RH} \beta(f_{cm}) \beta(t_0)$$

For $f_{cm} > 35$ MPa (here $f_{cm} = 48$ MPa, from 2-1-1/Table 3.1)

$$\varphi_{RH} = \left[1 + \frac{1 - RH / 100}{0.1 \sqrt[3]{h_0}} \alpha_1 \right] \alpha_2$$

$$\alpha_1 = \left[\frac{35}{f_{cm}} \right]^{0.7} = \left[\frac{35}{48} \right]^{0.7} = 0.802$$

$$\alpha_2 = \left[\frac{35}{f_{cm}} \right]^{0.2} = \left[\frac{35}{48} \right]^{0.2} = 0.939$$

For a 250 thick slab ($h_0 = 250$)

$$\varphi_{RH} = \left[1 + \frac{1 - 70 / 100}{0.1 \sqrt[3]{250}} 0.802 \right] 0.939 = 1.298$$

$$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}} = \frac{16.8}{\sqrt{48}} = 2.42$$

$$\beta(t_0) = \frac{1}{(0.1 + t_0^{0.20})} = \frac{1}{(0.1 + 21^{0.20})} = 0.516$$

Hence

$$\varphi_0 = 1.298 \times 2.42 \times 0.516 = 1.621$$

For permanent loads, $\psi_L = 1.1$ and thus:

$$n_L = 6.0(1 + 1.621 \times 1.1) = 6.0 \times 2.79 = 16.7$$

$$\text{Long term modulus} = 210/16.7 = 12.6 \text{ GPa}$$

2-1-1/
Table 3.1

4-2/5.4.2.2

2-1-1/B.1

2-1-1/
Table 3.1
2-1-1/(B.3a)

2-1-1/(B.8c)

2-1-1/(B.5)

4-2/5.4.2.2(2)



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If the design effects need to be determined at the time of opening, the creep coefficient will need to be modified to reflect the short duration of loading. In this example, the average age at which the permanent actions are imposed is 21 days and the age at opening to traffic is 56 days. The creep coefficient is modified by the parameter $\beta_c(t, t_0)$ and in this case, with $t - t_0 = 35$ days, $\beta_c = 0.418$

2-1-1/ (B.7)

Thus, for permanent loads at opening:

$$n_L = 6.0(1 + 0.418 \times 1.621 \times 1.1) = 6.0 \times 1.75 = 10.5$$

And modulus = $210/10.5 = 20.0$ GPa

Shrinkage

The shrinkage strain on the concrete deck and the appropriate modular ratios are given by EN 1992-2. The values depend on the age since casting; in this example two ages are considered - at bridge opening, for which an average age of 56 days is assumed, and at the end of the design life, for which it is assumed that $t = \infty$.

4-2/5.4.2.2(4)

For shrinkage, the age at loading (i.e. at age $t_s = 1$, the beginning of drying shrinkage in 2-1-1/3.1.4) is assumed to be one day.

The autogenous shrinkage strain at $t = \infty$ is:

2-1-1/3.1.4

$$\varepsilon_{ca}(\infty) = 2.5(f_{ck} - 10) \times 10^{-6} = 2.5(40 - 10) \times 10^{-6} = 7.5 \times 10^{-5}$$

At $t = 56$ days, the strain is given by:

$$\varepsilon_{ca}(t) = \beta_{as}(t) \varepsilon_{ca}(\infty)$$

$$\text{Where } \beta_{as}(t) = 1 - \exp(-0.2t^{0.5}) = 1 - e^{-1.5} = 0.777$$

$$\text{Thus } \varepsilon_{ca}(56) = 0.777 \times 7.5 \times 10^{-5} = 5.8 \times 10^{-5}$$

The drying shrinkage depends on the nominal unrestrained drying shrinkage, given by expression (B.11) in B.2 (or by interpolation in Table 3.2 of EN 1992-1-1).

2-1-1/B.2

$$\varepsilon_{cd,0} = 0.85 \times [220 + 110 \times \alpha_{ds1} \times \exp(-\alpha_{ds2} f_{cm}/f_{cmo}) \times \beta_{RH}] \times 10^{-6}$$

2-1-1/(B.11)

For 70% relative humidity, $f_{ck} = 40$ MPa and class N cement:

$$f_{cmo} = 10, \alpha_{ds1} = 4 \text{ and } \alpha_{ds2} = 0.12$$

2-1-1/(B.12)

$$\beta_{RH} = 1.55[1 - (RH/100)^3] = 1.55[1 - 0.7^3] = 1.018$$

$$\varepsilon_{cd,0} = 0.85 \times [220 + 110 \times 4 \times \exp(-0.12 \times 40/10) \times 1.018] \times 10^{-6} = 32 \times 10^{-5}$$

The drying shrinkage at time t is given by:

$$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) k_h \varepsilon_{cd,0}$$

2-1-1/(3.10)

$$\text{Where } k_h = 0.80 \text{ (from Table 3.3, with } h_0 = 250) \text{ and } \beta_{ds}(t, t_s) = \frac{t - t_s}{t - t_s + 0.04\sqrt{h_0^3}}$$



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For $t = 56$ and $t_s = 1$ (see 4-2/5.4.2.2(4)), $\beta_{ds} = 0.258$

For $t = \infty$, $\beta_{ds} = 1$

Thus the drying shrinkage is:

At $t = 56$ days $\epsilon_{cd} = 0.258 \times 0.80 \times 32 \times 10^{-5} = 6.60 \times 10^{-5}$

At $t = \infty$ $\epsilon_{cd} = 0.80 \times 32 \times 10^{-5} = 25.6 \times 10^{-5}$

The total shrinkage is thus:

At $t = 56$ days $\epsilon_{cd} = 5.8 \times 10^{-5} + 6.60 \times 10^{-5} = 12.4 \times 10^{-5}$

At $t = \infty$ $\epsilon_{cd} = 7.5 \times 10^{-5} + 25.6 \times 10^{-5} = 33.1 \times 10^{-5}$

For the modular ratio, the creep factor is calculated as for long term loading but the age at first loading is assumed to be 1 day. Thus:

4-2/5.4.2.2(4)

$$\beta(t_0) = \frac{1}{(0.1 + t_0^{0.20})} = \frac{1}{(0.1 + 1^{0.20})} = 0.91$$

2-1-1/(B.5)

The final creep coefficient is calculated as above for long term effects but with $\beta(t_0) = 0.91$, and thus $\varphi_0 = 1.298 \times 2.42 \times 0.91 = 2.86$

For shrinkage, $\psi_L = 0.55$ and thus:

4-2/5.4.2.2(2)

$n_L = 6.0(1 + 2.86 \times 0.55) = 6.0 \times 2.57 = 15.4$

At opening to traffic ($t = 56$ days) the creep coefficient is modified by the parameter $\beta_c(t, t_0)$ and in this case $\beta_c = 0.475$ and $n_L = 10.5$.

2-1-1/(B.7)

In this example, the shrinkage effects will be taken into account at their long term values where they are unfavourable. Where the effects are favourable, the lesser values at 56 days could be considered but it is conservative to neglect shrinkage in that case.



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3 Actions on the bridge

3.1 Permanent actions

Self weight of structural elements

The 'density' of steel is taken as 77 kN/m³ and the density of reinforced concrete is taken as 25 kN/m³. The self weights are based on nominal dimensions.

1-1-1/Table A.1

Self weight of surfacing

The total nominal thickness of the surfacing, including waterproofing layer is 130 mm. Assume that the 'density' is 23 kN/m³ for the whole thickness.

The self weight generally produces adverse effects and for that case the self weight is based on nominal thickness +55%. Thus:

1-1-1/Table NA.1

$$g_k = 1.55 \times 0.13 \times 23 = 4.63 \text{ kN/m}^2$$

Self weight of footway construction

The nominal thickness of the footway (comprising concrete fill and a thin surfacing) is 200 mm and a uniform density of 24 kN/m³ is assumed. The self weight is based on the nominal dimensions and thus:

$$g_k = 1.0 \times 0.2 \times 24 = 4.80 \text{ kN/m}^2$$

Self weight of parapets

A nominal value of 2 kN/m is assumed for each parapet.

Self weight of soil

The density of the granular fill behind the integral abutments is taken as 21 kN/m³.

3.2 Construction loads

Construction loads are classed as variable loads.

1-1-6/2.2

For global analysis, a uniform construction load of $Q_{ca} = 0.75 \text{ kN/m}^2$ is assumed during casting and the weight of temporary formwork is assumed to be $Q_{cc} = 0.50 \text{ kN/m}^2$. Additionally, wet concrete is assumed to have a density of 1 kN/m³ greater than that of hardened concrete; for a slab thickness of 250 mm this adds $Q_{cf} = 0.25 \text{ kN/m}^2$

1-1-6/
Table 4.2

The total construction load is thus: $Q_c = 0.75 + 0.50 + 0.25 = 1.5 \text{ kN/m}^2$

3.3 Traffic loads

Road traffic

Normal traffic is represented by Load Model 1 (LM1).

For the road carried by this bridge, the highway authority specifies that abnormal traffic be represented by special vehicle SV100, as defined in the UK National Annex.

1-2/
NA.2.16.1.2

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<p>Pedestrian traffic</p> <p>Pedestrian traffic is represented by the reduced value given by the NA to BS EN 1991-2, Table NA.3 and clause NA.2.36. Thus $0.6q_{fk}$ is applied ($= 0.6 \times 5.0 = 3 \text{ kN/m}^2$). The reduction for longer loaded lengths is not made.</p> <p>Fatigue loads</p> <p>For fatigue assessment, Fatigue Load Model 3 (FLM3), defined in 1-2/4.6.4, is used, as recommended by 3-2/9.2.2</p> <p>3.4 Thermal actions</p> <p>Shade temperatures</p> <p>Maximum and minimum shade air temperatures for the UK, for a 50-year return period are defined in EN 1991-1-5 NA.2.20. For this bridge location, the values are: Maximum 33°C Minimum -17°C</p> <p>Thermal range of effective bridge temperature (for determination of soil pressures)</p> <p>For the purposes of determining soil pressure, the total range of effective bridge temperature for a 50 year return period is relevant, not the range from an assumed initial restraint position.</p> <p>The values of maximum/ minimum uniform bridge temperatures are given by EN 1991-1-5, 6.1.3.1; these are referred to as $T_{e,\min}$ and $T_{e,\max}$</p> <p>For Type 2 deck (concrete slab on steel girders)</p> $T_{e,\max} = T_{\max} + 4 \text{ (EN 1991-1-5, Figure 6.1)}$ $T_{e,\min} = T_{\min} + 5$ <p>Hence the total range = $(33 + 4) - (-17 + 5) = 49^\circ\text{C}$</p> <p><i>(The adjustments for surfacing thickness over 100 mm given by the NA would result in a small reduction to the range and have been neglected.)</i></p> <p>Thermal range (for determination of extreme value of thermal movement)</p> <p>For determination of the maximum movement at ULS, the values for a 120 year design life are relevant but according to EN 1990:A2, these are determined by applying $\gamma_Q = 1.55$ to characteristic values for a 50 year return period.</p> <p>For change of length in composite sections, the coefficient of linear thermal expansion is 12×10^{-6} per $^\circ\text{C}$.</p> <p>Vertical temperature difference</p> <p>The vertical temperature difference given in EN 1991-1-5, Table 6.2b will be used and temperature difference will be considered to act simultaneously with uniform temperature change, as recommended in NA.2.12, if that is more onerous. For surfacing thickness other than 100 mm, interpolate in 1-1-5/Table B.2, as follows:</p>		<p>1-2/Table NA.3</p> <p>1-1-5/NA.2.20</p> <p>PD 6694-1^[3] clause 7.4.2 (draft)</p> <p>1-1-5/NA.2.4</p> <p>Table NA.A2.4(B) Note 6</p> <p>4-2/5.4.2.5</p>	



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CALCULATION SHEET

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Surfacing thickness (mm)	ΔT_1 for slab thickness (mm)	
	200	300
100	13	16
150	10.5	12.5

Interpolating for slab thickness 250 mm, surfacing thickness 130 mm, gives $\Delta T_1 = 12.7^\circ$.

(The 55% increase over nominal thickness, where surfacing load is adverse, is ignored.)

For temperature difference in composite sections, the coefficient of thermal expansion is 10×10^{-6} per $^\circ\text{C}$. 4-2/5.4.2.5

3.5 Geotechnical actions

Design values of thermal movements giving rise to geotechnical actions

At the ULS that designs the superstructure, traffic loads are the leading action and thermal actions may be considered as an accompanying action. In that case the movement (from mean position) is:

$$(1.2 \times 10^{-5} \times 28000 \times 49/2) \times \gamma_Q \times \psi_0 = 8.23 \times 1.55 \times 0.60 = 7.65 \text{ mm}$$

For maximum axial force due to restraint of temperature, the thermal action would be the leading action and the movement (from mean position) would be:

$$8.23 \times \gamma_Q = 8.23 \times 1.55 = 12.8 \text{ mm.}$$

Accompanying LM1 traffic loads would be at 75% of their value as a leading action (since $\psi_1 = 0.75$ for gr1a). (LM3 is not considered as an accompanying action.) Sheet 3

Soil pressure coefficients

Soil pressure coefficient K^*

To determine the maximum soil pressure on the endscreen wall, PD 6694-1 requires the value of the total thermal movement range, the height of the wall above the pivot point and the K_0 value for the soil. PD 6694-1^[3]
(draft)

The PD gives:

$$d = \alpha L_x (T_{e,\max} - T_{e,\min})$$

NOTE - The PD does not refer to characteristic values or to frequent values, it simply ignores the design basis; it could be argued that it is the frequent value as a leading action that determines the pressure coefficient, in which case the partial factor γ is unity and the factor ψ_1 should be applied ($\psi_1 = 0.6$ according to the UK NA)

Here, following the PD

$$d = 1.2 \times 10^{-5} \times 28000 \times 49 = 16.5 \text{ mm}$$

(i.e. ± 8.25 about the mean position)



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For the present configuration, where the piles are much more flexible than the deck, the movement is essentially translational and thus the expression for K^* for movement by sliding is more applicable.

Thus the multiplier on K_p is $(40 \times 16.5/2250)^{0.4} = 0.612$

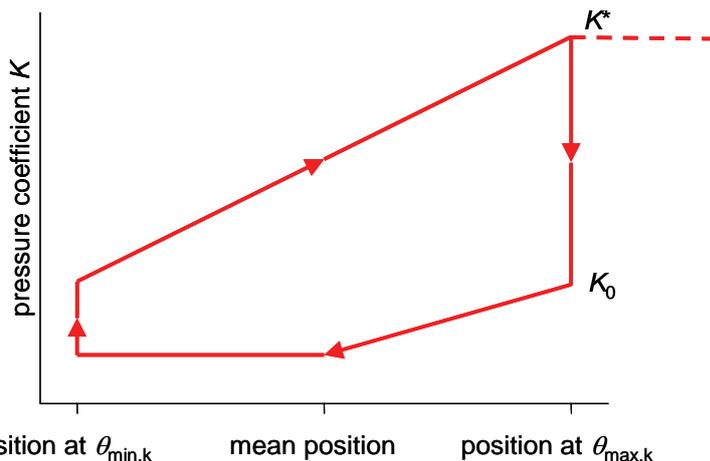
And thus $K^* = K_0 + 0.612K_p$

Assume for a granular fill that $K_0 = 0.5$ and $K_p = 4.6$ (K_p from PD) and apply modelling factor $\gamma_{sd;K}$ to K_p .

$K^* = 0.5 + 0.612 \times (4.6 \times 1.2) = 3.88$ (not more than K_p - OK)

Movement - pressure coefficient diagram

At any time, the soil pressure coefficient for characteristic values of actions lies within the envelope shown diagrammatically below.



The most unfavourable pressure, in terms of the greatest horizontal force in the deck, is given by the pressure coefficient K^* . This value applies at the characteristic value of thermal expansion from the mean position. At the ULS design value of expansion, the displacement is greater but the value of K^* may still be used (see PD 6694-1).

When thermal expansion is an accompanying action at ULS, the movement from the mean position is γ_Q/ψ_0 times the characteristic value = $1.55 \times 0.6 = 0.93$ times characteristic. The value of $K^* = 3.88$ will be used here for both leading and accompanying thermal actions.

Vertical soil pressures

Traffic surcharge loading does not need to be considered in conjunction with K^* pressures (see PD 6694-1, clause 7.5.1).

	γ_6 or γ_0	Characteristic value kN/m ²	Design value (ULS) kN/m ²
Road surfacing	1.2	4.63	5.6
Soil (at 2.25 below top of wall)	1.35	2.25×21	63.8
Total at top of wall			5.6
Total at bottom of wall			69.4

PD 6994-1,
7.4.6

PD 6994-1,
7.4.5

PD 6694-1
(draft)

PD 6694-1
(draft)



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Horizontal soil pressures

The design value of horizontal pressure at ULS when thermal actions is either a leading or an accompanying action is:

Pressure

At top of slab	5.6×3.88	=	22 kN/m ²
At bottom of wall	69.4×3.88	=	269 kN/m ²

The pressure is applied to the end diaphragms as a hydrostatic pressure.

AT SLS, the movement is 0.6 times characteristic, so the pressure coefficient is:

$$K_0 + (K^* - K_0) \times 1.6/2.0 = 0.5 + 3.38 \times 0.8 = 3.21 (= 83\% \text{ of } K^*)$$

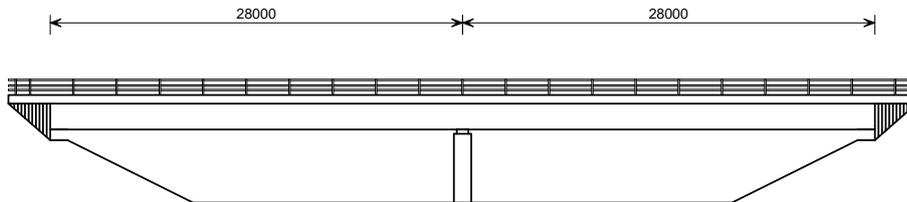


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CALCULATION SHEET

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4 Girder make-up and slab reinforcement

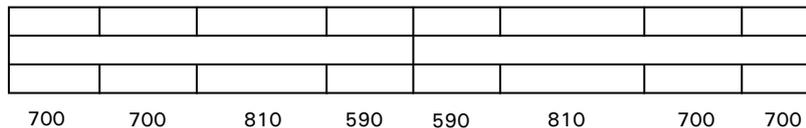


	21.7 m span girder		12.6 m pier girder	21.7 m span girder	
Top flange	500 × 40		500 × 40	500 × 40	
Web	12	10	14	10	12
Bottom flange	500 × 40		600 × 60	500 × 40	
Top rebars	B16 @ 150 mm crs		B25 @ 150 mm	B16 @ 150 mm crs	
Bottom rebars	B16 @ 150 mm crs		B25 @ 150 mm	B16 @ 150 mm crs	

The overall girder depth is 1100 mm, as shown in Section 1 of this example.

The cover to the top longitudinal bars is 55 mm (35 mm + 20 mm transverse bars); this is appropriate to XC3. The cover to the bottom bars is 60 mm (40 mm + 20 mm); this is appropriate to XC4. (See 2-1-1/4.4.1.2 for minimum cover and also 2-2/4.2 and the respective NAs.)

Bracing arrangements



The above bracing arrangements are nominal and might be adjusted during detailed design. The model nodes and intermediate bracings other than either side of the central support coincide with nodes in the FE model. The splice positions coincide with model nodes; the bracings are 400 mm closer to the supports than the splice positions; these positions might also be adjusted during final design.



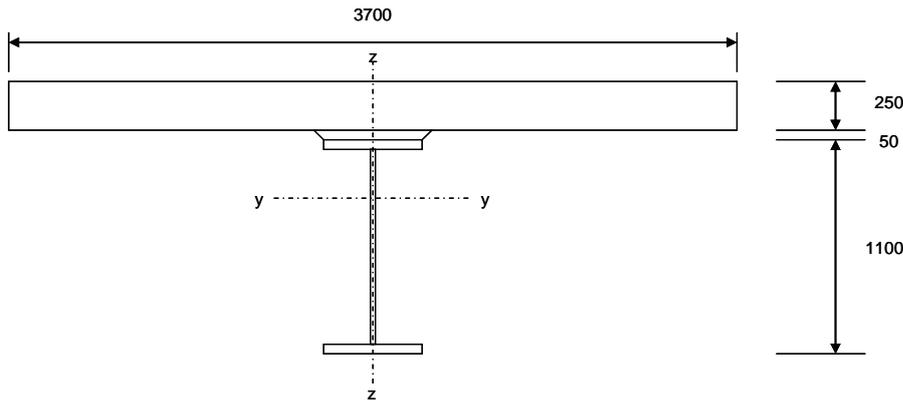
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CALCULATION SHEET

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5 Beam cross sections

5.1 Section properties (internal main girders)



Gross section properties are needed for global analysis. For section analysis, consider the effective section, allowing for shear lag:

The equivalent spans for effective width are:

Abutment and midspan sections: $L_c = 0.85 L_1 = 0.85 \times 28 = 23.8 \text{ m}$

Hogging section: $L_c = 0.25 (L_1 + L_2) = 0.25 \times 56 = 14.0 \text{ m}$

At mid-span, $b_{\text{eff}} = b_0 + \sum b_{\text{ei}}$

where $b_{\text{ei}} = L_c / 8$ each side, but not more than geometric width

$b_{\text{ei}} = 23800 / 8 = 2975 \text{ mm}$, so the section is fully effective,

At the abutment $b_{\text{eff}} = b_0 + \sum \beta_i b_{\text{ei}}$

where $\beta_i = (0.55 + 0.025 L_c / b_{\text{ei}}) \leq 1$

Here, assuming the width between shear studs is 400 mm (i.e. $b_i = 1650 \text{ mm}$)

$\beta_i = (0.55 + 0.025 \times 23800 / 1650) = 0.91$ and thus:

$b_{\text{eff}} = 400 + 2 \times 1650 \times 0.91 = 3403 \text{ mm}$

At the pier, $b_{\text{ei}} = 1400 / 8 = 1750 \text{ mm}$ each side, so the section is fully effective.

Properties for gross sections (which are also the effective sections) at the pier and in the span are tabulated below. Values for the abutment are not shown.

4-2/Figure 5.2

4-2/(5.3)

4-2/(5.4)



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Bare steel cross sections

		Span girder	Pier girder	
Area	A	50200	70000	(mm ²)
Height of NA		550	436	(mm)
Second moment of area	I _y	1.212E+10	1.562E+10	(mm ⁴)
Elastic modulus, centroid top flange	W _{tf,y}	2.287E+07	2.425E+07	(mm ³)
Elastic modulus, centroid bottom flange	W _{bf,y}	2.287E+07	3.847E+07	(mm ³)
Section class		4*	3 (hogging)	
Plastic bending resistance	M _{pl}	8237	9882	(kNm)

* The section is only marginally Class 4. For stress build-up during construction the bare steel section may be treated as Class 3, since the final composite section is Class 3 or better. See sheet 31 for moduli of the effective section at the wet concrete stage.

Note: As an example, the classification for the span girder is as follows:

Flange outstand $c = (500 - 10)/2 = 245$ mm (welds neglected)

and thus $c/t = 245/40 = 6.12$

Outstand limit for class 1 is $c/t \geq 9\varepsilon = 9 \times \sqrt{235/345} = 7.5$, so flange is class 1

Depth of web $c = 1100 - 2 \times 40 = 1020$ mm and thus $c/t = 1020/10 = 102$

Limit for class 3 internal part is $c/t \geq 124\varepsilon = 124 \times \sqrt{235/355} = 100.2$, so web is class 4

Properties calculated by spreadsheet

3-1-1/
Table 5.2(2)

3-1-1/
Table 5.2(1)

Composite cross sections (short term) - sagging ($n_0 = 6.0$)

		Span girder	Pier girder	
Area	A	209800		(mm ²)
Height of NA		1098	1016	(mm)
Second moment of area	I _y	3.288E+10	5.040E+10	(mm ⁴)
Elastic modulus, top of slab	W _c	6.534E+08		(mm ³)
Elastic modulus, centroid top flange	W _{tf,y}	1.827E+09		(mm ³)
Elastic modulus, centroid bottom flange	W _{bf,y}	3.050E+07		(mm ³)
Plastic bending resistance	M _{pl}	13070		(kNm)

The cross section of the span girder is class 1, provided that the top flange is restrained by shear connectors within the spacing limits in 4-2/6.6.5.5 (in this case, max spacing 730 mm, max edge distance 299 mm).

Uncracked pier girder section properties are needed for calculation of shear flow.

Value of M_{pl} calculated using f_y/γ_{M0} values for steel, 0.85f_{ck}/γ_C for concrete

Composite cross sections (long term) - sagging ($n_L = 16.7$)

		Span girder		
Area	A	107500		(mm ²)
Height of NA		934		(mm)
Second moment of area	I _y	2.634E+10		(mm ⁴)
Elastic modulus, top of slab	W _c	9.439E+08		(mm ³)
Elastic modulus, centroid top flange	W _{tf,y}	1.804E+08		(mm ³)
Elastic modulus, centroid bottom flange	W _{bf,y}	2.882E+07		(mm ³)



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Composite cross sections (long term shrinkage) - sagging ($n_L = 15.4$)

		Span girder		
Area	A	112400		(mm ²)
Height of NA		948		(mm)
Second moment of area	I_y	2.684E + 10		(mm ⁴)
Elastic modulus, top of slab	W_c	9.145E + 08		(mm ³)
Elastic modulus, centroid top flange	$W_{tf,y}$	2.033E + 08		(mm ³)
Elastic modulus, centroid bottom flange	$W_{bf,y}$	2.892E + 07		(mm ³)

Cracked composite sections - hogging (cracked)

		Span girder	Pier girder	
Area	A	74450	94250	(mm ²)
Height of NA		788	653	(mm)
Second moment of area	I_y	2.092E + 10	2.845E + 10	(mm ⁴)
Elastic modulus, top rebars	W	3.806E + 07	4.184E + 07	(mm ³)
Elastic modulus, centroid top flange	$W_{tf,y}$	7.164E + 07	6.663E + 07	(mm ³)
Elastic modulus, centroid bottom flange	$W_{bf,y}$	2.724E + 07	4.567E + 07	(mm ³)
Section class		3	3	
Plastic bending resistance	M_{pl}		16990	(kNm)

Value of M_{pl} calculated using f_y/γ_{M0} values for steel, f_{yk}/γ_S for rebar

5.2 Primary effects of temperature difference & shrinkage

Temperature difference

For calculation of primary effects, use the short-term modulus for concrete:

$E_{cm} = 35 \text{ GPa}$ (For steel, $E = 210 \text{ GPa}$)

Note: For each element of section, calculate stress as strain \times modulus of elasticity, then determine force and centre of force for that area.

For a fully restrained section, the restraint force and moment in the span girder, inner beam, due to the characteristic values of temperature difference noted on Sheet 8 are:

	Av strain	Force (kN)	Centre of force		Moment (kNm)
			Below top	Above NA	
Top part of slab	0.000084	1632	62	240	392
Bottom part of slab	0.000036	466	198	104	48
Haunch	0.000030	34	274	28	1
Top flange	0.000026	109	320	-18	-2
Web (to 400 below slab)	0.000012	5	410	-108	-1
		<u>2246</u>			<u>438</u>

Sheet 3



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CALCULATION SHEET

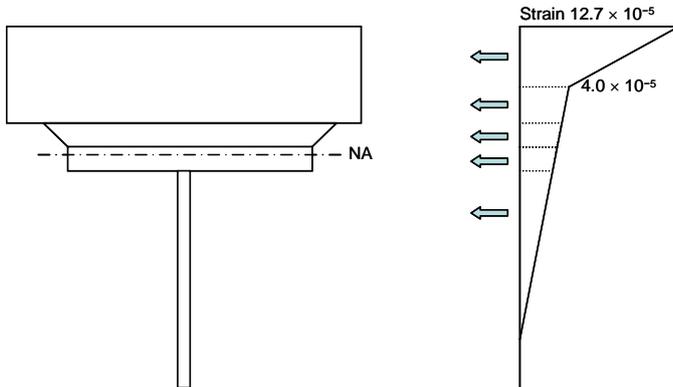
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Job Title **Composite highway bridges: Worked examples**

Subject **Example 1: Multi-girder two-span bridge**
Section 5: Beam cross sections

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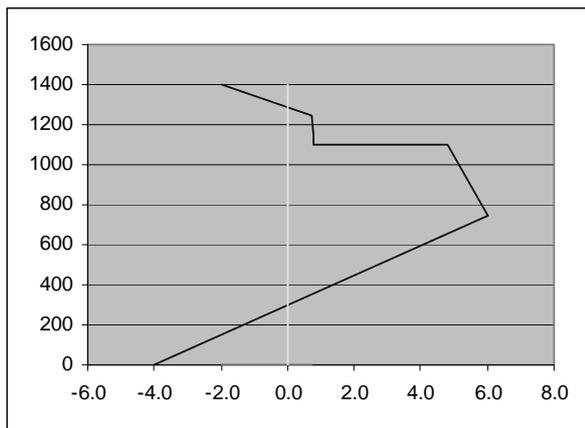
The strains and forces are illustrated diagrammatically below:



Hence the primary effects (stresses) are given by:

	W (steel units)	Restraint	Release of restraint		Total
			Bending	Axial	
Top of slab	1.09E+08	-4.4	0.7	1.8	-1.9
0.6 into slab	2.16E+08	-1.4	0.3	1.8	0.7
bottom of slab	6.32E+08	-1.1	0.1	1.8	0.8
bottom of haunch	1.64E+10	-1.0	0.0	1.8	0.8
Top of top flange	1.64E+10	-5.9	0.0	10.7	4.8
400 below slab	-9.45E+07	0.0	-4.6	10.7	6.1
bottom flange	-2.99E+07	0.0	-14.6	10.7	-3.9

Diagrammatically:



The release of the restraint moments is applied along the span, in the uncracked regions, as a separate loadcase, to determine the secondary effects of vertical temperature difference.

Note that the omission of restraint moments in cracked regions is not mentioned in EN 1994-2 but the view has been taken that the omission permitted for shrinkage (see EN 1994-2, 5.4.2.2(8)) may be used for the calculation of secondary effects of temperature difference.



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Shrinkage

For complete verification, shrinkage effects should be calculated at the time of opening to traffic and at the end of the service life and the more onerous values used. Here, primary and secondary effects are calculated only for the long-term situation (the values are greater than those at opening) and where the total effects of shrinkage are advantageous, they are neglected.

The characteristic value of shrinkage strain is given on Sheet 6 as $\epsilon_{cd} = 33.1 \times 10^{-5}$ and the modular ratio is $\eta_L = 15.4$. This is very close to the value for long-term effects generally and for determining the secondary effects, the long-term properties will be used for both.

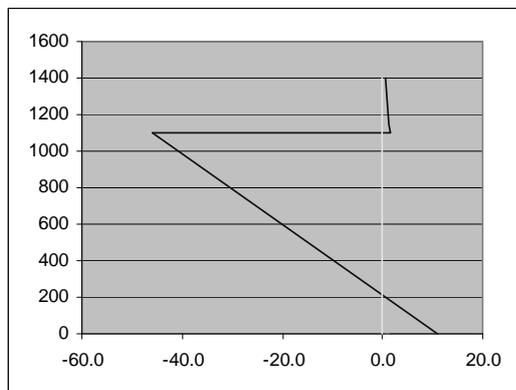
For a fully restrained section, the restraint force and moment in the span girder, inner beam, due to the characteristic values of shrinkage strain are given by:

	Strain	Force (kN)	Centre of force		moment (kNm)
			Below top	Above NA	
Slab	-0.000331	-4164	125	327	-1362
Haunch	-0.000331	-146	275	177	-26
		<u>-4310</u>			<u>-1388</u>

Hence the primary effects are:

	W (steel units)	Restraint	Release of restraint		Total
			Bending	Axial	
Top of slab	5.93E+07	4.5	-1.5	-2.5	0.5
bottom of slab	1.33E+08	4.5	-0.7	-2.5	1.3
bottom of haunch	1.76E+08	4.5	-0.5	-2.5	1.5
Top of top flange	1.76E+08	0.0	-7.9	-38.4	-46.3
bottom flange	-2.83E+07	0.0	49.0	-38.4	10.6

Diagrammatically:



The release of the restraint moments is applied along the span, in the uncracked regions, as a separate loadcase, to determine the secondary effects of shrinkage.

4-2/
5.4.2.2(8)



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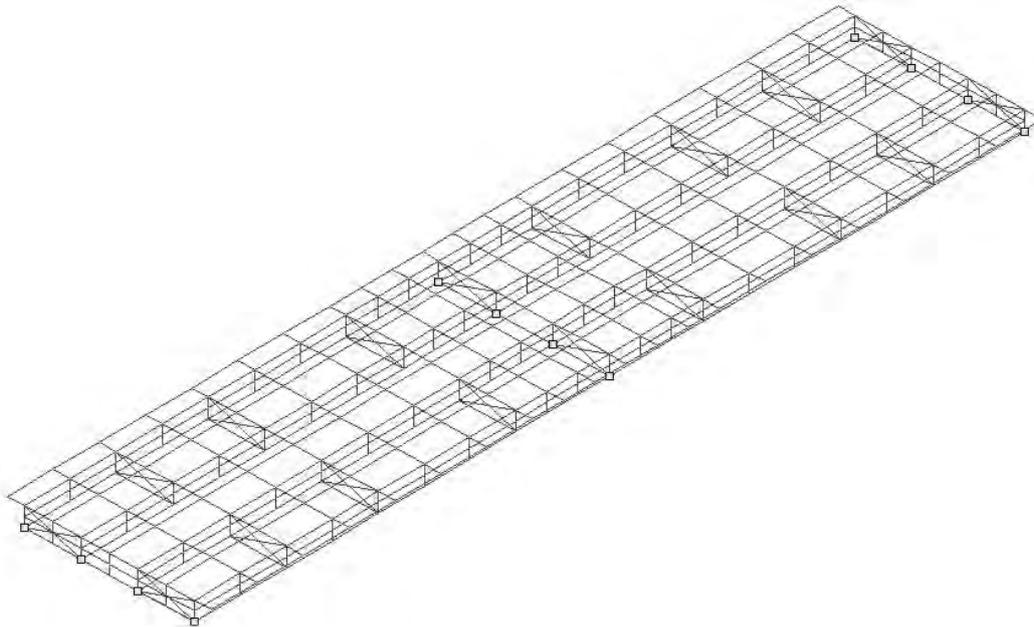
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6 Global analysis

6.1 3D FE model

A 3D model of the structure was created, with FE elements for the deck slab and for the girder webs, and with beam elements for the girder flanges and for the RC edge beams.



Concrete diaphragms were provided at both abutments, with vertical support only (soil pressures were applied as equal and opposite hydrostatic pressures at the two ends).

For the cracked regions over the intermediate support (15% of each span), the slab elements were given anisotropic properties (cracked stiffness longitudinally, uncracked stiffness transversely).

The use of FE elements means that shear lag does not need to be explicitly allowed for, since shear lag effects are taken into account in the analysis.

The results of the analysis, in terms of stresses in all the elements, are converted by the software into equivalent forces and moments on longitudinal composite beams (each comprising a steel girder and a width of slab). In general this means that, even without the application of external horizontal forces, the effects on the individual composite beams include axial forces as well as moments. This is a consequence of the 3D behaviour and the verification of the composite beams must take account of these axial forces.

4-2/5.4.2.3



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6.2 Construction stages

It is presumed that the deck will be concreted in two stages - the whole of span 1, followed by the whole of span 2. The edge beams will be concreted after span 2. Separate analytical models are therefore provided for:

- Stage 1 All steelwork, wet concrete in span 1
- Stage 2 Composite structure in span 1 (long-term properties), wet concrete in span 2
- Stage 3 Composite structure in both spans (long-term properties)
- Stage 4 Composite structure (short term properties)

(For simplicity, the weight of the edge beams is applied to the stage 3 model, which includes the long-term properties of the edge beams, rather than introduce another model. The difference between the two approaches is negligible, in relation to the design of the main beams.)

A further model, a modification of Stage 1, without the wet slab, was analysed to determine the rotational stiffness of the beams at that stage.

6.3 Analysis results

All the following results are for design values of actions, i.e. after application of appropriate partial factors on characteristic values of actions.

For construction loading, results are given for the total effects at each of the three construction stages. For traffic loading the results are given for the combination of traffic and pedestrian loading for worst bending effects at three locations - at the pier, at a girder splice (the same position as the first bracing adjacent to the intermediate support) and at 'mid-span' (taken to be at the bracing position, 12.4 m from the end support).

Stage 1

- Self weight of steelwork
- Self weight of concrete on span 1
- Construction loads on span 1

Distance from pier (m)	ULS			SLS		
	M_y (kNm)	F_x (kN)	F_z (kN)	M_y (kNm)	F_x (kN)	F_z (kN)
0	-2573	0	689	-1958	0	519
6.3	1024	-2	415	752	-1	312
15.6	3132	-3	43	2343	-2	33
28	25	1	-521	19	0	-391

Note: F_x is axial force, F_z is vertical shear

Stage 2

- Self weight of concrete on span 2
- Construction loads on span 2
- Removal of construction loads on span 1



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Subject **Example 1: Multi-girder two-span bridge
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Distance from pier (m)	ULS			SLS		
	M_y (kNm)	F_x (kN)	F_z (kN)	M_y (kNm)	F_x (kN)	F_z (kN)
0	-2499	-22	-13	-1851	-16	-10
6.3	-2354	31	45	-1744	23	33
15.6	-1714	-5	96	-1270	-3	71
28	-5	3	182	-4	2	135

Stage 3

- Self weight of concrete edge beams
- Self weight of parapets
- Self weight of carriageway surfacing
- Self weight of footway construction
- Removal of construction loads on span 2

Distance from pier (m)	ULS			SLS		
	M_y (kNm)	F_x (kN)	F_z (kN)	M_y (kNm)	F_x (kN)	F_z (kN)
0.0	-1705	-230	308	-1444	-168	260
6.3	89	-87	193	51	-63	163
15.6	1265	114	46	1033	84	38
28.0	165	-17	-198	117	-12	-166

Long term shrinkage (restraint moments applied in uncracked regions)

Values apply at both ULS and SLS since $\gamma_{sh} = 1.0$

Distance from pier (m)	characteristic		
	M_y (kNm)	F_x (kN)	F_z (kN)
0.0	-1552	-43	45
6.3	-1189	16	53
15.6	-695	2	54
28.0	-3	33	50

Stage 4 – transient actions

Traffic loads for worst hogging at intermediate support (gr5 loads)

(The effects due to gr5 loads without footway loading are greater than those due to gr1a, including footway loading.)

Distance from pier (m)	ULS			SLS		
	M_y (kNm)	F_x (kN)	F_z (kN)	M_y (kNm)	F_x (kN)	F_z (kN)
0	-3621	622	499	-2682	461	370
6.3	-1139	432	348	-844	320	258
15.6	781	66	83	579	49	61
28	-269	26	-201	-199	19	-149

Sheet 2



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Traffic loads for worst hogging at splice position (gr5 loads)

(The effects due to gr5 loads without footway loading are greater than those due to gr1a, including footway loading.)

Distance from pier (m)	ULS			SLS		
	M_y (kNm)	F_x (kN)	F_z (kN)	M_y (kNm)	F_x (kN)	F_z (kN)
0	-2895	479	184	-2145	355	136
6.3	-1980	479	183	-1467	355	135
15.6	-528	207	117	-391	153	86
28	-82	-7	-78	-61	-5	-57

Traffic loads for worst sagging at splice position (gr5 loads)

(The effects due to gr5 loads without footway loading are greater than those due to gr1a, including footway loading.)

Distance from pier (m)	ULS			SLS		
	M_y (kNm)	F_x (kN)	F_z (kN)	M_y (kNm)	F_x (kN)	F_z (kN)
0	-2457	264	1268	-1820	196	939
6.3	2839	-160	303	2103	-118	225
15.6	2058	-296	-151	1524	-219	-112
28	-370	53	-198	-274	39	-147

Traffic loads for worst sagging at 'mid-span' (12.4 m from abutment) (gr5 loads)

(The effects due to gr5 loads without footway loading are greater than those due to gr1a, including footway loading.)

Distance from pier (m)	ULS			SLS		
	M_y (kNm)	F_x (kN)	F_z (kN)	M_y (kNm)	F_x (kN)	F_z (kN)
0	-2425	381	661	-1796	282	490
6.3	1040	12	525	771	9	389
15.6	4952	-692	156	3668	-513	116
28	-1180	145	-691	-874	107	-512

gr5 traffic loads for maximum shear forces

	ULS			SLS		
	M_y (kNm)	F_x (kN)	F_z (kN)	M_y (kNm)	F_x (kN)	F_z (kN)
Pier	-3206	387	1482	-2375	287	1098
Splice +	2367	-11	669	1754	-8	496
Splice-	733	-101	-24	543	-75	-18
Span-	2805	-502	-396	2078	-372	-293
Abut-	-1347	119	-1354	-998	88	-1003



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Effects of thermal actions

Distance from pier (m)	Vertical temperature difference (restraint moments applied in uncracked regions) characteristic			Soil pressures due to characteristic value of thermal expansion characteristic		
	M_y (kNm)	F_x (kN)	F_z (kN)	M_y (kNm)	F_x (kN)	F_z (kN)
0	423	3	-10	1058	1211	-50
6.3	340	-12	-14	-25	1187	-55
15.6	215	27	-15	-537	1253	-58
28	17	-9	-14	-1265	1204	-54

Note that the effect of the soil pressure (due to restraint of thermal expansion) introduces hogging moments at the abutments and sagging moments at the intermediate support, as well as axial force. The total effect at the pier is therefore favourable, in terms of stresses in the bottom flange (and, in the rebars, the moment and axial force both reduce tension). Similarly, there is a hogging moment at the 'midspan' position and again the total effect is favourable, both in the bottom flange and the slab.

Range of effects due to passage of fatigue vehicle

Worst bending effects

	Pier		Splice		Span	
	M_y (kNm)	F_x (kN)	M_y (kNm)	F_x (kN)	M_y (kNm)	F_x (kN)
Lane 1 pos	0	0	407	-29	653	60
Lane 1 neg	-428	76	-267	85	-135	15
range	428	-76	674	-114	788	45
Lane 2 pos	0	0	387	-17	626	49
Lane 2 neg	-401	89	-253	98	-125	15
	401	-89	640	-115	751	34

Worst shear effects

	Pier F_z (kN)	Splice F_z (kN)	Span F_z (kN)	Abut F_z (kN)
Lane 1 pos	265	93	51	14
Lane 1 neg	-6	-6	-25	-269
range	271	99	76	283
Lane 2 pos	235	88	45	13
Lane 2 neg	0	-2	-23	-247
	235	90	68	260



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7 Design values of the effects of combined actions

Design values of effects are given below for certain design situations, for the design of the inner beams. In practice, further situations for other parts of the structure would also need to be considered.

7.1 Effects of construction loads (ULS)

Generally, the effects of construction loads apply to different cross section properties, although for span 1, the cross sections for the inner beam are the same at stages 2 and 3. The following tabulations summarize the forces and moments at each stage and the stresses due to those effects, for selected cross sections.

Elastic section moduli are tabulated in Section 5.1

Stresses at pier

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Stage 1	-2573	0	689	38.47	-67	24.25	106			70.0	0
Stage 2	-2499	-22	-13	45.67	-55	66.63	38	41.84	60	94.3	0
Stage 3	-1705	-230	308	45.67	-37	66.63	26	41.84	41	94.3	2
Shrinkage ($\gamma_{sh} = 1$)	-1552	-43	45	45.67	-34	66.63	23	41.84	37	94.3	0
	<u>-8329</u>	<u>-295</u>	<u>1029</u>		<u>-193</u>		<u>193</u>		<u>138</u>		<u>2</u>

Using steel and cracked section properties

Stress at splice (pier girder, cracked section)

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Stage 1	1024	-2	415	38.47	27	24.25	-42			70.0	0
Stage 2	-2354	31	45	45.67	-52	66.63	35	41.84	56	94.3	0
Stage 3	89	-87	193	45.67	2	66.63	-1	41.84	-2	94.3	1
Shrinkage ($\gamma_{sh} = 1$)	-1189	16	53	45.67	-26	66.63	18	41.84	28	94.3	0
	<u>-2430</u>	<u>-42</u>	<u>706</u>		<u>-49</u>		<u>10</u>		<u>82</u>		<u>1</u>

Using steel and cracked section properties

Stress at mid-span (span girder)

	M_y	F_x	F_z	Bottom flange		Top flange		Top of slab		Axial	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Stage 1	3132	-3	43	22.87	137	22.87	-137			50.2	0
Stage 2	-1714	-5	96	28.82	-59	180.4	10	943.9	1.8	107.5	0
Stage 3	1265	114	46	28.82	44	180.4	-7	943.9	-1.3	107.5	-1
Shrinkage ($\gamma_{sh} = 1$)	(not adverse)										
	<u>2683</u>	<u>106</u>	<u>185</u>		<u>122</u>		<u>-134</u>		<u>0.5</u>		<u>-1</u>

Using steel and long-term section properties



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Stress at abutment (abutment girder)

	M_y	F_x	F_z	Bottom flange		Top flange		Top of slab		Axial	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Stage 1	25	1	-521	22.87	1	22.87	-1			50.2	0
Stage 2	-5	3	182	28.82	0	180.4	0	943.9	0.0	107.5	0
Stage 3	165	-17	-198	28.82	6	180.4	-1	943.9	-0.2	107.5	0
	<u>185</u>	<u>-13</u>	<u>-537</u>		<u>7</u>		<u>-2</u>		<u>-0.2</u>		<u>0</u>

Using steel and long-term section properties

7.2 Effects of traffic loads plus construction loads (ULS)

Loading for maximum hogging at pier

The worst effects are due to gr5 traffic loads. Effects due to temperature difference are not adverse.

Effects at pier position

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial (steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Construction	-8329	-295	1029		-193		193		138		2
Gr 5 traffic	-3621	622	499	45.67	-79	66.63	54	41.84	87	94.3	-7
	<u>-11950</u>	<u>327</u>	<u>1528</u>		<u>-272</u>		<u>247</u>		<u>225</u>		<u>-5</u>

Using cracked properties for effects of variable actions

The effects of soil pressure are an axial force plus a sagging moment and the total effects are not adverse.

Coexistent effects at splice position

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial(steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Construction	-2430	-42	706		-49		10		82		1
Gr 5 traffic	-1139	432	348	45.67	-25	66.63	17	41.84	27	94.3	-5
	<u>-3569</u>	<u>390</u>	<u>1054</u>		<u>-74</u>		<u>27</u>		<u>109</u>		<u>-4</u>

Using cracked properties for effects of variable actions

Loading for maximum sagging bending

The maximum sagging moments on the composite beam occur at approximately midspan; the results for the node position 12.4 m from the abutment give the maximum values from the global analysis, for construction loads and for traffic loads.

	M_y	F_x	F_z	Bottom flange		Top flange		Top of slab		Axial (steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Construction	2683	106	185		122		-134		0.5		-1
Traffic gr5	4952	-692	156	30.50	162	1827	3	653.4	7.6	209.8	3
Temp difference*	200	25	-14	30.50	7	1827	0	653.4	0.3	209.8	0
	<u>7835</u>	<u>-561</u>	<u>327</u>		<u>291</u>		<u>-131</u>		<u>7.4</u>		<u>2</u>

Using short term composite properties for effects of variable actions

* $\gamma_0 = 1.55$ and $\psi_0 = 0.6$ applied to characteristic values

The effects of soil pressure due to thermal expansion are not adverse at bottom flange level and have only a very small adverse effect on the stress in the slab



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Loading for maximum hogging at splice

The maximum hogging moment at the splice position is much greater than the maximum sagging moment, so this will govern the design of the splice.

Pier side of splice

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial(steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Construction	-2430	-42	706	-49		10		82		1	
Gr 5 traffic	-1980	479	183	45.67	-43	66.83	30	41.84	47	94.3	-5
Temp (soil press)	-25	1187	55	45.67	-1	66.63	0	41.84	1	94.3	-13
	<u>-4435</u>	<u>1624</u>	<u>944</u>		<u>-93</u>		<u>40</u>		<u>130</u>		<u>-17</u>

Using cracked properties for effects of variable actions

No results are tabulated for the bracing position, 0.4 m closer to the centre support, but values may be interpolated linearly with sufficient accuracy. In practice, model nodes might be positioned at bracing locations.

Span side of splice

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial(steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Construction	-2430	-42	706		-82	4		91		1	
Gr 5 traffic	-1980	479	183	27.24	-73	71.64	28	38.06	52	74.5	-6
Temp (soil press)	-25	1187	55	27.24	0	71.64	0	38.06	0	74.5	-16
	<u>-4435</u>	<u>1624</u>	<u>944</u>		<u>-155</u>		<u>32</u>		<u>143</u>		<u>-21</u>

Using cracked properties for effects of variable action

Loading for maximum shear

Maximum shear at pier position

The value of the maximum shear is needed to verify the shear resistance of the web and to determine the longitudinal shear on the stud connectors.

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial (steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Construction	-8329	-295	1029		-93	203		138		2	
Gr 5 traffic	-3206	387	1482	45.67	-70	66.63	48	41.84	77	94.3	-4
	<u>-11535</u>	<u>92</u>	<u>2511</u>								

Using cracked properties for effects of variable actions

Maximum shear at splice position

The value of the maximum shear is needed to determine the maximum longitudinal shear on the stud connectors.

	M_y	F_x	F_z
Construction	-2430	-42	706
Gr 5 traffic	2367	-11	669
	<u>-63</u>	<u>-53</u>	<u>1375</u>



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Maximum shear at midspan position

The value of the maximum shear is needed to determine the maximum longitudinal shear on the stud connectors.

	M_y	F_x	F_z
Construction	2683	106	185
Gr 5 traffic	2805	-502	-396
	<u>5488</u>	<u>-396</u>	<u>-211</u>

Maximum shear at abutment

	M_y	F_x	F_z
Construction	185	-13	-537
Gr 5 traffic	-1347	119	-1354
	<u>-1162</u>	<u>106</u>	<u>-1891</u>

7.3 Effects of traffic loads plus construction loads (SLS)

The values of effects at SLS are needed to verify crack control in the slab at the pier and to verify the slip resistance of the splice.

Effects at pier position

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial (steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Stage 1	-1964	44	494	38.47	-51	24.25	81			70.0	0
Stage 2	-1857	29	-2	45.67	-41	66.63	28	41.84	44	94.3	0
Stage 3	-1451	-178	290	45.67	-32	66.63	22	41.84	35	94.3	2
Shrinkage	-1546	59	47	45.67	-34	66.63	23	41.84	37	132.2	0
Gr 5 traffic	-2695	719	483	45.67	-59	66.63	40	41.84	64	94.3	-7
	<u>-9513</u>	<u>673</u>	<u>1312</u>		<u>-217</u>		<u>194</u>		<u>180</u>		<u>-5</u>

The effects of soil pressures due to thermal expansion are not adverse at this position.

Effects at splice position (worst shear)

Pier side

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial(steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Stage 1	740	-5	359	38.47	20	24.25	-31			70.0	0
Stage 2	-1745	30	39	45.67	-38	66.63	26	41.84	42	94.3	0
Stage 3	39	-98	189	45.67	1	66.63	-1	41.84	-1	94.3	1
Shrinkage	-1210	54	53	45.67	-26	66.63	18	41.84	28	132.2	0
Gr 5 traffic	-1474	390	141	45.67	-32	66.63	22	41.84	35	94.3	-4
	<u>-3650</u>	<u>371</u>	<u>781</u>		<u>-75</u>		<u>34</u>		<u>104</u>		<u>-3</u>



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Span side

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial(steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Stage 1	740	-5	359	22.87	33	22.87	-33			50.2	0
Stage 2	-1745	30	39	27.24	-64	71.64	24	38.06	46	74.5	0
Stage 3	39	-98	189	27.24	2	71.64	-1	38.06	-1	74.5	1
Shrinkage	-1210	54	53	27.24	-44	71.64	17	38.06	31	74.5	0
Gr 5 traffic	-1474	390	141	27.24	-54	71.64	20	38.06	39	74.5	-5
	<u>-3650</u>	<u>371</u>	<u>781</u>		<u>-127</u>		<u>27</u>		<u>115</u>		<u>-4</u>

7.4 Effects due to fatigue vehicle

The range of bending effects due to the passage of the fatigue vehicle in each lane is determined at the three locations already considered for static loading.

At pier

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial(steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Range, lane 1	428	-76		45.67	9.4	66.63	-6.4	41.84	-10.2	94.3	0.8
Range, lane 2	401	-89		45.67	8.8	66.63	-6.0	41.84	-9.6	94.3	0.9

Ratio lane 2/lane 1 moments = 0.937

At splice (span side, uncracked)

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial(steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Range, lane 1	674	-114		30.50	22.1	-1,827	0.4	653.4	-1.0	209.8	0.5
Range, lane 2	640	-115		30.50	21.0	-1,827	0.4	653.4	-1.0	209.8	0.5

Ratio lane 2/lane 1 moments = 0.950

At midspan

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial(steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Range, lane 1	788	45		30.50	25.8	-1,827	0.4	653.4	-1.2	209.8	-0.2
Range, lane 2	751	34		30.50	24.6	-1,827	0.4	653.4	-1.1	209.8	-0.2

Ratio lane 2/lane 1 moments = 0.953



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8 Verification of bare steel girder during construction

The paired beams are susceptible to lateral torsional buckling under the weight of the wet concrete (i.e. before it hardens and provides restraint to the top flanges).

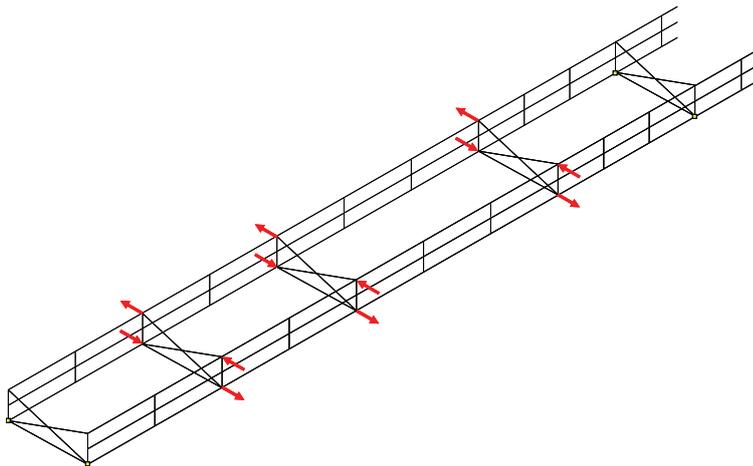
The beams are partially restrained against buckling by the bracing frames between each pair at three points in the span. This connection provides flexible torsional restraint to the beams.

Use the expressions in Appendix C of P356 to determine the non-dimensional slenderness and thus the buckling resistance. Initially, use gross section properties for verification of buckling resistance, even where the section is Class 4.

P356^[4]

8.1 Torsional flexibility of paired beams

In the global model, horizontal forces of 10 kN were applied at top and bottom flange levels at the three bracing locations, on each beam.



In the model, the forces at the top flange are applied at the model nodes, which are above the level of the steel flange, at the mid-thickness of the slab. The lever arm is thus $1400 - 250/2 - 40/2 = 1255$ mm

The total torque applied is thus:

$$3 \times 2 \times 10 \times 1.255 = 37.65 \text{ kNm}$$

The horizontal deflections at each beam given by the analysis were:

(bracings modelled at nodes rather than at positions indicated in Section 4)

- At 6.3 m from pier +0.525 mm, -0.243 mm
- At 15.6 m from pier +0.858 mm, -0.420 mm
- At 21.8 m from pier +0.622 mm, -0.308 mm

The bracings are not equally spaced but the torsional restraint that they provide is not sensitive to the spacing and they may be considered as equally spaced, for the application of the expressions in Appendix C of P356



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The rotations are thus:

- At 6.3 m from pier $(0.525 + 0.243) / 1255 = 6.12 \times 10^{-4}$ rad
- At 15.6 m from pier $(0.858 + 0.420) / 1255 = 10.18 \times 10^{-4}$ rad
- At 21.8 m from pier $(0.622 + 0.308) / 1255 = 7.41 \times 10^{-4}$ rad

Thus, use:

$$\theta_R = 10.18 \times 10^{-4} / (37.65 \times 10^6) = 2.705 \times 10^{-11} \text{ rad/Nmm}$$

8.2 Evaluation of non-dimensional slenderness

Geometrical parameters:

$L_w = 28000$ mm and for the span girder cross section, the section properties are:

$$I_{z,c} = I_{z,t} = 4.167 \times 10^8 \text{ mm}^4$$

$$I_T = 2.167 \times 10^7 \text{ mm}^4$$

$$i_z = 128.8 \text{ mm}$$

$$h = 1100 \text{ mm}$$

$$t_f = 40 \text{ mm}$$

$$d_f = 1060 \text{ mm}$$

$$\lambda_F = \frac{L_w}{i_z} \cdot \frac{t_f}{h} = \frac{28000}{128.8} \cdot \frac{40}{1100} = 7.91$$

$$a = \frac{I_{z,c}}{I_{z,c} + I_{z,t}} = 0.5 \text{ (equal flanges)}$$

$$\psi_a = 0.8(2a - 1) = 0.8 \times (2 \times 0.5 - 1) = 0$$

To determine V_{eq} , the following are needed:

$$\tau = 4a(1 - a) + \psi_a^2 = 4 \times 0.5 \times (1 - 0.5) + 0 = 1.0$$

$$\omega = \frac{\pi^2 d_f^2 EI_z}{GI_T L_w^2} = \frac{\pi^2 \times 1060^2 \times 8.33 \times 10^8 \times 2.6}{2.167 \times 10^7 \times 28000^2} = 1.414 \text{ (using } E/G = 2.6)$$

Thus:

$$V_{eq} = \left[\frac{2 a \omega}{\left[\sqrt{4 + \tau \omega + \psi_a \sqrt{\omega}} \right]^2} \right]^{0.25} = \left[\frac{2 \times 0.5 \times 1.414}{\left[\sqrt{4 + 1.414 + 0} \right]^2} \right]^{0.25} = 0.715$$

Thus the restraint parameter $V_{eq}^4 L_w^3 / [EI_{z,c} \theta_R d_f^2 (1 - a)] = 4316$

P356/C.4.3
and C.3.2

P356/C.4.5



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And using the expression $k = \left[1 + \frac{V_{eq}^4 L_w^3}{\pi^4 EI_{z,c} d_f^2 \theta_R (1-a)} \right]^{(-0,25)}$

The value of $k = 0.385$

The limiting (minimum) value of k is $(1.7 - 0.7V_{eq})L_r/L_w$

Taking $L_r = 8200$ (the longest unbraced length - this is conservative, the limit is:

$$(1.7 - 0.7 \times 0.715) \times 8.2/28.0 = 0.351, \text{ so use } k = 0.385$$

Assume $1/\sqrt{C_1} = 1.0$ (uniform moment - conservative assumption)

$U = 1.0$ (welded section)

$$V = \left\{ \left[4a(1-a) + 0.05\lambda_F^2 + \psi_a^2 \right]^{0.5} + \psi_a \right\}^{-0.5}$$

$$= \left\{ \left[4 \times 0.5(1-0.5) + 0.05 \times 7.91^2 + 0 \right]^{0.5} + 0 \right\}^{-0.5} = 0.702$$

Take $D = 1.2$ (destabilising loads)

$$\lambda_z = \frac{kL_w}{i_z} = \frac{0.385 \times 28000}{128.8} = 83.7$$

$$\lambda_1 = \pi \sqrt{\frac{E}{f_y}} = \pi \sqrt{\frac{210000}{345}} = 77.5$$

$$\beta_w = \frac{W_y}{W_{pl,y}} = 2.287 \times 10^7 / (8237 \times 10^6 / 345) = 0.958$$

Thus:

$$\bar{\lambda}_{LT} = \frac{1}{\sqrt{C_1}} UVD \frac{\lambda_z}{\lambda_1} \sqrt{\beta_w} = 1 \times 1 \times 0.702 \times 1.2 \times \frac{83.7}{77.5} \sqrt{0.958} = 0.89$$

Slenderness determined from buckling analysis

Alternatively, and less conservatively, slenderness could be derived from an elastic buckling analysis of the structure at the bare steel girder stage and then the value of

$\bar{\lambda}_{LT}$ would be given by $\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$ where M_{cr} is given by the analysis.

A buckling analysis was not available for this example.

P356/C.4.2
P356/C.4.3

P356/C.1

W_y and M_{pl}
from Sheet 14

P356/C.1



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8.3 Reduction factor

Since $h/b > 2$, use buckling curve d, $\alpha_{LT} = 0.76$

$$\phi_{LT} = 0.5 \left[1 + \alpha_{LT} (\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2 \right]$$

$$= 0.5 \left[1 + 0.76(0.89 - 0.2) + 0.89^2 \right] = 1.16$$

Hence

$$\chi_{LT} = 1 / \left(\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2} \right) = 1 / \left(1.16 + \sqrt{1.16^2 - 0.89^2} \right) = 0.525$$

8.4 Verification

$$M_{b,Rd} = \frac{\chi W_{el} f_y}{\gamma_{M1}} = \frac{0.525 \times 2.287 \times 10^7 \times 345}{1.1} \times 10^{-6} = 3766 \text{ kNm}$$

$$M_{Ed} = 3132 \text{ kNm (Sheet 23)} < M_{b,Rd} = 3766 \text{ kNm - OK}$$

The above calculations assume that the cross section is Class 3. In fact it is marginally Class 4, as noted on Sheet 14. The determination of the properties of the effective section for this particular cross section (neglecting fillet welds) gives the following parameters:

$$k_{\alpha} = 23.9$$

$$\bar{\lambda}_p = 0.907$$

$$\rho = 0.069$$

$b_{eff} = 494.2$, which means that there is a 'hole' in the web 15.8 mm vertically with its centroid $b_{eff}/2$ below the underside of the top flange.

The section moduli are then:

$2.281 \times 10^7 \text{ mm}^4$ at the mid-thickness of the top flange and

$2.288 \times 10^7 \text{ mm}^4$ at the mid-thickness of the bottom flange

The modulus for the effective section should be used in the expressions for β_w and $M_{b,Rd}$. Here the difference is negligible.

3-2/6.3.2.2
 3-1-1/6.3.2.2
 3-1-
 1/NA.2.16

3-1-1/6.3.2.1

3-1-5/4.4



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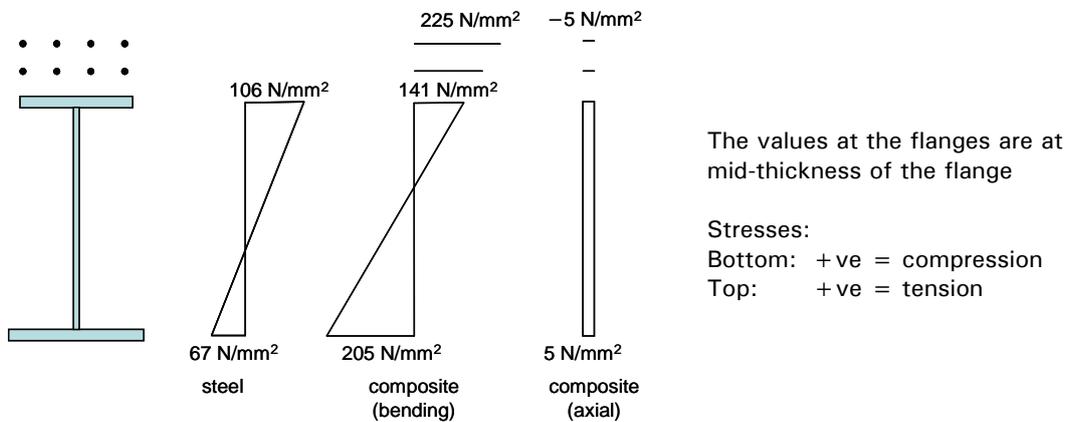
9 Verification of composite girder

9.1 In hogging bending with axial force

The composite cross section is Class 3.

The elastic design bending resistance for a beam constructed in stages depends on the design effects at the stages.

From Sections 7.1 and 7.2, the design moment on the steel section is 2573 kNm and the total moment is 11950 kNm, which means that the moment on the composite (cracked) section is 9377 kNm. The stresses are as shown below.



The primary effects of shrinkage do not need to be included.

4-2/6.2.1.5(5)

For verification of cross section resistance, the stresses should not exceed the limiting stresses f_{yd} and f_{sd} .

For this verification:

$$f_{yd} = f_y / \gamma_{M0} = 335 / 1.0 = 335 \text{ N/mm}^2 \text{ for the 60 mm bottom flange}$$

$$f_{sd} = f_{yk} / \gamma_s = 500 / 1.15 = 435 \text{ N/mm}^2 \text{ for the reinforcement}$$

By inspection, the stresses in both are OK

The member is subject to combined bending and axial force and for member resistance a linear interaction will be assumed (conservative).

For verification of buckling resistance in bending, the design resistance of the cross section (on which $M_{b,Rd}$ is based) has to be determined using:

$$M_{el,Rd} = M_a + kM_{c,Ed}$$

4-2/6.4.2

Where k is a factor such that a stress limit is reached due to bending alone.

In this case the bottom flange will reach its limit first and the limit is:

$$f_{yd} = f_y / \gamma_{M1} = 335 / 1.1 = 305 \text{ N/mm}^2$$

$$\text{Thus } M_{el,Rd} = 2573 + \frac{(305 - 67)}{205} \times 9377 = 13460 \text{ kNm}$$



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To evaluate $M_{b,Rd}$, determine the slenderness

The slenderness of the length of beam between the intermediate support and the bracing at 5.9 m into the span could be evaluated considering the LTB of a section comprising the effective width of slab and the steel girder but it is much simpler and a little less conservative to use the simplified method of EN 1993-2, as allowed by EN 1994-2.

4-2/6.4.3.2

Consider an effective Tee section comprising the bottom flange and one third of the depth of the part of the web in compression. Take the depth in compression as that under total effects, including axial force.

3-2/6.3.4.2

Flange area is 600×60 mm

Height to zero stress:

$$(277/532) \times 1050 + 30 = 590 \text{ mm}$$

Height of web in compression = 530 mm

	Stress	Mid-height
Top flange	255	1080
Bottom flange	277	30

$$\text{Area of Tee} = 600 \times 60 + (530 \times 14)/3 = 38470 \text{ mm}^2$$

$$\text{Lateral 2}^{\text{nd}} \text{ moment of area } 600^3 \times 60 / 12 = 1080 \times 10^6 \text{ mm}^4$$

$$\text{Radius of gyration} = \sqrt{1080 \times 10^6 / 38470} = 168 \text{ mm}$$

For a buckling length of 5900 mm (support to first bracing):

$$N_E = \pi^2 \frac{EI}{L^2} = \pi^2 \frac{210000 \times 1080 \times 10^6}{5900^2} \times 10^{-3} = 64300 \text{ kN}$$

3-2/(6.12)

Initially, take m conservatively as 1.0

$$\text{Then } N_{\text{crit}} = N_E = 64300 \text{ kN}$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{A_{\text{eff}} f_y}{N_{\text{crit}}}} = \sqrt{\frac{38470 \times 335}{64300 \times 10^3}} = 0.448$$

3-2/(6.10)

Since $h/b > 2$, use buckling curve d ($\alpha = 0.76$)

3-2/6.3.2.2

$$\phi_{LT} = 0.5 \left[1 + \alpha_{LT} (\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2 \right]$$

3-1-1/6.3.2.2

$$= 0.5 \left[1 + 0.76(0.448 - 0.2) + 0.448^2 \right] = 0.695$$

3-1-
1/NA.2.16

Hence

$$\chi_{LT} = 1 / \left(\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2} \right) = 1 / \left(0.695 + \sqrt{0.695^2 - 0.448^2} \right) = 0.815$$

$$M_{b,Rd} = \chi M_{el,Rd} = 0.815 \times 13460 = 10970 \text{ kNm}$$

3-1-1/6.3.2.1



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For verifying the contribution of axial resistance in the interaction criterion, consider the same Tee section (and thus the same slenderness and reduction factor).

No effective section for axial force is given in EN 1993-2 but it could be argued that the effective Tee that would buckle laterally should comprise half the area of the web: the slenderness with this amount of web is very little different from that derived above bending.

$$N_{b,Rd} = \chi A_{Tee} f_{yd} = 0.815 \times 38470 \times 305 = 9560 \text{ kN}$$

$$N_{Ed} = A_{Tee} \times stress = 38410 \times 5 = 192 \text{ kN}$$

Considering first the interaction for values of M_{Ed} and N_{Ed} at the support (using a linear interaction, since the buckling mode is the same for both) and with no allowance for variation over the buckling length:

$$\frac{M_{Ed}}{M_{b,Rd}} + \frac{N_{Ed}}{N_{b,Rd}} = \frac{11950}{10970} + \frac{192}{9560} = 1.11$$

This is inadequate, so consider the variation of moment and location for M_{Ed} .

Allowance for varying moment over buckling length

Evaluate the m parameter in 6.3.4.2(6)

Coexisting total moment at the splice = 3569 kNm and shear = 1054 kN

Assume values at the brace position of $M = 4000$ kNm and $V = 1080$ kN

(If the model had been given nodes at the bracing position as well as at the change of section, actual values could have been used but the result would be negligibly different for the small distance involved in this example.)

Using the Note to 6.3.4.2(7) and ignoring any contribution from the continuous restraint provided by the web (which will be very small) (i.e. take $\gamma = 0$)

$$M_2/M_1 = 4000/11950 = 0.335$$

$$\mu = V_2/V_1 - 1080/1528 = 0.707$$

$$\Phi = 2(1 - M_2/M_1)/(1 + \mu) = 2(1 - 0.335)/(1 + 0.707) = 0.78$$

$$m = 1 + 0.44(1 + \mu)\Phi^{1.5} = 1 + 0.44 \times (1 + 0.707) \times 0.78^{1.5} = 1.52$$

Hence

$$N_{crit} = 1.52 \times N_E = 1.52 \times 64300 = 97740 \text{ kNm}$$

And

$$\bar{\lambda}_{LT} = \sqrt{\frac{A_{eff} f_y}{N_{crit}}} = \sqrt{\frac{38470 \times 10^6 \times 335}{97740 \times 10^3}} = 0.363$$

$$\phi_{LT} = 0.5 \left[1 + 0.76(0.363 - 0.2) + 0.363^2 \right] = 0.628$$

$$\chi_{LT} = 1 / \left(0.628 + \sqrt{0.628^2 - 0.363^2} \right) = 0.877$$

3-2/6.3.4.2

3-1-1/6.3.2.2



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$$M_{b,Rd} = \chi M_{el,Rd} = 0.877 \times 13460 = 11800 \text{ kNm}$$

3-1-1/6.3.2.1

Consider the moment at a distance $0.25L_k$ from the support, where $L_k = L/\sqrt{m}$

3-2/6.3.4.2(7)

Distance = $0.25 \times 5900/\sqrt{1.52} = 1196$ mm from the support

$$M_{Ed} = 11950 - (11950 - 4000) \times 1196/5900 = 10340 \text{ kNm (conservative interpolation)}$$

The axial force and $N_{b,Rd}$ could also be reduced and the resistance $N_{b,Rd}$ enhanced but that adjustment is not made here (the difference in the result is negligible).

The utilisation is now:

$$\frac{M_{Ed}}{M_{b,Rd}} + \frac{N_{Ed}}{N_{b,Rd}} = \frac{10340}{11800} + \frac{192}{9560} = 0.90 \text{ Acceptable}$$

The results of the cross section verification and buckling resistance verification indicate that some economy could be achieved (reducing the cross section slightly).

Interaction with shear must also be considered.

9.2 Maximum shear at support

The maximum shear in the girder at the intermediate support = 2511 kN

Sheet 25

Assume that first transverse web stiffener is provided at 1967 mm from the support (i.e. divide the length to the first bracing into three panels).

For the web panel adjacent to the support

$$\begin{aligned} a_w &= 1967 \text{ mm} \\ h_w &= 1000 \text{ mm} \\ t &= 14 \text{ mm} \\ f_y &= 355 \text{ N/mm}^2 \end{aligned}$$

3-1-5/NA.2.4

The factor $\eta = 1.0$ according to the NA

From Equation (5.6):

3-1-5/5.3

$$\bar{\lambda}_w = \frac{h_w}{37.4t\epsilon\sqrt{k_t}} \text{ where } \epsilon = \sqrt{235/f_y} = \sqrt{235/355} = 0.81$$

Since $a_w > h_w$ and there are no longitudinal stiffeners:

3-1-5/A.3

$$k_t = 5.34 + 4.0(h_w/a)^2 = 5.34 + 4.0(1000/1967)^2 = 6.37$$

$$\bar{\lambda}_w = \frac{1000}{37.4 \times 14 \times 0.81 \sqrt{6.37}} = 0.934$$

Since the girder is continuous, consider as a 'rigid endpost' case; thus, from Table 5.1:

3-1-5/
Table 5.1

$$\chi_w = 0.83/\bar{\lambda}_w = 0.83/0.934 = 0.889$$



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$$V_{bw,Rd} = \frac{\chi_w f_{yw} h_w t}{\sqrt{3} \gamma_{M1}} = \frac{0.889 \times 355 \times 1000 \times 14}{\sqrt{3} \times 1.1} \times 10^{-3} = 2319 \text{ kN}$$

3-1-5/5.2

This resistance is adequate for the maximum moment situation but requires a contribution from the flanges for the maximum shear situation.

Maximum contribution from flanges is given by:

$$V_{bf,Rd} = \frac{b_f t_f^2 f_{yf}}{c \gamma_{M1}} \left(1 - \left(\frac{M_{Ed}}{M_{f,Rd}} \right)^2 \right)$$

3-1-5/5.4

$$c = a \left(0.25 + \frac{1.6 b_f t_f^2 f_{yf}}{t h_w^2 f_{yw}} \right) = 1967 \left(0.25 + \frac{1.6 \times 600 \times 60^2 \times 335}{14 \times 1000^2 \times 355} \right) = 950 \text{ mm}$$

$M_{f,Rd}$ is the resistance of the flanges alone (no web).

The axial resistance of the top bars and top flange is:

$$(2 \times 12,108) \times (500/1.15) + 20000 \times (345/1.0) = 17430 \text{ kN}$$

$$\text{And of the bottom flange is } 36000 \times (335/1.0) = 12060 \text{ kN}$$

3-1-5/5.4

Take the lever arm between top and bottom as 1159 mm and thus:

$$M_{f,Rd} = 12060 \times 1159 \times 10^{-3} = 13980 \text{ kNm}$$

For the design situation for maximum shear, the net axial force on the cross section is a small tensile force; no reduction is needed to $M_{f,Rd}$ for this axial force.

For the maximum shear situation, $M_{Ed} = 11535 \text{ kNm}$

Sheet 25

$$M_{Ed} / M_{f,Rd} = 11535/13980 = 0.83$$

$$V_{bf,Rd} = \frac{600 \times 60^2 \times 335}{950 \times 1.1} (1 - 0.83^2) = 692 \times (1 - 0.69) = 215 \text{ kN}$$

The total shear resistance is thus:

$$V_{b,Rd} = 2319 + 215 = 2534 \text{ kN} (\eta_3 = 2487/2534 = 0.99) \text{ Satisfactory}$$

9.3 Combined bending shear and axial force

When $M_{Ed} > M_{f,Rd}$ and when $V_{Ed} > 0.5 V_{bw,Rd}$ the design resistance to bending and axial force must be reduced for the coexisting shear force.

3-1-5/7.1

Maximum shear with coexisting moment

As noted above, $M_{Ed} / M_{f,Rd} = 0.83$, therefore the bending resistance does not need to be reduced for shear (instead, the shear resistance has already been reduced for coexisting moment).

Maximum moment with coexisting shear

Sheet 24

$$V_{Ed} = 1528 \text{ kN} \quad M_{Ed} = 11950 \text{ kNm} \quad F_{x,Ed} = 327 \text{ kN (axial compression)}$$



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The value of $M_{f,Rd}$ is reduced for axial force in accordance with 3-1-5/5.4(2) by applying the factor:

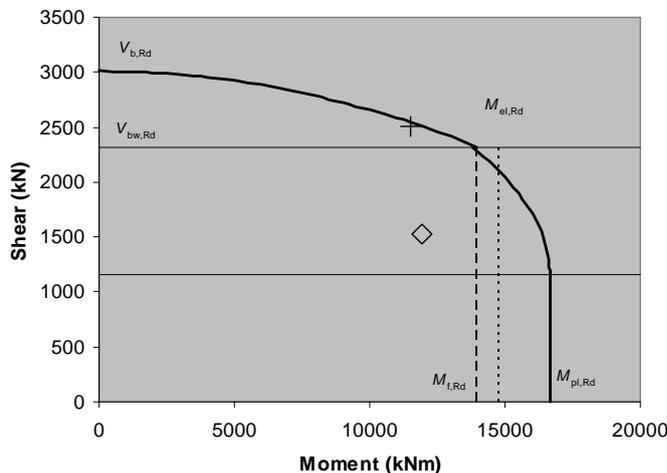
$$\left(1 - \frac{N_{Ed}}{N_{bf,Rd} + N_{tf,Rd}}\right) = \left(1 - \frac{327}{12060 + 17430}\right) = 0.99$$

And hence $M_{f,Rd} = 13700$ kNm

Hence, since $M_{Ed} < M_{f,Rd}$ ($11950 < 13700$), bending resistance does not need to be reduced for shear.

Note: PD 6696-2 and Hendy and Johnson^[5] suggest that, for use in 3-1-5/7.1, M_{Ed} should be determined as the product of the accumulated stress and the section modulus for the relevant fibre of the cross section. However, a proposed revision of EN 1994-2 would modify the wording of 4-2/6.2.2.4 to confirm that it is the total moment that should be used. For this example, in both cases the value is less than $M_{f,Rd}$.

Although interaction does not need to be evaluated, the limiting combinations of M and V given by 3-1-5/5.4 and 3-1-5/7.1 are plotted below, for information. The values of maximum moment with coexisting shear and maximum shear with coexisting moment are shown on the plot. (For the different design situations $M_{pl,Rd}$ and $M_{f,Rd}$ are slightly different but the differences are very small.)



9.4 In sagging bending

The composite cross section is Class 1 (pna in the top flange) so the plastic resistance can be utilised.

The plastic bending resistance of the short term composite section is 13070 kNm and the total design value of bending effects is 7835 kNm, with a very small axial tensile force, so the section is satisfactory by inspection.

It can also be seen that the stresses calculated elastically, taking account of construction in stages are also satisfactory, as follows:

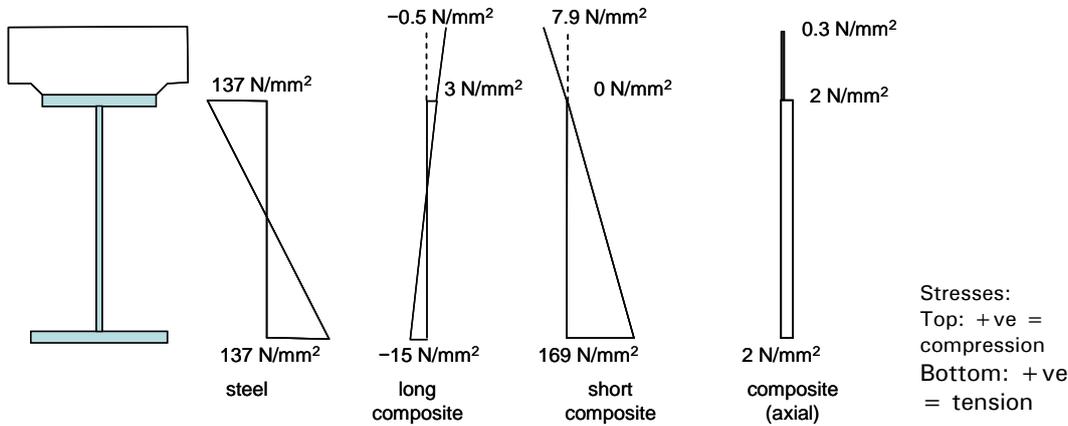
From Sections 7.1 and 7.2, the design value of stresses are as shown below.



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The above stresses include the secondary effects of temperature difference (as an accompanying action). The primary effects should be added (values as an accompanying action): they are 4 N/mm² compression at the bottom flange and 1.9 N/mm² compression at the top of the slab.

For verification of cross section resistance, the stresses should not exceed the limiting stresses f_{yd} and f_{sd} .

For this verification:

$$f_{yd} = f_y / \gamma_{M0} = 335 / 1.0 = 335 \text{ N/mm}^2 \text{ for the 40 mm bottom flange}$$

$$f_{cd} = f_{ck} / \gamma_c = 40 / 1.5 = 26.7 \text{ N/mm}^2 \text{ for the concrete}$$

By inspection, the stresses in both are OK ($\sigma_{bf} = 293 \text{ N/mm}^2$ $\sigma_{slab} = 7.3 \text{ N/mm}^2$)

9.5 Verification of crack control at SLS

Minimum reinforcement

The minimum required reinforcement is:

$$A_s = k_s k_c k_f f_{ct,eff} A_{ct} / \sigma_s$$

$$k_s = 0.9$$

$$k_c = \frac{1}{1 + h_c / 2z_0} + 0.3 \leq 1.0$$

Here $h_c = 250$ and $z_0 = 1275 - 1016 = 259$

$$k_c = \frac{1}{1 + 250 / (2 \times 259)} + 0.3 = 0.974$$

$$k = 0.8$$

$$f_{ctm} = 3.5 \text{ (Table 3.1)}$$

$$w_{max} = 0.3 \text{ mm and thus for 25 mm bars, } \sigma_s = 200 \text{ N/mm}^2 \text{ (Table 7.1)}$$

$$A_s = 0.9 \times 0.974 \times 0.8 \times 3.5 \times (250 \times 3700) / 200 = 11350 \text{ mm}^2$$

Area provided = $2 \times 491 \times 3700 / 150 = 24250 \text{ mm}^2$ - Satisfactory

Crack control

Requirements relate only to the quasi-permanent design situation and therefore local longitudinal stresses in the reinforcement are negligible.

Sheet 16

4-2/6.2.1.5

Sheet 24

4-2/7.4.2

2-1-1/3.1.2

4-2/7.4.2



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Global stresses due to permanent loads, including shrinkage are 116 N/mm² in the top rebars. The tensile stress including the effect of tension stiffening

$$\text{are: } \sigma_s = \sigma_{s,0} + \frac{0.4 f_{ctm}}{\alpha_{st} \rho_s}$$

$$\alpha_{st} = \frac{AI}{A_a I_a} = \frac{94250 \times 28450}{70000 \times 15620} = 2.452$$

$$\rho_s = A_s / A_{ct} = 24250 / (3700 \times 250) = 0.0262 \text{ (i.e. 2.62\%)}$$

$$\sigma_s = 116 + \frac{0.4 \times 3.5}{2.452 \times 0.0262} = 140 \text{ N/mm}^2$$

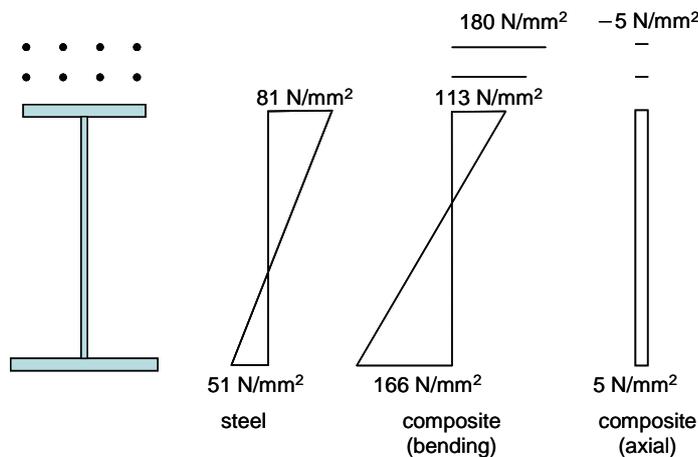
From Table 7.2, maximum bar spacing = 300 mm > 150 mm provided

∴ provision is satisfactory

4-2/Table 7.2

9.6 Limiting stresses at SLS

At the pier, the stresses in the cracked section are:



The primary stresses due to shrinkage do not need to be added (see 4-2/7.2.1(4)) and all stresses are less than $f_y / \gamma_{M,ser}$ (= 345 N/mm² for top flange, 335 N/mm² for bottom flange) and $k_3 f_{sk}$ (= 0.8 × 500 = 400 MPa).

4-2/7.2.2
 3-2/7.3
 2-1-1/7.2(5)

At midspan, the SLS stresses in the steel are satisfactory by inspection (see stresses at ULS on sheet 37) even with the addition of the primary shrinkage stresses (see sheet 17). There is no limit on concrete stress for the characteristic combination, for class XC exposure. For the quasi-permanent combination, the concrete stress limit (for linear creep) is $k_2 f_{ck}$ (= 0.45 × 40 = 18 MPa) and for that criterion the situation is also satisfactory by inspection.

4-2/7.2.2 and
 2-2/7.2



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10 Longitudinal shear

The resistance to longitudinal shear is verified for the web/flange weld, the shear connectors and the transverse reinforcement at the pier, at the splice and at mid-span

10.1 Shear forces

ULS values

	Pier	Splice	Span	Abutment
Shear on steel section (stage 1)	689	415	43	-521
Shear on long-term composite section	340	291	142	-16
Shear on short-term composite section (worst effects)	1482	669	-396	-1354

SLS values

	Pier	Splice	Span	Abutment
Shear on steel section (stage 1)	519	312	33	-391
Shear on long-term composite section	295	196	109	-31
Shear on short-term composite section (worst effects)	1098	496	-293	-1003

10.2 Section properties

To determine shear flows the parameter $\bar{A}z/I_y$ is needed for each section and stage.

For composite sections, uncracked unreinforced composite section properties can be used to determine shear flow.

	Pier girder		Span girder		Abutment girder	
	Web/top fl	Top fl/slab	Web/top fl	Top fl/slab	Web/top fl	Top fl/slab
	$\bar{A}z/I_y$ (m ⁻¹)					
Steel section	0.825		0.875		0.861	
Long term section	0.836	0.706	0.843	0.732	0.843	0.732
Short term section	0.831	0.805	0.826	0.836	0.836	0.844

At the pier, the values at the bottom flange/web junction are 0.936, 0.739 and 0.704 for steel, long-term and short-term cross sections respectively.

In addition to the shear flows determined from the vertical shear, the inequality of forces and moments on the four girders for any particular design situation leads to different axial forces on the composite beam sections that are verified to the design rules. These axial forces vary longitudinally and the variation is associated with a shear flow transferred between one composite section and the adjacent section. In this example, detailed interrogation of the analysis results identifies, for example, a negative shear flow of 34 kN/m at the pier due to permanent actions and a positive shear flow of 35 kN/m for the traffic loading for the worst shear case (both at the edge of the section, not at the steel/concrete interface). There is therefore no significant overall contribution. Values for other situations give only very small shear flows.

Sheet 25

4-2/
6.6.2.1(2)



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10.3 Shear flow at ULS

Force at web/top flange junction

At pier	$689 \times 0.825 + 340 \times 0.836 + 1482 \times 0.831 = 2084 \text{ kN/m}$
At splice (span girder)	$415 \times 0.875 + 291 \times 0.843 + 669 \times 0.826 = 1161 \text{ kN/m}$
At mid-span	$43 \times 0.875 + 142 \times 0.843 - 396 \times 0.826 = -169 \text{ kN/m}$
At abutment	$-521 \times 0.861 - 16 \times 0.843 - 1354 \times 0.836 = 1594 \text{ kN/m}$

Force at flange/slab junction

At pier	$340 \times 0.706 + 1482 \times 0.805 = 1433 \text{ kN/m}$
At splice (span girder)	$291 \times 0.732 + 669 \times 0.836 = 772 \text{ kN/m}$
At mid-span	$142 \times 0.734 - 396 \times 0.843 = -230 \text{ kN/m}$
At abutment	$-16 \times 0.732 - 1354 \times 0.844 = -1155 \text{ kN/m}$

At the web/bottom flange junction

At pier	$689 \times 0.936 + 340 \times 0.739 + 1482 \times 0.704 = 1939 \text{ kN/m}$
---------	---

10.4 Shear flow at SLS

Force at flange/slab junction

At pier	$295 \times 0.706 + 1098 \times 0.805 = 1092 \text{ kN/m}$
At splice (span girder)	$196 \times 0.732 + 496 \times 0.836 = 558 \text{ kN/m}$
At mid-span	$109 \times 0.734 - 293 \times 0.843 = -167 \text{ kN/m}$
At abutment	$-31 \times 0.732 - 1003 \times 0.844 = -870 \text{ kN/m}$

The shear flow at SLS is required for verification of the shear connectors

10.5 Web/flange welds

Design weld resistance given by the simplified method of EN 1993-1-8, 4.5.3.3 is: 3-1-8/4.5.3.3

$$F_{w,Rd} = f_{vw,d} a \text{ where } f_{vw,d} = \frac{f_u / \sqrt{3}}{\beta \gamma_{M2}}$$

For 6 mm throat fillet weld (8.4 mm leg length) $a = 6 \text{ mm}$

For web and flange grade S355 in thickness range 3 - 100 mm, $f_u = 470 \text{ N/mm}^2$

From Table 3-1-8/4.1 $\beta = 0.9$

$$\text{Thus } F_{w,Rd} = \frac{6 \times 470 / \sqrt{3}}{0.9 \times 1.25} = 1447 \text{ N/mm (kN/m)}$$

Resistance of 2 welds = 2890 kN/m > 2084 kN/m shear flow in pier girder at top flange - OK

By inspection, 5 mm throat welds would be satisfactory at the splices and in the span regions.



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Shear flows at bottom flange are slightly less and are OK by inspection but the interaction with vertical effects at the bearing stiffener need to be checked (see Section 13.3)

10.6 Shear connectors

Stud shear connectors 19 mm diameter 150 mm long (type SD1 to EN ISO 13918) are assumed, with $f_u = 450 \text{ N/mm}^2$

The resistance of a single stud is given by 4-2/6.6.3.1 as the lesser of:

$$P_{Rd} = \frac{0.8 \times f_u \times \pi \times d^2 / 4}{\gamma_V}$$

6.6.3.1(1)
Eq (6.18)

$$P_{Rd} = \frac{0.29 \times \alpha \times d^2 \sqrt{f_{ck} \times E_{cm}}}{\gamma_V}$$

Eq (6.19)

$$\alpha = 1.0 \text{ as } \frac{h_{sc}}{d} = \frac{150}{19} > 4$$

Eq (6.21)

$$P_{Rd} = \frac{0.8 \times 450 \times \pi \times (19^2 / 4)}{1.25} \times 10^{-3} = 81.7 \text{ kN}$$

Eq (6.18)

$$P_{Rd} = \frac{0.29 \times 1.0 \times 19^2 \times \sqrt{40 \times 35 \times 10^3}}{1.25} \times 10^{-3} = 99.1 \text{ kN}$$

Eq (6.19)

Therefore the design resistance of a single headed shear connector is

$$P_{Rd} = 81.7 \text{ kN}$$

If studs are grouped and spaced at 150 mm spacing along the beam (to suit transverse reinforcement), then a row of 3 studs has a design resistance of:

$$F_{Rd} = 81.7 \times 3 / 0.150 = 1630 \text{ kN/m}$$

This is adequate at the pier ($F_{Rd} = 1630 > F_{Ed} = 1433 \text{ kN/m}$)

Rows of 2 studs would be adequate at the splice position ($F_{Rd} = 1090 > F_{Ed} = 772$). The change from 3 studs per row to 2 studs per row can be made on the pier side of the splice (where the shear is a little higher), taking advantage of the permission in 6.6.5.5 to consider groups of connectors but that option is not explored here. Rows of 2 studs would not quite be adequate at the abutment.

The shear flow calculated above is based on elastic section properties and in this example the elastic bending resistance in the span is adequate, even though the composite section is class 2. If plastic bending resistance were utilised, the shear flow would need to be determined between the position where the elastic resistance is just mobilised and the position where the plastic resistance is developed (based on the difference in slab force over that length).

4-2/6.6.2.2

Resistance at SLS

At SLS the shear connector resistance is limited to $k_s P_{Rd}$ with $k_s = 0.75$.

4-2/6.8.1
4-2/NA.2.11

The resistance of 3 studs at 150 mm spacing is thus $0.75 \times 1630 = 1220 \text{ kN}$ and the resistance with 2 studs per row is 815 kN.



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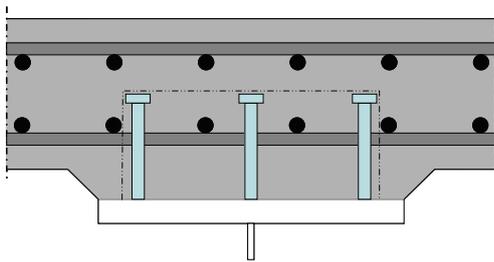
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The SLS shear flows in Section 10.4 are all between 72% and 77% of the ULS values and since the ULS peak shear flows are all less than the ULS design resistances, the SLS requirement is satisfactory by inspection.

10.7 Transverse reinforcement

Consider the transverse reinforcement required to transfer the full shear resistance of the studs at the pier, i.e. 1630 kN/m.



For a critical shear plane around the studs (type b-b in 4-2/Figure 6.15 and shown dotted above) the shear resistance is provided by twice the area of the bottom bars.

The shear force to be resisted is given by 4-2/(6.21) as $1630/\cot\theta$,

Take $\cot\theta = 1$, hence required resistance = 1630 kN/m

Assume B20 bars at 150 mm spacing:

$$\text{Resistance} = A_s f_y d / s_f = (2 \times 314) \times (500/1.15) / 150 \times 10^{-3} = 1821 \text{ kN/m}$$

The transverse bars are adequate. (If they were also required to provide resistance to transverse sagging moment, the resistance would need to be adequate for coexisting combined effects.)

The underside of the heads of the studs need to be at least 40 mm above the transverse bars. In this case an overall stud height of 175 mm should be sufficient, if the haunches are only 50 mm deep.

4-2/6.6.6.1

2-1-1/6.2.4

2-1-1/6.2.4

4-2/6.6.5.4



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11 Fatigue assessment

11.1 Assessment of structural steel details

The design value of the stress range in structural steel is given as

$$\gamma_{FF} \Delta \sigma_{E2} = \gamma_{FF} \lambda \phi_2 \Delta \sigma_p$$

where $\phi_2 = 1.0$ and γ_{FF} is given by the NA as 1.0

The value of $\lambda = \lambda_1 \lambda_2 \lambda_3 \lambda_4$

For intermediate supports in spans up to 30 m, $\lambda_1 = (2 - 0.3 \times (L - 10) / 20)$ where L is the length of the critical influence line (in m) and here $L = 28$ (mean of adjacent spans)

$$\text{Thus } \lambda_1 = (2 - 0.3 \times (L - 10) / 20) = 1.73$$

For span regions, $\lambda_1 = (2.55 - 0.7 \times (L - 10) / 70)$ and here $L = 28$ as before

$$\text{Thus } \lambda_1 = (2.55 - 0.7 \times (L - 10) / 70) = 2.37$$

The value of λ_2 is given by $\lambda_2 = \left(\frac{Q_{m1}}{Q_0} \right) \times \left(\frac{N_{Obs}}{N_0} \right)^{1/5}$

Where $Q_0 = 480$ kN and $N_0 = 0.5 \times 10^6$

From 3-2/NA.2.39, $Q_{m1} = 260$ kN

From 1-2/Table NA.4, $N_{Obs} = 1 \times 10^6$

$$\text{Hence } \lambda_2 = \left(\frac{260}{480} \right) \times \left(\frac{1.0}{0.5} \right)^{0.2} = 0.62$$

For a 120 year design life the value of λ_3 given by Table 9.2 is 1.037:

The value of λ_4 depends on the relative magnitude of the stress range due to the passage of FLM3 in the second lane and is given by:

$$\lambda_4 = \left(1 + \frac{\text{effect in lane 2}}{\text{effect in lane 1}} \right)^{0.2}$$

Design stress ranges at pier

At the pier, the stress range $\Delta \sigma_p$ in top and bottom flanges (at their mid thickness) is:

Top flange: 7.3 N/mm²

Bottom flange: 9.7 N/mm²

The ratio of lane 2/lane 1 effects = 0.940 and thus $\lambda_4 = 1.14$

$$\lambda = 1.73 \times 0.62 \times 1.037 \times 1.14 = 1.274$$

The design stress ranges are thus:

Top flange: $1.0 \times 1.274 \times 7.3 = 9$ N/mm²

Bottom flange: $1.0 \times 1.274 \times 9.7 = 12$ N/mm²

The partial factor for fatigue strength $\gamma_{Mf} = 1.1$

3-2/9.4.1,
9.5.1
3-2/NA.2.35

3-2/9.5.2

Figure 9.5
3-2/9.5.2(2)a

Figure 9.5

3-2/9.5.2(5)

3-2/9.5.2(5)

3-1-9/ NA.2.5.3



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The worst detail category that might apply is for a bearing plate welded to the underside of the bottom flange, which, for a flange plate over 50 mm thick, is category 36 (3-1-9/Table 8.5, detail 6).

Design value of fatigue strength $\Delta\sigma_c / \gamma_{Mf} = 36/1.1 = 33 \text{ N/mm}^2$ OK

Design stress ranges at bracing position

At the bracing position, there is negligible stress range in the top flange. The range in the bottom flange (on the span girder side) is 22.2 N/mm^2 . The ratio of lane 2/lane 1 effects = 0.947 and thus $\lambda_4 = 1.14$

$$\lambda = 2.37 \times 0.62 \times 1.037 \times 1.14 = 1.747$$

The design stress range is thus:

Bottom flange: $1.0 \times 1.747 \times 22.2 = 38 \text{ N/mm}^2$

The most onerous detail at a bolted splice would be category 112 (3-1-9/Table 8.1, detail 8, at the bolt holes); the stress range is OK, by inspection.

For a welded splice, a flange butt weld would be category 80 (with size effect factor of $(25/t)^{0.2} = (25/40)^{0.2} = 0.91$). An open cope hole would introduce category 71 in the flange and a stress concentration factor of 2.4 in the web (for which a cut edge is category 125 or, if a butt weld terminates at the cope hole, category 112).

3-1-9/
Table 8.3

Thus the fatigue strength is either:

Flange butt: $80 \times 0.91/1.1 = 66 \text{ N/mm}^2 > 38 \text{ N/mm}^2$ OK

Flange at cope: $71 \times 0.91/1.1 = 59 \text{ N/mm}^2 > 38 \text{ N/mm}^2$ OK

Web at cope: $125/1.1 = 114 \text{ N/mm}^2 > 38 \times 2.4 = 91 \text{ N/mm}^2$ OK

Web butt at cope: $112/1.1 = 102 \text{ N/mm}^2 > 38 \times 2.4 = 91 \text{ N/mm}^2$ OK

Where a transverse web stiffener is attached to the bottom flange, the detail category would be 80 (3-1-9/Table 8.4, detail 7) and thus the fatigue strength is

$$80/1.1 = 73 \text{ N/mm}^2 > 38 \text{ N/mm}^2 \text{ OK}$$

Design stress ranges in mid-span

At mid-span, there is negligible stress range in the top flange. The range in the bottom flange is 26.2 N/mm^2 . The ratio of lane 2/lane 1 effects = 0.942 and thus $\lambda_4 = 1.14$

$$\lambda = 2.37 \times 0.62 \times 1.037 \times 1.14 = 1.747$$

The design stress range is thus:

Bottom flange: $1.0 \times 1.75 \times 26.2 = 46 \text{ N/mm}^2$

The most onerous detail would be a transverse web stiffener, for which the fatigue strength would be 73 N/mm^2 (as above) and this is OK even for stiffeners welded to the bottom flange.



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11.2 Assessment of reinforcing steel

The design value of the stress range in reinforcing steel is $\gamma_{F,fat} \Delta\sigma_{S,eq}$ where the value of $\gamma_{F,fat}$ is given by Table NA.1 as $\gamma_{F,fat} = 1.0$

2-1/6.8.5

$\Delta\sigma_{S,eq}$ is referred to in EN 1994-2 as $\Delta\sigma_E$, given by:

$$\Delta\sigma_E = \lambda\phi|\sigma_{max,f} - \sigma_{min,f}|$$

4-2/6.8.6.1

The value of $\lambda = \lambda_s$

and $\lambda_s = \phi_{fat} \lambda_{s,1} \lambda_{s,2} \lambda_{s,3} \lambda_{s,4}$

2-1-1/NN.2.1

Where ϕ_{fat} is a damage equivalent impact factor

The value ϕ effectively duplicates ϕ_{fat} but since $\phi = 1.0$, this is not significant

4-2/6.8.6.1

The value of the stress range due to FLM3 needs to be increased by a factor of 1.75 (in regions of intermediate supports) in accordance with NN.2.1(101). Stresses also need to be increased for the effect of tension stiffening in accordance with 4-2/7.4.3

2-1-1/NN.2.1

Based on cracked section properties, the stress in the top rebars due to permanent actions is 118 N/mm² (see SLS values in Section 7.3). (Coexisting global plus local effects do not govern.)

1-2/Annex B

The stress range due to the FLM3 fatigue vehicle in lane 1 is 0 to 8.6 N/mm² (see Section 7.4) and this is increased by the 1.75 factor, giving a range of 15 N/mm² and thus, ignoring tension stiffening, gives $\sigma_{max,f} = 135$ N/mm².

Since FLM3 does not cause sagging bending, $\sigma_{min,f} = 118$ N/mm².

To determine the effect of tension stiffening, the following parameters are needed:

4-2/7.4.3

$A_a = 70000$, $I_s = 1.562 \times 10^{10}$ (for the bare steel section)

Sheet 13

$A = 94250$, $I = 2.845 \times 10^{10}$ (for the cracked section)

2-1-1/

$\rho_s = A_s/A_{ct} = 24250/(3700 \times 250) = 0.0262$

Table 3.1

$\alpha_{st} = AI/A_a I_s = (94250 \times 2.845 \times 10^{10}) / (70000 \times 1.562 \times 10^{10}) = 2.45$

See

$f_{ctm} = 3.5$ MPa (for C40/50 concrete)

4-2/6.8.5.4

$$\Delta\sigma_s = \frac{0.2f_{ctm}}{\alpha_{st}\rho_s} = \frac{0.2 \times 3.5}{2.45 \times 0.0262} = 11 \text{ N/mm}^2$$

for 0.2 factor

Thus, the maximum and minimum stresses including tension stiffening are:

$$\sigma_{s,max,f} = \sigma_{max,f} + \Delta\sigma_s = 135 + 11 = 146 \text{ N/mm}^2$$

$$\text{And } \sigma_{s,min,f} = \sigma_{s,max,f} \frac{M_{E,d,min,f}}{M_{E,d,max,f}}$$

4-2/6.8.5.4

Using the ratio of stresses, rather than directly using moments:

$$\sigma_{s,min,f} = 124 \times \frac{118}{135} = 128 \text{ N/mm}^2$$

For intermediate support region and span of 28 m, $\lambda_{s,1} = 0.97$

2-2/Figure
NN.1



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For $N_{Obs} = 1 \times 10^6$, medium distance traffic and straight bars ($k_2 = 9$): $\bar{Q} = 0.94$ and

$$\lambda_{s,2} = \bar{Q} k_2 \sqrt{\frac{N_{Obs}}{2.0}} = 0.94 \sqrt[9]{\frac{1.0}{2.0}} = 0.87$$

For 120 year design life:

$$\lambda_{s,3} = k_2 \sqrt{\frac{N_{Years}}{100}} = \sqrt[9]{\frac{120}{100}} = 1.020$$

For 2 slow lanes:

$$\lambda_{s,4} = k_2 \sqrt{\frac{\sum N_{Obs,i}}{N_{Obs,1}}} = \sqrt[9]{\frac{2.0}{1.0}} = 1.080$$

For road surface of good roughness $\phi_{fat} = 1.2$

Thus $\lambda = 1.2 \times 0.94 \times 0.87 \times 1.02 \times 1.08 = 1.08$

$$\Delta\sigma_E = 1.08 \times 1.0 \times |146 - 128| = 1.08 \times 18 = 19 \text{ N/mm}^2$$

$$\gamma_{F,fat} \Delta\sigma_{S,eq} = 1.0 \times 19 = 19 \text{ N/mm}^2$$

For straight bars, $\Delta\sigma_{Rsk} = 162.5 \text{ MPa}$

$$\frac{\Delta\sigma_{Rsk}}{\gamma_{s,fat}} = \frac{162.5}{1.15} = 141 \text{ N/mm}^2 > 22 \text{ mm}^2 \text{ OK}$$

Note: If the bars were bent, as they might be at the abutment, the value of $\Delta\sigma_{Rsk}$ would be significantly reduced - see 2-1-1/Table 6.3.N.

11.3 Assessment of shear connection

The design value of the stress range in shear studs is given as $\gamma_{Ff} \Delta\tau_{E2}$ where

$$\Delta\tau_{E,2} = \lambda_v \Delta\tau$$

In which $\Delta\tau$ is the range of shear stress in the cross section of the stud.

EN 1994-2 refers to EN 1993-2 for the value of γ_{Ff} , which is given by the NA as 1.0

The value of $\lambda_v = \lambda_{v,1} \lambda_{v,2} \lambda_{v,3} \lambda_{v,4}$

Since the span is less than 100 m, $\lambda_{v,1} = 1.55$

The value of λ_2 , λ_3 and λ_4 are calculated in the same manner as for structural steel but with an exponent of 1/8 rather than 1/5

$$\text{Hence } \lambda_2 = \left(\frac{260}{480}\right) \times \left(\frac{1.0}{0.5}\right)^{0.125} = 0.591$$

$$\lambda_3 = \left(\frac{120}{100}\right)^{0.125} = 1.023$$

2-2/Table NN.1

1-2/Annex B

2-1-1/
Table 6.3N

2-1/6.8.5

4-2/6.8.7.2

3-2/NA.2.35

4-2/6.8.6.2

6.8.6.2(4)

6.8.6.2(4)



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The value of λ_4 depends on the relative magnitude of the stress range due to the passage of FLM3 in the second lane and is given by:

$$\lambda_4 = \left(1 + \frac{\text{effect in lane 2}}{\text{effect in lane 1}} \right)^{0.125}$$

Shear at pier

The range of vertical shear force at the pier is 271 kN and the ratio of lane 2/lane 1 effects is 0.867.

At the pier, the studs are 19 mm diameter, in rows of 3 at 150 mm spacing

Thus the stress range = Range of vertical shear $\times \bar{A}_z / I_y \times 0.150 / (3 \times \pi d^2 / 4)$

$$\bar{A}_z / I_y = 0.805 \text{ m}^{-1}$$

$$\text{Stress range } \Delta\tau = 271 \times 0.805 \times 0.150 / (3 \times 284) = 38 \text{ N/mm}^2$$

$$\lambda_4 = (1 + 0.867)^{0.125} = 1.081$$

$$\lambda_v = 1.55 \times 0.591 \times 1.023 \times 1.081 = 1.013$$

$$\Delta\tau_{E,2} = 1.013 \times 38 = 39 \text{ N/mm}^2$$

The reference value of fatigue strength for a shear stud is $\Delta\tau_c = 90$

The partial factor on fatigue strength $\gamma_{Mf} = 1.1$.

The design strength is thus $90/1.1 = 81 \text{ N/mm}^2 > 39 \text{ N/mm}^2$ OK

Additionally, since the flange is in tension, the interaction with normal stress in the steel flange must be verified, using:

$$\frac{\gamma_{Ff} \Delta\sigma_{E,2}}{\Delta\sigma_c / \gamma_{Mf}} + \frac{\gamma_{Ff} \Delta\tau_{E,2}}{\Delta\tau_c / \gamma_{Mf,s}} \leq 1.3$$

With $\Delta\sigma_c = 80$.

Coexistent stresses should be used but conservatively one can consider the most onerous values for each of $\Delta\sigma_c$ and $\Delta\tau_c$

$$\frac{1.0 \times 9}{80/1.1} + \frac{1.0 \times 39}{90/1.1} = 0.60 \text{ OK}$$

Shear at splice

The range of vertical shear force at the splice is 99 kN and the ratio of lane 2/lane 1 effects is 0.909

At the splice, the studs are 19 mm diameter, in rows of 2 at 150 mm spacing

$$\text{Stress range} = 99 \times 0.836 \times 0.150 / (2 \times 284) = 22 \text{ N/mm}^2$$

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4-2/6.8.3

3-1-9/
NA.2.5.3

4-2/6.8.7.2



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$$\lambda_4 = (1 + 0.909)^{0.125} = 1.084$$

$$\lambda_v = 1.55 \times 0.591 \times 1.023 \times 1.084 = 1.016$$

$$\Delta\tau_{E,2} = 1.016 \times 22 = 22 \text{ N/mm}^2 < 81 \text{ N/mm}^2 \text{ OK}$$

Shear at midspan

The range of vertical shear force at midspan is 76 kN and the ratio of lane 2/lane 1 effects is 0.895

At midspan, the studs are 19 mm diameter, in rows of 2 at 150 mm spacing

$$\text{Stress range} = 76 \times 0.836 \times 0.150 / (2 \times 284) = 17 \text{ N/mm}^2$$

$$\lambda_4 = (1 + 0.895)^{0.125} = 1.083$$

$$\lambda_v = 1.55 \times 0.591 \times 1.023 \times 1.083 = 1.015$$

$$\Delta\tau_{E,2} = 1.015 \times 17 = 17 \text{ N/mm}^2 < 81 \text{ N/mm}^2 \text{ OK}$$

Shear at abutment

The range of vertical shear force at the abutment is 283 kN and the ratio of lane 2/lane 1 effects is 0.919

At the splice, the studs are 19 mm diameter, in rows of 2 at 150 mm spacing

$$\text{Stress range} = 283 \times 0.836 \times 0.150 / (2 \times 284) = 62 \text{ N/mm}^2$$

$$\lambda_4 = (1 + 0.919)^{0.125} = 1.085$$

$$\lambda_v = 1.55 \times 0.591 \times 1.023 \times 1.085 = 1.017$$

$$\Delta\tau_{E,2} = 1.017 \times 62 = 63 \text{ N/mm}^2 < 81 \text{ N/mm}^2 \text{ OK}$$



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12 Main girder splices

12.1 Forces and moments at splice position

Design effects to be considered:

Worst hogging moment at splice, at ULS and SLS

Worst shear at splice, at ULS and SLS

(The worst sagging moment is much less than maximum hogging moment, as noted before)

Consider the stresses in the pier girder side of the splice. The stress distribution will be different on the span girder side of the splice but the total moments and forces at the splice position must be the same. Because the bottom flange is smaller, more force will be carried in the web on the span side but since the moment on the bolt group on the pier side is increased by its eccentricity from the centreline of the splice and the moment on the group on the span side is decreased, it can be shown that the total effects on the bolt group are less on the span side. A symmetric arrangement of bolts, designed for the pier side, will thus be satisfactory.

The in-service design combinations of actions considered are:

- (1) Construction load + traffic load for worst hogging + force due to temperature expansion
- (2) Construction load + traffic load for worst shear + force due to temperature expansion.

	ULS hog	SLS hog	ULS shear	SLS shear
Top flange stress	23	21	-38	-23 N/mm ²
Bottom flange stress	-110	-88	-10	-14 N/mm ²
Shear force	834	650	1320	1011 N/mm ²

From the above stresses, the forces in each flange, the axial force and moment in the web are as follows:

	ULS hog	SLS hog	ULS shear	SLS shear
Top flange force	460	420	-760	-460 kN
Bottom flange force	-3960	-3168	-360	-504 kN
Web force	-644	-497	-336	-259 kN
Web moment	161	132	-33	-11 kNm

Actions at the construction stage

It is noted that the compressive stress in the top flange is higher at construction stage 1, under wet concrete load in span 1. At that stage, the splice must provide continuity of stiffness, without slipping, and because the beams are slender at that stage it is appropriate to amplify the design force to ensure adequate continuity of resistance.

The maximum stress in the top flange during construction is 42 N/mm²

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The slenderness of the beam at that stage is $\bar{\lambda}_{LT} = 0.89$ which means that

$$M_{c,Rk}/M_{cr} = 0.89^2 = 0.79$$

The midspan bending moment at the bare steel stage is $M_{Ed} = 3132$ kNm (Sheet 23) and the resistance of the cross section $M_{c,Rk} = 2.287 \times 10^7 \text{ mm}^3 \times 345 = 7890$ kNm.

$$\text{Hence } \frac{M_{Ed}}{M_{cr}} = \frac{M_{c,Rk}}{M_{cr}} \times \frac{M_{Ed}}{M_{c,Rk}} = 0.79 \times \frac{3132}{7890} = 0.316$$

Second order effects can thus be determined by multiplying by $\frac{1}{(1 - 0.316)} = 1.46$

Thus the design force for the top flange is $840 \times 1.46 = 1226$ kN

(Clearly this is more onerous than in the final situation, where the stresses are of similar magnitude but the flange is restrained against buckling and no amplification is needed.)

12.2 Slip resistance of bolts

Use M24 grade 8.8 preloaded bolts in double shear in normal clearance holes with class A friction surface:

- $d = 24$ mm
- $d_0 = 26$ mm
- $f_{ub} = 800$ N/mm²
- $A_s = 353$ mm²
- $\mu = 0.5$
- $k_s = 1.0$

$$\text{Preload force } F_{p,C} = 0.7f_{ub}A_s = 0.7 \times 800 \times 353 \times 10^{-3} = 198 \text{ kN}$$

ULS Slip resistance of bolts (double shear)

$$F_{s,Rd} = \frac{k_s n \mu}{\gamma_{M3}} F_{p,C} = \frac{1.0 \times 2 \times 0.5}{1.25} \times 198 = 158 \text{ kN}$$

For SLS slip resistance, use the same equation but divide by $\gamma_{M3,ser}$ (= 1.1)

$$\text{Slip resistance in double shear} = 198/1.1 = 180 \text{ kN}$$

12.3 Shear resistance of bolts

ULS shear resistance of bolt (assuming shear through threads):

$$F_{v,RD} = \frac{\alpha_v f_{ub} A_s}{\gamma_{M2}} = \frac{0.6 \times 800 \times 353}{1.25} = 136 \text{ kN}$$

$$\text{Resistance in double shear} = 272 \text{ kN}$$

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3-1-8/3.9.1

3-1-8/
Table 3.4



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12.4 Bolt spacing and edge distances

Limiting spacings for M24 bolts, for strength:

- End and edge distances: $1.2d_0 = 1.2 \times 26 = 31.2$ mm
- Spacing in direction of force: $2.2d_0 = 2.2 \times 26 = 57.2$ mm
- Spacing perpendicular to force $2.4d_0 = 2.4 \times 26 = 62.4$ mm

3-1-8/
Table 3.3

Limiting spacings for M24 bolts, for fatigue classification:

- End and edge distances: $1.5d = 1.5 \times 26 = 39$ mm
- Spacing: $2.5d = 2.5 \times 24 = 60$ mm

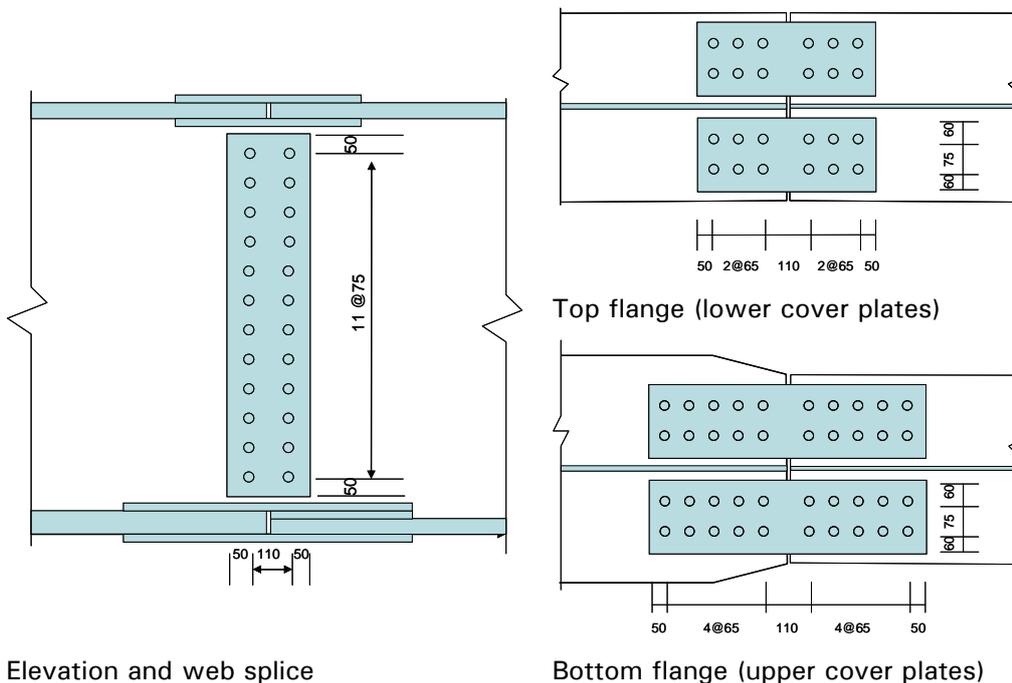
3-1-9/
Table 8.1

(The parameter d is not specified in Table 8.1 but GN 5.08 (P185^[6]), suggests use of hole diameter for edge distances and bolt diameter for spacings.)

For detailing purposes, use minima of 40 mm, 65 mm and 70 mm respectively

12.5 Splice configuration

Consider the following splice configuration:



Elevation and web splice

Bottom flange (upper cover plates)

Top flange splice

(Dimensions for lower covers)

Bolt spacing:

- In line of force: $e_1 = 50$ mm, $p_1 = 65$ mm
- Perpendicular to force: $e_2 = 60$ mm, $p_2 = 75$ mm
- Overall dimension 470×195 mm
- Thickness 10 mm

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<p>The length of the cover is sufficiently short that stud shear connectors do not need to be welded to the upper cover (the maximum permitted longitudinal spacing is 800 mm).</p> <p>Bottom flange splice (Dimensions for upper covers)</p> <p>Bolt spacing:</p> <p>In line of force: $e_1 = 50$ mm, $p_1 = 65$ mm</p> <p>Perpendicular to force: $e_2 = 60$ mm, $p_2 = 75$ mm</p> <p>Overall dimension 210×195 mm.</p> <p>Thickness 20 mm</p> <p>Web splice</p> <p>Bolt spacing:</p> <p>In line of force: $e_1 = 50$ mm (only a single column, so no p_1 value)</p> <p>Perpendicular to force: $e_2 = 50$ mm, $p_2 = 75$ mm</p> <p>Overall dimension 730×925 mm.</p> <p>Thickness 10 mm</p> <p>The web depth on the support side of the splice is 1000 mm and if the web splice is positioned symmetrically within this depth, the centreline of the lowest bolt will be 87.5 mm above the flange and 67.5 mm above the cover plate. This is adequate for the tightening of the bolt (see GN 2.06, P185^[6]).</p> <p>12.6 Verification of connection resistances</p> <p>Top flange splice</p> <p>There are 3 rows of bolts, with 4 bolts per row across the flange.</p> <p>A category C connection is required (the design situation is for resistance against buckling of the beam during construction).</p> <p>Slip resistance at ULS = $12 \times 158 = 1896$ kN > 1226 kN adequate</p> <p>Bottom flange splice</p> <p>There are 5 rows of bolts, with 4 bolts per row across the flange.</p> <p>A category B connection is required (the design situation is for resistance against compression in the flange in service).</p> <p>Resistance at ULS</p> <p>ULS slip resistance = $20 \times 158 = 3160$ kN < 3960 kN so the splice will slip into bearing at ULS</p> <p>ULS shear resistance of bolt group = $20 \times 272 = 5440$ kN - adequate</p>			4-2/6.6.5.5



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ULS bearing resistance per bolt is given by:

$$F_{b,Rd} = \frac{k_1 \alpha_b f_u d t}{\gamma_{M2}} =$$

Bolt spacings, for determination of factors k_1 and α_b

In line of force: $e_1 = 50$ mm, $p_1 = 65$ mm

Perpendicular to force: $e_2 = 60$ mm, $p_2 = 75$ mm

Since $f_{ub} > f_u$, $\alpha_b = \alpha_d$ (but ≤ 1)

For end bolts: $\alpha_d = e_1/3d_0 = 50/(3 \times 26) = 0.64$

For inner bolts: $\alpha_d = p_1/d_0 - 1/4 = 65/(3 \times 26) - 0.25 = 0.58$

For edge bolts k_1 is the smaller of $2.8e_2/d_0 - 1.7$ and 2.5

$k_1 = \min(2.8 \times 60/26 - 1.7; 2.5) = 2.5$

In the upper cover plates there is no 'inner' line of bolts (in the direction of force) and for the flange and lower cover, the mean value of p_2 that would apply is sufficient to ensure that $k_1 = 2.5$

The value of f_u is given by the product standard for S355 plates as 470 kN/mm^2

EN 10025-2

Conservatively, using $\alpha_b = 0.58$ the resistance of the bolt in 20 mm covers is:

$$F_{b,Rd} = \frac{2.50 \times 0.58 \times 470 \times 24 \times 20}{1.25} = 262 \text{ kN}$$

Bearing resistance of group, with double covers = $20 \times 2 \times 262 = 10480 \text{ kN}$

The ULS bearing resistance is adequate and the connection resistance is determined by the shear resistance of the bolts. Note that, on the span side, 20 mm packing is used. This would reduce the bearing/shear resistance on the upper shear plane by about 15% (see 3-1-8/3.6.1(12)) but the resistance would still be adequate.

Resistance at SLS

SLS slip resistance of group = $20 \times 180 = 3600 \text{ kN} > 3312 \text{ kN}$ satisfactory

Web splice

The splice has a single column of 12 bolts at 75 mm spacing

For this group the 'modulus' for the outer bolts = $\sum r_i^2 / r_{\max}$

where r_i is the distance of each bolt from the centre of the group and r_{\max} is the distance of the furthest bolt.

Here, the modulus = 1950 mm

The extra moment due to the shear = shear force \times eccentricity of group from the centreline of the splice



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Hence the force on the outer bolts at ULS and SLS are

	ULS hog	SLS hog	ULS shear	SLS shear	
Shear V	834	650	1320	1011 kN	
Longitudinal force F_L	609	469	-336	-259 kN	
Moment	155	127	-33	-11 kNm	
Moment due to $e = 55$ mm	46	36	73	56 kNm	
Total Moment M	201	163	40	45 kNm	
Force per bolt due to M	103	84	21	23 kN	(= $M/1950$)
Force per bolt due to F_L	51	39	-28	22 kN	(= $F_L/12$)
Total horizontal force	154	123	49	45 kN	
Vertical force due to V	70	54	110	84 kN	(= $V/12$)
Resultant force	169	134	120	95 kN	(Vector sum)

Bearing resistance for web bolts

Note: The directions of the resultant forces are not parallel to an edge. Table 3.4 suggests that in such cases the parallel and normal components could be verified separately but no interaction relationship is suggested. Here the direction of the resultant force being not normal to the long edge of the cover plate edge is considered not to have an adverse effect on the factors, since the edge distance is less than the end distance. The factors for resistance in a horizontal direction are therefore used.

For end bolts (there is only a single row, transverse to the force):

$$\alpha_d = e_1/3d_0 = 50/(3 \times 26) = 0.64$$

For edge bolts $k_1 = \min(2.8 \times 50/26 - 1.7; 2.5) = 2.5$

For inner bolts $k_1 = \min(1.4 \times 75/26 - 1.7; 2.5) = 2.34$

With two 10 mm covers, the bearing stress on the 14 mm web is higher (and is higher again on the 10 mm web, although the values for the design forces on that side of the splice are lower and are not shown here)

$$F_{b,Rd} = \frac{2.50 \times 64 \times 470 \times 24 \times 14}{1.25} = 202 \text{ kN (for end bolts, 192 kN for inner bolts)}$$

The bearing resistance is less than the resistance of the bolts in double shear (272 kN), so bearing resistance governs.

The maximum resultant force at ULS (164 kN) exceeds the slip resistance (158 kN) but is less than the resistance in bearing and shear (202 kN) therefore the bolt group is satisfactory. The maximum force at SLS is 134 kN and the resistance is 180 kN so there is no slip at SLS. The forces on the inner bolts are less and are satisfactory by inspection.

12.7 Forces in cover plates

The cover plates are verified as members in tension or compression, in accordance with EN 1993-1-1.



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Top flange

The covers are in tension. Assume half of the load is carried in the lower cover plates. The force per cover plate thus = $1302/4 = 326$ kN

Area of gross cross section = $195 \times 10 = 1950$ mm²

Area of net section = $1950 - 2 \times 26 \times 10 = 1430$ mm²

This is a Category C slip resistant connection, therefore the design tension resistance is given by:

$$N_{\text{net,Rd}} = \frac{A_{\text{net}} f_y}{\gamma_{M0}} = \frac{1430 \times 355}{1.0} \times 10^{-3} = 508 \text{ kN Satisfactory}$$

3-1-1/6.2.3

The maximum spacing of bolts is 110 mm and the limiting spacing is given by Table 3.3 as the smaller of $14t (= 140 \text{ mm})$ and 200 mm. Since $p_1/t = 65/20 = 3.25$, which is less than $9\varepsilon (= 7.2)$ buckling does not need to be checked. The spacing is satisfactory.

3-1-1/
Table 3.3

Bottom flange

The covers are in compression. Assume half of the load is carried in the upper cover plates. The force per cover plate thus = $3960/4 = 990$ kN

Fastener holes do not need to be deducted (unless oversize holes are allowed), therefore $A = 1950 \times 20 = 3900$ mm²

$$N_{\text{pl,Rd}} = \frac{A f_y}{\gamma_{M0}} = \frac{3900 \times 345}{1.0} \times 10^{-3} = 1346 \text{ kN Satisfactory}$$

The maximum spacing of bolts is 110 mm and the limiting spacing is given by Table 3.3 as the smaller of $14t (= 280 \text{ mm})$ and 200 mm. The spacing is satisfactory.

3-1-1/
Table 3.3

Web

Consider the stresses in the cover plate on a line through the vertical row of bolts.

The moment on each cover plate = $201/2 = 101$ kN

The axial force = $609/2 = 305$ kN

The shear force = $834/2 = 417$ kN

The stress at the bottom of the cover plate is thus:

$$(101 \times 10^6) / (10 \times 925^2/6) + (305 \times 10^3) / (925 \times 10) = 71 + 33 = 104 \text{ N/mm}^2$$

The value of $p_1/t = 115/10 = 11$, which is greater than $9\varepsilon (= 7.2)$ so buckling must be checked. Using a buckling length of $0.6p_1 = 66$ mm and $i = 10/\sqrt{12} = 2.89$ mm, the slenderness is:

3-1-8/
Table 3.3
3-1-1/6.3.1.3

$$\lambda = \frac{L_{\text{cr}}}{i\lambda_1} = \frac{66}{2.98 \times 76.5} = 0.30$$



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From buckling curve a, $\chi = 0.98$, so the limiting stress = $0.98 \times 355/1.1 = 316 \text{ N/mm}^2$, which is satisfactory. The spacing also complies with the limit of $14t$ (= 140 mm).

The shear stress is:

$$417 \times 10^3 / (10 \times 925) = 45 \text{ N/mm}^2$$

This is satisfactory and is low enough that the resistance to direct stress does not need to be reduced.



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13 Transverse web stiffeners

13.1 Intermediate stiffeners

The intermediate stiffeners are required to have adequate stiffness and strength.

Choose flat stiffeners 200×20 mm for both the pier girder and span girder.

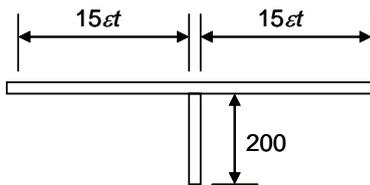
The limiting outstand to prevent torsional buckling is given by 9.2.1(8) and for flat stiffeners this equates to a limit of $h_s/t_s \leq 13\varepsilon$ (see P356, Section 8.3).

For the yield strength of the stiffener ($f_y = 345$), $\varepsilon = 0.825$ and the limit is:

$$h_s/t_s \leq 10.7 \text{ - satisfactory}$$

Stiffness

The effective section is



For the web, $f_y = 355$ and, $\varepsilon = 0.81$

	pier girder	span girder
$15\varepsilon t$	170	122
Area of Tee	9043	6630
I_{st}	39.0×106	30.9×106

Since, for the web panels, $a/h_w = 1967/1000 = 1.967 > \sqrt{2}$ the stiffness requirement is $I_{st} \geq 0.75h_w t^3$

Required $I_{st} = 0.75 \times 1020 \times 14^3 = 2.06 \times 10^6 \text{ mm}^4$ (for the pier girder).

The stiffener is satisfactory for both girders.

Strength

The stiffener is required to sustain an axial force, applied in the plane of the web, given by:

$$V_{Ed} - \frac{1}{\lambda_w^2} \frac{f_{yw} h_w t}{\sqrt{3} \gamma_{M1}}$$

Here, for the panel adjacent to the support at the pier, $\bar{\lambda}_w = 0.934$

Take max shear at support (the value $0.5h_w$ from the support may be used):

$$V_{Ed} = 2511 \text{ kN}$$

$$\frac{1}{\lambda_w^2} \frac{f_{yw} h_w t}{\sqrt{3} \gamma_{M1}} = \frac{1}{0.934^2} \times \frac{355 \times 1000 \times 14}{\sqrt{3} \times 1.1} \times 10^{-3} = 2990 \text{ kN}$$

3-1-5/9.2.1
P356^[4]

3-1-5/
9.3.3(5)

3-1-5/9.3.3(3)

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Therefore, the stiffener does not need to be designed for an axial force.

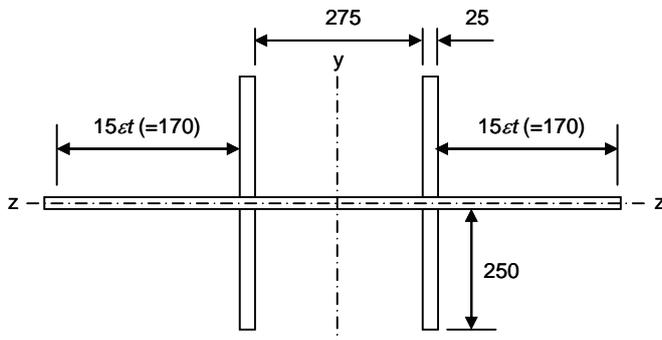
Since the web is Class 3, there is no destabilising effect of the web on the stiffener, so the requirements of 9.2.1(5) do not need to be applied.

The intermediate stiffeners are satisfactory.

13.2 Bearing stiffeners

Consider the adequacy of double flat stiffeners, 250 × 25 mm on both sides of the web at the intermediate support.

The outstand/thickness ratio is 10, as for intermediate stiffeners and is therefore satisfactory.



For the web, $t = 14$ mm, $f_y = 355$ and, $\varepsilon = 0.81$

	pier girder	
$15\varepsilon t$	170	mm
Area of effective stiffener	34310	mm ²
I_y	907×10^6	mm ⁴
I_z	566×10^6	mm ⁴
i_z	128	mm

For buckling out of the plane of the web, the critical buckling length $L_{cr} = 1050$ mm (taken to mid-thickness of flanges)

$$\bar{\lambda} = \frac{L_{cr}}{\lambda_1 i}$$

3-1-1/(6.50)

$\lambda_1 = 93.9\varepsilon$ and for the stiffener $f_y = 345$ and $\varepsilon = 0.825$, so

$$\lambda_1 = 93.9 \times 0.825 = 77.5$$

3-1-1/
6.3.1.2(4)

$$\text{Thus } \bar{\lambda} = \frac{1050}{77.5 \times 128} = 0.11$$

Since this value is < 0.2 , buckling can be ignored



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The reactions at the support due to the construction stages are:

	ULS
Stage 1	894
Stage 2	636
Stage 3	533

The maximum reaction due to variable load is 1976 kN and occurs with gr5 loading and the following effects:

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial (steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Construction	-8329	-295	1029		-193		193		138		2
Traffic gr5	-2476	269	874	45.67	-54	66.63	37	41.84	59	94.3	-3
	-10805	-26	1903		-247		230		197		-1

Although there should be no thermal movement longitudinally (the bridge is a symmetric integral bridge and the bearing is at the mid-length of the bridge) and the transverse movement is very small (with one of the inner main girders restrained laterally), allow for an eccentricity of 10 mm in each direction.

$$N_{Ed} = 894 + 636 + 533 + 1976 = 4039 \text{ kN}$$

$$M_{Ed} = 4039 \times 0.010 = 40.4 \text{ kNm}$$

$$W_z = 566 \times 10^6 / 257 = 2.20 \times 10^6 \text{ mm}^3 \text{ (tip of stiffener)}$$

$$W_y = 907 \times 10^6 / 150 = 6.05 \times 10^6 \text{ mm}^3 \text{ (at stiffener)}$$

$$W_y = 907 \times 10^6 / 331 = 2.73 \times 10^6 \text{ mm}^3 \text{ web (at edge of section)}$$

Interaction criterion

$$\frac{N_{Ed}}{N_{Rd}} + \frac{M_{y,Ed}}{M_{y,Rd}} + \frac{M_{z,Ed}}{M_{z,Rd}} \leq 1.0 \text{ (biaxial interaction)}$$

Use the value of $M_{y,Rd}$ based on the modulus at the stiffener, not on the web.

$$N_{Rd} = \frac{Af_y}{\gamma_{M0}} = \frac{34310 \times 345}{1.0} \times 10^{-3} = 11840 \text{ kN}$$

$$M_{y,Rd} = \frac{W_y f_y}{\gamma_{M0}} = \frac{6.05 \times 345}{1.0} = 2087 \text{ kNm}$$

$$M_{z,Rd} = \frac{W_z f_y}{\gamma_{M0}} = \frac{2.20 \times 345}{1.0} = 759 \text{ kNm}$$

$$\frac{4039}{11840} + \frac{40.4}{2087} + \frac{40.4}{759} = 0.34 + 0.02 + 0.05 = 0.41 \text{ OK}$$

A separate verification should also be made for the extreme fibre in the web, which is subject to axial force and uniaxial bending. The interaction values for that case are

$$0.34 + 0.04 = 0.38, \text{ which is also satisfactory.}$$



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No verification of the interaction of these vertical stresses with longitudinal stresses and shear stresses in the web is called for in EN 1993.

13.3 Bearing at loaded end of the stiffener

There is no explicit verification called for at the interface between the flange and the effective bearing stiffener but it should nevertheless be verified.

Web/flange interface

If the web is not fitted to the flange (which is the usual case) the force must be transferred through the weld. To transfer the full strength of the web, consider the strength of fillet welds loaded transversely to their length.

Using the simplified method of 3-1-8/4.5.3.3 and neglecting the longitudinal force on the weld, the resistance of a 6 mm throat fillet weld is:

$$F_{w,Rd} = a \frac{f_u / \sqrt{3}}{\beta_w \gamma_{M2}} = 6 \times \frac{470 / \sqrt{3}}{0.9 \times 1.25} = 1450 \text{ N/mm}$$

As noted above, the maximum utilisation in the web is 0.41, which is equivalent to a vertical stress of 141 N/mm². The design force in the web at that position is therefore 141 × 14 = 1974 N/mm. The two 6 mm welds are adequate.

Stiffener/flange interface

The ends of the 25 mm flats should be fitted and welded to the flange, because it is impractical to provide a sufficiently heavy fillet weld. The fillet weld must then be checked for fatigue, as follows.

Range of reaction due to passage of FLM3 = 293 kN (lane 1) and 251 kN (Lane 2)

The stress range at the tip of the flat due to this range is:

$$293000/34310 + 2930 \times (1/2730 + 1/6050) = 11 \text{ N/mm}^2$$

The force per unit length = 25 × 11 = 275 N/mm

The fatigue resistance should be checked at the toe of the weld (on the stiffener) and at the root of the weld.

At the toe of the weld, the detail category is 71 (Table 8.1, for 60 mm flange) and the stress range is satisfactory by inspection.

At the root of the weld, the stress range is given by dividing the force/unit length by the weld throat (see 3-1-9/Figure 5.1) and the detail category is 36 (3-1-9/Table 8.5, detail 3)

Since there is no longitudinal or transverse shear force, for a 6 mm throat fillet weld,

$$\sigma_{wf} = \sigma_{\perp f} = 275 / (2 \times 6) = 23 \text{ N/mm}^2$$



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As for intermediate support regions in Section 11.1:

$$\lambda_1 = 1.73$$

$$\lambda_2 = 0.62$$

$$\lambda_3 = 1.037$$

$$\lambda_4 = \left(1 + \left(\frac{251}{293} \right)^5 \right)^{0.2} = 1.08$$

Design value is: $23 \times 1.73 \times 0.62 \times 1.037 \times 1.08 = 28 \text{ N/mm}^2$

Fatigue strength = 36 N/mm^2 (Table 8.5, constructional detail 3)

Design value of fatigue strength = $36/\gamma_{Mf} = 36/1.1 = 33 \text{ N/mm}^2$

The weld is satisfactory.

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3-1-9/
Table 8.5



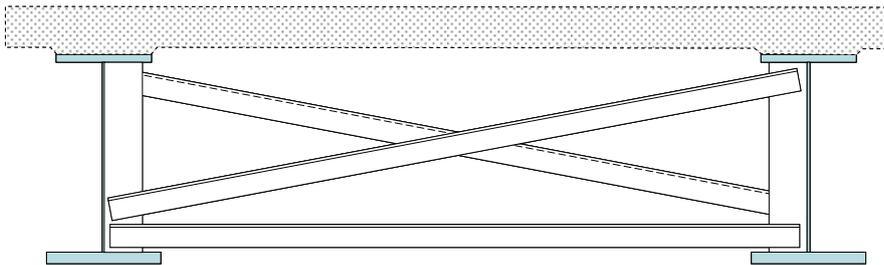
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14 Bracing

The configuration of the intermediate bracing systems between girder pairs is as shown below.



Assume the use of 120 × 120 × 12 angle sections.

Consider requirements for stiffness and strength

To perform as a fully effective intermediate restraint to the bottom flange adjacent to the support, the stiffness needs to be at least the value given by:

$$C_D = \frac{4N_E}{L} = \frac{4\pi^2 EI}{L^3}$$

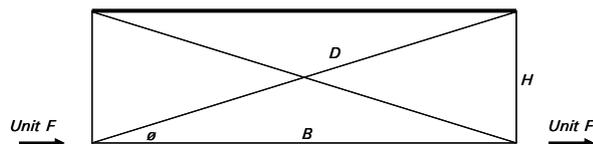
where

I is the lateral second moment of area of the effective bottom flange (in the simplified method considered in Section 9.1)

L is the length of flange restrained by the bracing

$$C_D = \frac{4\pi^2 \times 210000 \times 1.08 \times 10^9}{5900^3} = 44 \text{ kN/mm}$$

The stiffness of the bracing system can be determined from a simple plane frame model that reflects the actual geometry, including eccentric end connections, and the effective section of the intermediate stiffeners or a value can be determined from the simple triangulated system below:



$$\begin{aligned} B &= 3700 \text{ mm} \\ H &= 1100 \text{ mm} \\ D &= 3860 \text{ mm} \\ \theta &= 0.289 \text{ rad} \end{aligned}$$

(The use of a diagonal system between top and bottom flanges will generally give a greater flexibility than that with the more detailed plane frame model and the shallower inclination of the angles.)



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For a unit force, the lateral displacement is given by consideration of equilibrium and axial stiffness of bracing members as:

$$\delta = \frac{(1/\cos \phi) \times D}{EA_{\text{brace}}} \times \left(\frac{1}{\cos \phi} \right) + \frac{(1 \times \tan \phi)H}{EA_{\text{stiff}}} \times \left(\frac{1}{\tan \phi} \right)$$

$$A_{\text{brace}} = 2750 \text{ mm}^2 \text{ (for a 120 x 120 x 12 angle)}$$

$$A_{\text{stiff}} = 9043 \text{ mm}^2$$

$$\delta = \frac{3860}{210 \times 2750 \times 0.959^2} + \frac{1100}{210 \times 9043} = 0.00785 \text{ mm/kN}$$

Hence stiffness = $1/0.00785 = 127 \text{ kN/mm}$ - satisfactory

The strength of the bracing system must be sufficient to restrain the lateral force F_{Ed} . Since $L_k = 1196 \text{ mm} < 1.2\ell = 1.2 \times 5900 = 6980 \text{ mm}$, the restraint force is given by:

$$F_{\text{Ed}} = \frac{N_{\text{Ed}}}{100}$$

For the value of N_{Ed} , use the stress in the bottom flange at the pier and multiply by the area of the effective flange in the simplified model for buckling resistance.

$$F_{\text{Ed}} = \frac{277 \times 38470}{100} \times 10^{-3} = 107 \text{ kN}$$

The buckling resistance of the 3860 mm diagonal is easily adequate for this force and two-bolt end connections will also be adequate.

3-2/6.3.4.2(5)

WORKED EXAMPLE 2:

Ladder deck three-span bridge

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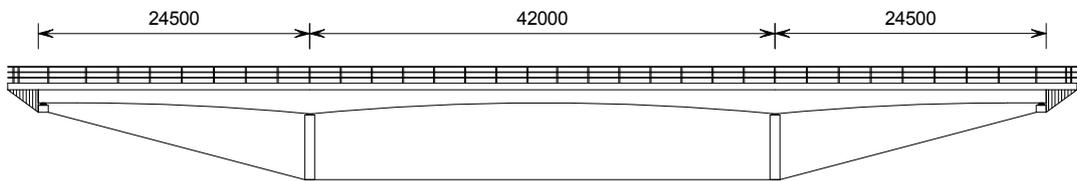
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1 Structural arrangement

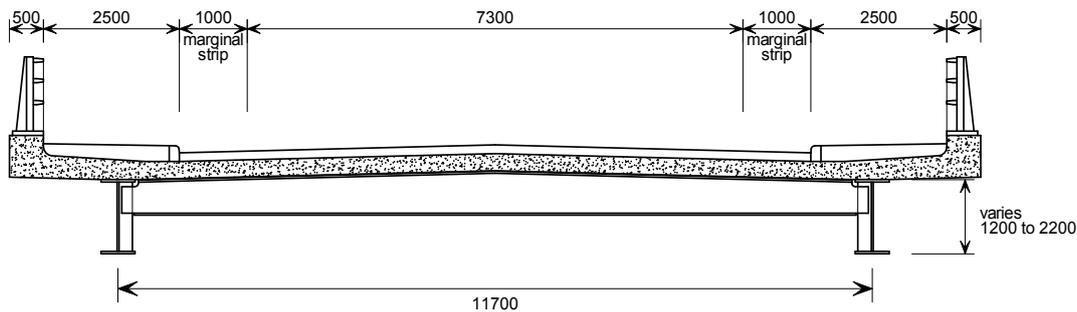
The bridge carries a 2-lane single carriageway rural road over a flood plain. The carriageway has 1.0 m wide marginal strips, in accordance with TD 27/05 and has a 2 m wide footway on either side (this width is slightly less than the width for footways given by TA 90/05). A ladder deck girder arrangement has been chosen, and a deck slab thickness of 250 mm has been assumed. The deck cantilevers 1.3 m outside the centrelines of the outer girders; a 250 mm thick slab is likely to be adequate for this length, carrying footway loading or accidental traffic loading.

TD 27/05^[1]

TA 90/05^[2]



Elevation



Cross section (at intermediate cross girder)



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2 Design basis

The bridge is to be designed in accordance with the Eurocodes, as modified by the UK National Annexes.

The basis of design set out in EN 1990 is verification by the partial factor method.

Design situations to be considered are as given in Example 1 and for brevity, they are not repeated here; see Sheets 2 and 3 in Example 1 for details.

Example 1

2.1 Partial factors and combination factors

For a full summary of factors for all types of action, see Example 1.

The values of the principal factors used in this Example are:

At ULS:

$\gamma_G = 1.35$ for concrete self weight, 1.20 for steel self weight and superimposed dead load (factors for adverse effects). $\gamma_Q = 1.35$ for traffic loads, 1.55 for thermal loads.

At SLS, factors of unity apply.

A combination factor $\psi_0 = 0.75$ applies to traffic actions where they are accompanying actions and $\psi_0 = 0.60$ to thermal actions where they are accompanying actions.

The values tabulated in Section 7 are after application of the relevant factors.

2.2 Structural material properties

It is assumed that the same structural material grades as in Example 1 will be used:

- Structural steel: S355 to EN 10025-2
- Concrete: C40/50 to EN 206-1
- Reinforcement: B500 to EN 10080 and BS 4449

For structural steel, the value of f_y depends on the product standard.

(Use 355 N/mm² for $t \leq 16$ mm; 345 N/mm² for $16 \text{ mm} > t \leq 40$ mm; and 335 N/mm² for $40 \text{ mm} < t \leq 63$ mm)

For concrete, $f_{ck} = 40$ MPa

For reinforcement $f_{yk} = 500$ N/mm²

The modulus of elasticity of both structural steel and reinforcing steel is taken as 210 GPa (as permitted by EN 1994-2, 3.2).

3-1-1/NA.2.4
2-1-1,
Table 3.1



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For concrete, it is assumed that the average age at first loading is the same as in Example 1 and thus the values of the modulus of elasticity of the concrete and long-term shrinkage strain are:

	Short term	Long term	Shrinkage (long-term)
E_{cm}	35 GPa	12.6 GPa	
Modular ratio	$n_0 = 6.0$	$n_L = 16.7$	$n_L = 15.4$
Drying shrinkage			$\epsilon_{cd} = 33.1 \times 10^{-5}$

In this example, the shrinkage effects will be taken into account at their long term values where they are unfavourable. Where the effects are favourable, lesser values at 56 days could be considered but it is conservative to neglect shrinkage in that case.

Example 1



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3 Actions on the bridge

3.1 Permanent actions

Self weight of structural elements

The 'density' of steel is taken as 77 kN/m^3 and the density of reinforced concrete is taken as 25 kN/m^3 . The self weights are based on nominal dimensions.

1-1-1/Table A.1

Self weight of surfacing

The total nominal thickness of the surfacing, including waterproofing layer is 130 mm. Assume that the 'density' is 23 kN/m^3 for the whole thickness.

The self weight generally produces adverse effects and for that case the self weight is based on nominal thickness +55%. Thus:

1-1-1/Table NA.1

$$g_k = 1.55 \times 0.13 \times 23 = 4.63 \text{ kN/m}^2$$

Self weight of footway construction

The nominal thickness of the footway (comprising concrete fill and a thin surfacing) is 200 mm and a uniform density of 24 kN/m^3 is assumed. The self weight is based on the nominal dimensions and thus:

$$g_k = 1.0 \times 0.2 \times 24 = 4.80 \text{ kN/m}^2$$

Self weight of parapets

A nominal value of 2 kN/m is assumed for each parapet.

3.2 Construction loads

Construction loads are classed as variable loads.

1-1-6/2.2

For global analysis, a uniform construction load of $Q_{ca} = 0.75 \text{ kN/m}^2$ is assumed during casting. The use of permanent precast planks is assumed and thus there is no extra load for formwork. Additionally, wet concrete is assumed to have a density of 1 kN/m^3 greater than that of hardened concrete; for a slab thickness of 250 mm this adds $Q_{cf} = 0.25 \text{ kN/m}^2$

1-1-6/
Table 4.2

The total construction load is thus: $Q_c = 0.75 + 0.25 = 1.0 \text{ kN/m}^2$

3.3 Traffic loads

Road traffic

Normal traffic is represented by Load Model 1 (LM1).

1-2/
NA.2.16.1.2

For the road carried by this bridge, the highway authority specifies that abnormal traffic be represented by special vehicle SV100, as defined in the UK National Annex.

Pedestrian traffic

Pedestrian traffic is represented by the reduced value given by the NA to BS EN 1991-2, Table NA.3 and clause NA.2.36. Thus $0.6q_{rk}$ is applied ($= 0.6 \times 5.0 = 3 \text{ kN/m}^2$). The reduction for longer loaded lengths is not made.

1-2/Table NA.3



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Fatigue loads

For fatigue assessment, Fatigue Load Model 3 (FLM3), defined in 1-2/4.6.4, is used, as recommended by 3-2/9.2.2

3.4 Thermal actions

Shade temperatures

Maximum and minimum shade temperatures, based on a 50-year return period are defined in BS EN 1991-1-5 NA.2.20. For this bridge location, the values are: 1-1-5/NA.2.20

Maximum 33°C
 Minimum -17°C

Thermal range (for determination of extreme value of thermal movement)

For determination of the maximum movement at ULS, the values for a 120 year design life are relevant but according to EN 1990:A2, these are determined by applying $\gamma_Q = 1.55$ to characteristic values for a 50 year return period. Table NA.A2.4(B) Note 6

The values of maximum/ minimum uniform bridge temperatures are given by EN 1991; these are referred to as $T_{e,min}$ and $T_{e,max}$ 1-1-1/6.1.3.1

For Type 2 deck 1-1-5/
Figure 6.1

$$T_{e,max} = T_{max} + 4 \text{ (Figure 6.1)}$$

$$T_{e,min} = T_{min} + 5$$

3-2/A.4.2.1(4)

The characteristic value ΔT_K is thus $\frac{1}{2}[(33 + 4) - (-17 + 5)] = 49/2 = 24.5^\circ\text{C}$

The design value of temperature difference is given by

$$\Delta T_d^* = \Delta T_K + \Delta T_\gamma + \Delta T_0 \quad 3-2/(A.6)$$

Assuming that bearing installation will be with estimated temperature and without correction by resetting, $\Delta T_0 = 15^\circ\text{C}$ 3-2/ Table A.4 NA.2.50

According to the UK NA, $\Delta T_\gamma = 5^\circ\text{C}$

Thus

$$\Delta T_d^* = 24.5 + 5 + 15 = 44.5^\circ\text{C}$$

For change of length in composite sections, the coefficient of linear thermal expansion is 12×10^{-6} per °C. 4-2/5.4.2.5

Thus, if the fixed bearing is at one end of the bridge, the characteristic value of displacement at the second intermediate support is:

$$v_x = (24500+42000) \times 44.5 \times 12 \times 10^{-6} = 35.5 \text{ mm}$$



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Vertical temperature difference

The vertical temperature difference given in Table 6.2b will be used and temperature difference will be considered to act simultaneously with uniform temperature change, as recommended in NA.2.12, if that is more onerous. For surfacing thickness other than 100 mm, interpolate in Table B.2, as follows:

Surfacing thickness (mm)	ΔT for slab thickness (mm)	
	200	300
100	13	16
150	10.5	12.5

Interpolating for slab thickness 250 mm, surfacing thickness 130 mm, gives $\Delta T = 12.7^\circ\text{C}$.
 (The 55% increase over nominal thickness, where surfacing load is adverse, is ignored.)

Only the heating difference is considered here; it is more onerous than the cooling difference situation.

For temperature difference in composite sections, the coefficient of thermal expansion is 10×10^{-6} per $^\circ\text{C}$

1-1-5/
Table 6.2b

4-2/5.4.2.5



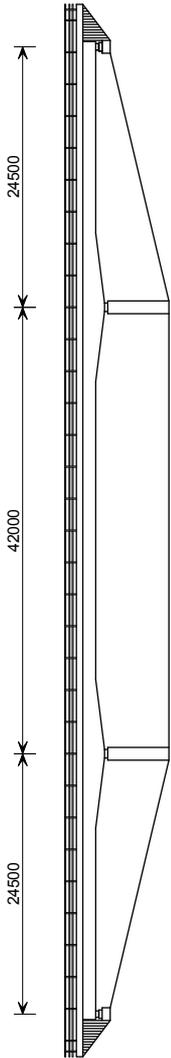
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4 Girder make-up and slab reinforcement

4.1 Main girders



	16.5 m span girder	16 m pier girder	26 m span girder	16 m pier girder	16.5 m span girder
Top flange	800 × 40	800 × 50	800 × 40	800 × 50	800 × 40
Web	12	20	12	20	12
Bottom flange	800 × 50	800 × 60	800 × 50	800 × 60	800 × 50
Top rebars	B20 @ 150 crs	B20 @ 150 crs	B20 @ 150 crs	B20 @ 150 crs	B20 @ 150 crs
Bottom rebars	B20 @ 150 crs	B20 @ 150 crs	B20 @ 150 crs	B20 @ 150 crs	B20 @ 150 crs

The overall girder depth varies from 2200 mm at the intermediate supports to 1200 in midspan and at the abutments.
 The cover to the top longitudinal bars is 35 mm, appropriate to XC3.
 For cross section properties, the cover to the bottom bars is taken conservatively as 40 mm, appropriate to XC4; where precast permanent formwork is used, some bars will be slightly higher.

The make-up shown above uses only two different combinations of flange sizes and web thicknesses. In practice, more variation might be used, for greater economy.

Cross girders are positioned at 3500 mm centres in all three spans, connected to the main girders by bolting to flat transverse web stiffeners on the main girders.



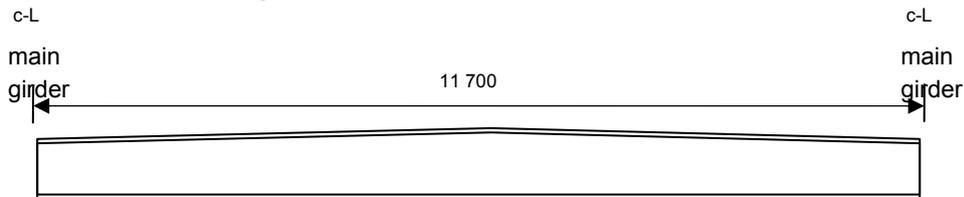
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4.2 Cross girders

Intermediate cross girders



Overall depth 750 mm at the ends, 896 mm at the centre

Flanges: 300 × 25

Web: 15 mm

The web is unstiffened, except possibly at the cross girder mid-span if the cross girders need to be braced for the construction condition.

Pier crosshead

At the intermediate supports, a 2000 mm deep crosshead girder is provided, with jacking stiffeners close to the main girders for bearing replacement. The design of the crosshead is not covered in the example.



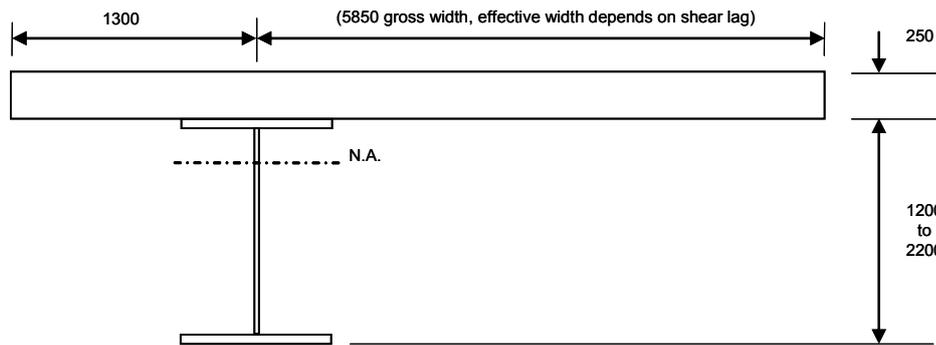
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5 Beam cross sections

5.1 Section properties - main girders



For determination of stresses in the cross section and resistances of the cross section, the effective width of the slab, allowing for shear lag is needed. The following calculations summarize the effective section properties for the sections considered.

The FE analysis will automatically take account of shear lag, so the gross section dimensions are used in the model.

The equivalent spans for effective width are:

Midspan section: $L_c = 0.70 L_1 = 0.70 \times 42 = 29.4$ m

Hogging section: $L_c = 0.25 (L_1 + L_2) = 0.25 \times 66.5 = 16.625$ m

At mid-span, $b_{eff} = b_0 + \sum b_{ei}$

where $b_{ei} = L_c / 8$ each side, but not more than geometric width

Assume that the outer stud connectors are 300 mm from the centreline of the girder.

$b_{ei} = 29400 / 8 = 3675$ mm, so the width from the centreline is 3975 mm

At the pier, $b_{ei} = 16625 / 8 = 2078$ mm, so the width from the centreline is 2378 mm

Classification of cross sections is determined separately for bending and for axial compression, in accordance with EN 1993-1-5. Where a web is slender, the Class 4 effective section will be different for bending and for axial compression.

Bare steel cross sections - gross section properties

		Span girder	Pier girder	
Area	A	85320	129800	(mm ²)
Height of NA		549	1037	(mm)
Second moment of area	I_y	2.515E+10	1.160E+11	(mm ⁴)
Elastic modulus, centroid top flange	$W_{bf,y}$	3.986E+07	1.020E+08	(mm ³)
Elastic modulus, centroid bottom flange	$W_{tf,y}$	4.800E+07	1.152E+08	(mm ³)
Section class		4 (sagging)	3 (hogging)	
Plastic bending resistance	M_{pl}	15110	38900	kNm

4-2/Figure 5.2

4-2/(5.3)

3-1-5/4.3

Properties calculated by spreadsheet

Value of M_{pl} calculated using f_y / γ_{M0} values for steel

For the span girder, the values for the elastic moduli of the gross section will be used for build up of stresses in the span girder during construction, since classification for the total stresses will be at least Class 3.



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For the pier section, gross section properties will be used for the build up of bending stresses (since the section is Class 3 in bending) but an effective area will be used the effects of any axial compression at this stage, since the bare section is Class 4 in compression (for which $A_{eff} = 105000 \text{ mm}^2$)

Bare steel cross sections - effective section properties in bending

The values for the effective section moduli are needed for verification of the span girder at the bare steel stage (when the cross section is Class 4 in bending).

The effective breadth of the Class 4 web is given by Table 4.1, with:

$$\psi = -499/611 = -0.817 \text{ (and thus } k_{\sigma} = 19.5 \text{ and } \bar{\lambda}_p = 0.911), \text{ which gives:}$$

$$\rho = [\bar{\lambda}_p - 0.055(3 + \psi)] / \bar{\lambda}_p^2 = [0.911 - 0.55 \times 3.911] / 0.911^2 = 0.953$$

$$b_{eff} = \rho \bar{b} / (1 - \psi) = 0.953 \times 1110 / 1.817 = 0.953 \times 611 = 582 \text{ mm}$$

There is thus a hole in the web $611 - 582 = 29 \text{ mm}$ long centred 913 mm above the soffit

		Span girder	
Height of NA		547	(mm)
Second moment of area	I_y	2.510E+10	(mm ⁴)
Elastic modulus, centroid top flange	$W_{bf,y}$	3.965E+07	(mm ³)
Elastic modulus, centroid bottom flange	$W_{tf,y}$	4.808E+07	(mm ³)

Composite cross sections (short term) ($n_0 = 6.0$)

		Span girder	Pier girder	
Area	A	305100		(mm ²)
Height of NA		1108	1735	(mm)
Second moment of area	I_y	6.336E+10	2.332E+11	(mm ⁴)
Elastic modulus, top of slab	W_c	1.112E+09		(mm ³)
Elastic modulus, centroid top flange	$W_{bf,y}$	8.800E+08		(mm ³)
Elastic modulus, centroid bottom flange	$W_{tf,y}$	5.850E+07		(mm ³)
Plastic bending resistance	M_{pl}	22519		kNm

The cross section of the span girder is Class 1, provided that the top flange is restrained by shear connectors within the spacing limits in 4-2/6.6.5.5 (in this case, max spacing 730 mm, max edge distance 299 mm).

Uncracked pier girder section properties are needed for calculation of shear flow.

Composite cross sections (long term) - sagging, midspan ($n_L = 16.7$)

		Span girder	
Area	A	164300	(mm ²)
Height of NA		922	(mm)
Second moment of area	I_y	5.028E+10	(mm ⁴)
Elastic modulus, top of slab	W_c	1.590E+09	(mm ³)
Elastic modulus, centroid top flange	$W_{tf,z}$	1.949E+08	(mm ³)
Elastic modulus, centroid bottom flange	$W_{bf,z}$	5.605E+07	(mm ³)

3-1-5/4.4

Value of M_{pl} calculated using f_y/γ_{M0} values for steel, $0.85f_{ck}/\gamma_c$ for concrete



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Cracked composite sections (hogging) - pier girder

		Gross	Effective*	
Area	A	145200	120660	(mm ²)
Height of NA		1174	1160	(mm)
Second moment of area	I _y	1.390E+11	1.380E+11	(mm ⁴)
Elastic modulus, top rebars	W	1.129E+08	1.129E+08	(mm ³)
Elastic modulus, centroid top flange	W _{tf,y}	1.389E+08	1.391E+08	(mm ³)
Elastic modulus, centroid bottom flange	W _{bf,y}	1.215E+08	1.197E+08	(mm ³)
Section class		4	4	

The cracked section is class 4, in bending and under axial compression.

* The effective area is that for a Class 4 section under axial compression only; the effective elastic section moduli are for a Class 4 section in bending only, as permitted by 3-1-5/4.3 (3) & (4).

Cracked composite sections (hogging) -at first cross girder in main span

		Gross	Effective	
Area	A	141300	122640	(mm ²)
Height of NA		1032		(mm)
Second moment of area	I _y	1.033E+11		(mm ⁴)
Elastic modulus, top rebars	W	9.861E+08		(mm ³)
Elastic modulus, centroid top flange	W _{tf,y}	1.234E+08		(mm ³)
Elastic modulus, centroid bottom flange	W _{bf,y}	1.031E+08		(mm ³)
Section class		3*	4	

* The cross section is on the Class 3/4 boundary in bending. It is class 4 under axial compression.

5.2 Primary effects of temperature difference & shrinkage

Temperature difference

For calculation of primary effects, use the short-term modulus for concrete:

$$E_{cs} = 35 \text{ GPa} \quad (\text{For steel, } E = 210 \text{ GPa})$$

Note: For each element of section, calculate stress as strain \times modulus of elasticity, then determine force and centre of force for that area.

For a fully restrained section comprising the full half-width of the slab, the restraint force and moment in the span girder due to the characteristic values of temperature difference noted on Sheet 6 are:

	Av strain	Force (kN)	Centre of force		Moment (kNm)
			Below top	Above NA	
Top part of slab	0.000084	3153	62	236	744
Bottom part of slab	0.000036	901	198	100	90
Top flange	0.000026	202	270	28	6
Web (to 400 below slab)	0.000012	9	410	-112	-1
		<u>4265</u>			<u>839</u>

Note that the full width of slab from the centreline to the edge of the cantilever is used to determine the full effects of restraint.

Sheet 2

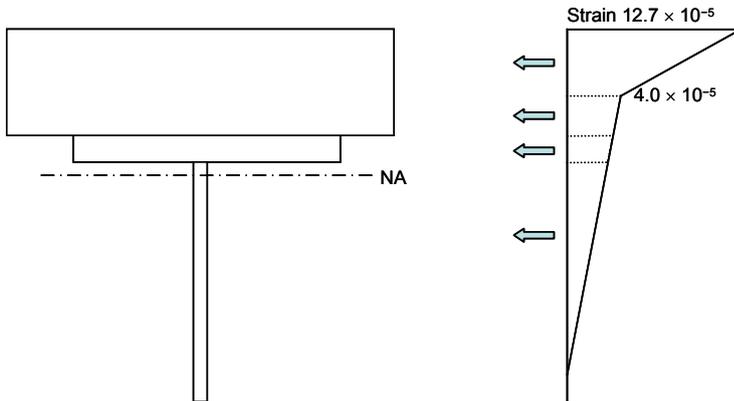


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The strains and forces are illustrated diagrammatically below.

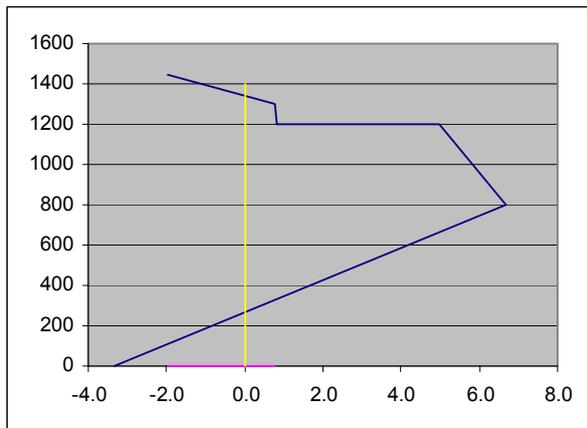


The primary effects (stresses) are given by:

	W (steel units)	Restraint ($\Delta T \alpha E$)	Release of restraint		Total
			Bending (M/W)	Axial (F/A)	
Top of slab	2.24E+08	-4.4	0.6	1.9	-1.9
0.6 into slab	4.50E+08	-1.4	0.3	1.9	0.8
Bottom of slab	1.39E+09	-1.1	0.1	1.9	0.9
Top of top flange	1.39E+09	-6.7	0.6	11.1	5.0
400 below slab	-1.89E+08	0.0	-4.4	11.1	6.7
Bottom flange	-5.79E+07	0.0	-14.5	11.1	-3.4

For the above calculation, short-term section properties for the full width of slab are used, not those tabulated on Sheet 10. (Area = 383200 mm², steel units)

Diagrammatically:





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The release of the restraint moments is applied along the span, in the uncracked regions, as a separate loadcase. Since the girder depth varies along the span, the value of the restraint moment will vary along the span. For simplicity, a uniform 'average' value has been applied to the model in this example.

Note that the omission of restraint moments in cracked regions is not mentioned in EN 1994-2 but the view has been taken that the omission permitted for shrinkage (see EN 1994-2, 5.4.2.2(8)) may be used for the calculation of secondary effects of temperature difference.

Shrinkage

For complete verification, shrinkage effects should be calculated at the time of opening to traffic and at the end of the service life and the more onerous values used. Here, primary and secondary effects are calculated only for the long-term situation (the values are greater than those at opening) and where the total effects of shrinkage are advantageous, they are neglected.

The characteristic value of shrinkage strain is given on Sheet 3 as $\epsilon_{cd} = 33.1 \times 10^{-5}$ and the modular ratio is $\eta_L = 15.4$. (This is very close to the value for long-term effects generally and for determining the secondary effects, the one set of long-term properties will be used for both.)

For a fully restrained section, the restraint force and moment in the span girder, inner beam, due to the characteristic values of shrinkage strain are given by:

	Strain	Force (kN)	Centre of force		moment (kNm)
			Below top	Above NA	
Slab	-0.000331	-8049	125	329	-2647
		<u>-8049</u>			<u>-2647</u>

The release of the restraint moments is applied along the span, in the uncracked regions, as a separate loadcase. Since the girder depth varies along the span, the value of the restraint moment will vary along the span. For simplicity, a uniform 'average' value has been applied to the model in this example.

4-2/5.4.2.2(8)

Hence the primary effects are:

	W (steel units)	Restraint ($\epsilon_{cd} E_{cm}$)	Release of restraint		Total
			Bending (M/W)	Axial (F/A)	
Top of slab	1.22E+08	4.5	-1.4	-2.6	0.5
Bottom of slab	2.72E+08	4.5	-0.6	-2.6	1.3
Top of top flange	2.72E+08	0.0	-9.7	-40.0	-49.7
Bottom flange	-5.56E+07	0.0	47.6	-40.0	7.6

(Area = 201391 mm², steel units)

For the above calculation, section properties for the full width of slab are used.

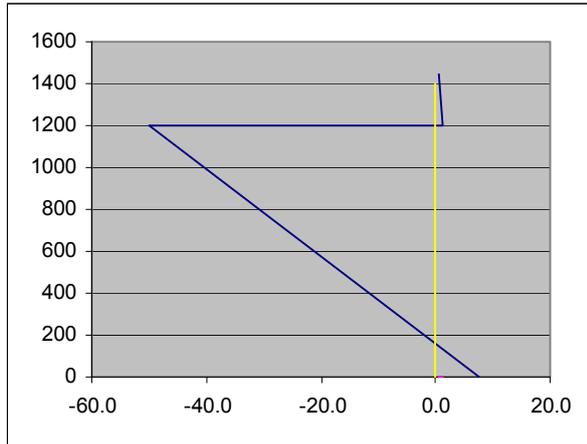


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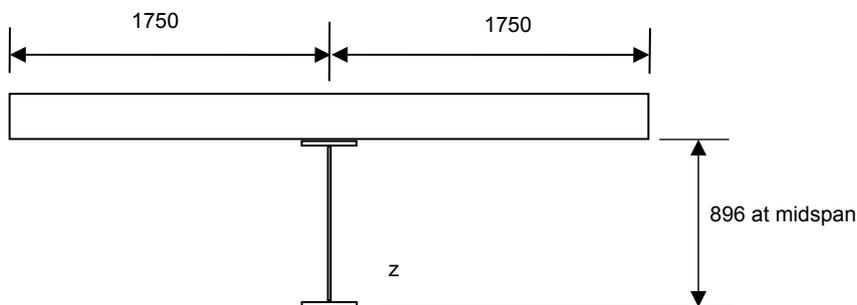
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Diagrammatically:



5.3 Section properties - cross girders

The gross composite section of the cross girder includes half the width of slab to the adjacent cross girders.



For the effective composite section, taking account of shear lag, the cross girder is effectively simply supported and thus $L_e = 11700$ mm and $b_{ei} = 11700/8 = 1438$ mm each side. Assume that there is only a single row of connectors on the beam centreline.

The effective section properties of the cross girder at midspan are:

		Bare steel	Short-term comp	Long-term comp	
Area	A	27690	149600	71477	(mm ²)
Height of NA		448	915	799	(mm)
Second moment of area	I _y	3.603E+09	1.163E+10	9.404E+09	(mm ⁴)
Top of slab			3.021E+08	4.526E+08	
Elastic modulus, centroid top flange	W _{bf,y}	8.273E+06	-3.692E+08	1.113E+08	(mm ³)
Elastic modulus, centroid bottom flange	W _{tf,y}	8.273E+06	1.289E+07	1.196E+07	(mm ³)
Section class		2 (bending)	1 (sagging)	1 (sagging)	
Plastic bending resistance	M _{pl}	3207	6232 (sagging)		kNm

Values of M_{pl} calculated using f_y/γ_{M0} values for steel, 0.85f_{ck}/γ_C for concrete.



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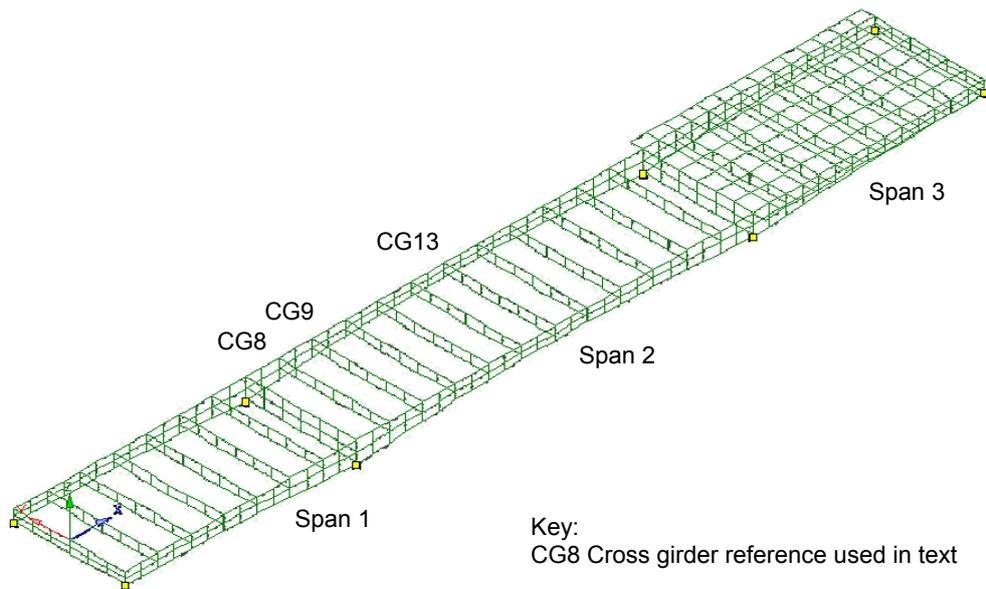
6 Global analysis

6.1 3D FE model

A 3D finite element model of the structure was created, as shown below.

The steel girders were represented by beam elements for the flanges and shell elements for the web. The deck slab was represented by a mesh of shell elements. The mesh for the deck slab was divided longitudinally by lines of nodes along each cross girder and midway between; transversely the mesh was divided into six elements between each main girder and one for each cantilever portion of slab. This division for the deck slab was intended to give an accurate representation of shear lag effects, without the need for the application of empirical allowances in the model.

Cracked section properties were used for the slab elements either side of the intermediate support (approximately 15% of the span either side of the support). The elements were given anisotropic properties (cracked longitudinally, uncracked transversely).



Analysis model, showing main girders, cross girders and slab mesh on span 3 only

The global analysis effectively gives a comprehensive pattern of stresses in all elements in the model. The software then converts the stresses into equivalent moments and forces on notional composite beams, for verification in accordance with the Eurocode rules. Each notional composite beam comprises the steel elements of flanges and web plus a width of deck slab.

For the cross beams, the notional beam includes half the width of slab either side, i.e. the forces and moments are based on the stresses in one deck slab element either side. For the main beams, the notional beams include the cantilever element and the width of slab to the bridge centreline.

Because shear lag effects are automatically accounted for in the global analysis model, the moments and forces per beam thus depend on the 'actual' shear lag rather than on any notional pattern of shear lag, such as that given by the rules in EN 1994-2.



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The extracted effects on each main girder section (as defined above) were combinations of moment and axial force. The axial forces arise mainly as a result of the edge beam being modelled separately but also as a result of unequal loading on the two main girders (there is a small longitudinal shear across the bridge centreline when the loading is not symmetrical).

The application of the extracted moment plus axial force on the effective cross sections (allowing for shear lag) is a realistic combination of effects on those sections. The variation of axial force in the edge beam gives rise to (relatively small) additional longitudinal shear which can be taken into account in verifying the shear connection (see Section 10.1)

6.2 Construction stages

For simplicity, it is presumed that the deck will be concreted in three stages - the whole of span 1, followed by the whole of span 3, followed by the whole of span 2. The edge beams will be concreted after span 2. Separate analytical models are therefore provided for:

- Stage 1 All steelwork, wet concrete in span 1
- Stage 2 Composite structure in span 1 (long-term properties), wet concrete in span 3
- Stage 3 Composite structure in spans 1 & 3, wet concrete in span 2
- Stage 4 Composite structure in both spans (long-term properties)
- Stage 5 Composite structure (short term properties)

(For simplicity, the weight of the edge beams is applied to the stage 4 model, which includes the long-term properties of the edge beams, rather than introduce another model. The difference between the two approaches is negligible, in relation to the design of the main beams.)

A further model, a modification of Stage 3, without the wet slab, was analysed to determine the rotational stiffness of the beams at that stage.

6.3 Analysis results

The following results are for design values of actions, i.e. after application of appropriate partial factors on characteristic values of actions, except for the individual load cases for shrinkage and temperature difference (for which characteristic values are given).

For construction loading, results are given for the total effects at each of the construction stages. For traffic loading the results are given for the combination of traffic and pedestrian loading for worst bending effects at two locations - at an intermediate support and at the middle of the central span. For verification of buckling resistance adjacent to the intermediate support, additional results were extracted for coexistent effects at the position of the first cross girder in the central span.



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Stage 1

Self weight of steelwork
 Self weight of concrete on span 1
 Construction loads on span 1

Position	ULS			SLS		
	M_y (kNm)	F_x (kN)	F_z (kN)	M_y (kNm)	F_x (kN)	F_z (kN)
At pier	-4432	4	311	-3476	3	255
At CG8	-3291		309	-2548	0	250
At CG13	121		82	161	0	61

Note: F_x is axial force, F_z is vertical shear
 CG8 is the first intermediate cross girder on the main span side of the pier. CG13 is at midspan.

Stage 2

Self weight of concrete on span 3
 Construction loads on span 3
 Removal of construction loads on span 1

Position	ULS			SLS		
	M_y (kNm)	F_x (kN)	F_z (kN)	M_y (kNm)	F_x (kN)	F_z (kN)
At pier	1307	-1	-77	968	-1	-57
At CG8	1004		-95	744	0	-70
At CG13	-584		-91	-433	0	-67

Stage 3

Self weight of concrete on span 2
 Construction loads on span 2
 Removal of construction loads on span 3

Position	ULS			SLS		
	M_y (kNm)	F_x (kN)	F_z (kN)	M_y (kNm)	F_x (kN)	F_z (kN)
At pier	-9374	48	1209	-6943	35	895
At CG8	-5267	38	1083	-3902	28	802
At CG13	4371	40	2	3237	30	2

Stage 4

Self weight of concrete edge beams
 Self weight of parapets
 Self weight of carriageway surfacing
 Self weight of footway construction
 Removal of construction loads on span 2

Position	ULS			SLS		
	M_y (kNm)	F_x (kN)	F_z (kN)	M_y (kNm)	F_x (kN)	F_z (kN)
At pier	-6944	303	936	-5693	257	769
At CG8	-3825	227	828	-3130	195	681
At CG13	2904	-457		2388	-370	0



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Long term shrinkage (restraint moments applied in uncracked regions)

The following characteristic values apply at both ULS and SLS, since $\gamma_{Sh} = 1.0$

Position	characteristic		
	M_y (kNm)	F_x (kN)	F_z (kN)
At pier	-3414	175	-15
At CG8	-3397	235	0
At CG13	-3737	-35	0

Sheet 2
and
Example 1

Stage 5 - variable actions

Traffic loads for worst hogging at intermediate support

Load Group	Position	ULS			SLS		
		M_y (kNm)	F_x (kN)	F_z (kN)	M_y (kNm)	F_x (kN)	F_z (kN)
Gr1,	At pier	-11113	598	1463	-8231	444	1083
Gr1,	At CG8	-6034	532	1400	-4469	395	1037
Gr5,	At pier	-10828	719	1368	-8020	532	1013
Gr5	At CG8	-5969	661	1379	-4421	490	1021

Traffic loads for worst shear at intermediate support

Load Group	Position	ULS			SLS		
		M_y (kNm)	F_x (kN)	F_z (kN)	M_y (kNm)	F_x (kN)	F_z (kN)
Gr1	At pier	-8190	492	2029	-6021	371	1509
	At CG8	-2405	341	1206	-1753	259	885
Gr5	At pier	-7719	563	2169	-5675	422	1612
	At CG8	-1493	351	1419	-1084	265	1046

Traffic loads for worst sagging at middle of centre span

Load Group	Position	ULS			SLS		
		M_y (kNm)	F_x (kN)	F_z (kN)	M_y (kNm)	F_x (kN)	F_z (kN)
Gr1	At CG13	8120	-1728	-113	6015	-1278	-83
	At CG12	7052	-1553	488	5224	-1149	361
Gr5	At CG13	8585	-1880	-69	6359	-1391	-51
	At CG12	7434	-1612	662	5507	-1192	490

CG13 is at midspan, CG12 is at the adjacent cross girder

Effects of thermal actions

Effects due to the characteristic values of vertical temperature difference (restraint moments applied in uncracked regions)

Position	Characteristic		
	M_y (kNm)	F_x (kN)	F_z (kN)
At pier	1233	-57	5
At CG8	1224	-89	-1
At CG13	1352	37	



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Worst shear at midspan

Position	ULS		SLS	
	F_z (kN)	ΔF_x over 3.5m	F_z (kN)	ΔF_x over 3.5m
Gr 1 traffic	672	427	498	316
Gr5 traffic	755	530	559	393

Range of effects due to passage of fatigue vehicle

Worst bending effects

	Pier M_y (kNm)	F_x (kN)	Mid-span M_y (kNm)	F_x (kN)
Lane 1 pos	-1560	146	1278	-289
Lane 1 neg	243	-14	-179	56
Range	-1803	161	1457	-344
Lane 2 pos	-1109	50	891	-227
Lane 2 neg	170	-7	-126	33
	-1279	57	1017	-260

Worst shear effects

	Pier F_z (kN)	Mid-span F_z (kN)	Associated change of F_x	
			Pier	Mid-span
Lane 1 pos	403	105	41	-99
Lane 1 neg	-21	-114	0	45
Range	423	219	41	-144
Lane 2 pos	227	68	20	-57
Lane 2 neg	-14	-76	1	23
	241	143	19	-80

The change of axial force F_x is the value over a panel of 3.5 m

Effects on intermediate cross girder

With the traffic load positioned for worst effects on the cross girders, the worst sagging occurs in the middle cross girder in the central span. The values are:

	At mid-span of CG13		
	M_y	F_x	F_z
Dead Loads at Stage 1 (ULS)	48	0	0
Dead Loads at Stage 2 (ULS)	0	0	0
Dead Loads at Stage 3 (ULS)	523	2	
Dead Loads at Stage 4 (ULS)	272	104	
	843	106	0
Gr1 worst sagging (ULS)	1960	-93	43
Gr5 worst sagging (ULS)	2192	-78	184



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For consideration of the effects on the bolted end connection, the values at the first and second cross girders adjacent to the pier and the middle cross girder are:

	CG8			CG9			CG13		
	M_y	F_x	F_z	M_y	F_x	F_z	M_y	F_x	F_z
Stage 1	-19	10	16	-10	7	15	0	0	16
Stage 2	-1	-4	0	0	1	0	0	0	0
Stage 3	-86	40	180	-27	11	180	-17	1	180
Stage 4	-106	29	99	-91	29	113	-68	-26	108
Construction	-212	75	295	-128	48	308	-85	-25	304
Gr1 traffic (for max shear)	-436	49	901	-281	-24	843	-201	-187	806
Gr5 traffic(for max shear)	-447	94	933	-276	-22	879	-184	-239	859
Total (gr5)	-659	169	1228	-404	26	308	-269	-264	1163



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7 Design values of the effects of combined actions

Design values of effects are given below for certain design situations for parts of the central span. In practice, further situations for other parts of the structure would also need to be considered.

7.1 Effects of construction loads (ULS)

Generally, the effects of construction loads apply to different cross section properties, as construction progresses. For the pier girder, on the main span side, the effects are on the bare steel section for stages 1, 2 and 3; long term effects are on the effective cracked composite section. For the section at the middle of the central span, effects at stages 1, 2 and 3 are on the bare steel gross section and stage 4 effects on the long-term composite section.

Stresses at pier (main span side)

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Stage 1	-4432	4	311	115.2	-38	102.0	43			105	0
Stage 2	1307	-1	-77	115.2	11	102.0	-13			105	0
Stage 3	-9374	48	1209	115.2	-81	102.0	92			105	0
Stage 4	-6944	303	936	119.7	-58	139.1	50	112.9	62	121	-3
Shrinkage ($\gamma_{sh} = 1$)	-3414	176	-15	119.7	-29	139.1	25	112.9	30	121	-1
	<u>-22857</u>	<u>530</u>	<u>2364</u>		<u>-195</u>		<u>197</u>		<u>92</u>		<u>-4</u>

Using steel and cracked section properties

Stress at CG8 (pier girder, cracked section)

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Stage 1	-3291	0	309	96.9	-34	85.3	39			105	0
Stage 2	1004	0	-95	96.9	10	85.3	-12			105	0
Stage 3	-5267	38	1083	96.9	-54	85.3	62			105	0
Stage 4	-3825	227	828	103.1	-37	123.4	31	96.8	40	123	-2
Shrinkage ($\gamma_{sh} = 1$)	-3397	235	0	103.1	-33	123.4	28	96.8	35	123	-2
	<u>-14776</u>	<u>500</u>	<u>2125</u>		<u>-148</u>		<u>148</u>		<u>75</u>		<u>-4</u>

Using steel and cracked section properties

Stress at mid-span (span girder)

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Stage 1	121	0	82	48.0	3	39.9	-3			85.3	0
Stage 2	-584	0	-91	48.0	-12	39.9	15			85.3	0
Stage 3	4371	40	2	48.0	91	39.9	-110			85.3	0
Stage 4	2904	-457	0	56.1	52	194.9	-15	1590	-1.8	164	3
Shrinkage ($\gamma_{sh} = 1$)	(not adverse)										
	<u>6812</u>	<u>-417</u>	<u>-7</u>		<u>134</u>		<u>-113</u>		<u>-1.8</u>		<u>3</u>

Using steel and long term section properties



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7.2 Effects of traffic loads plus construction loads (ULS)

Effects due to traffic actions and temperature difference are determined from the cracked cross section at the pier and the short term composite section in midspan.

Loading for maximum hogging at pier

The worst effects are due to gr1 traffic loads. Effects due to temperature difference are not adverse.

Effects at pier position

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial (steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Construction	-22857	530	2364		-195		197		92		-4
Gr 1 traffic	-11113	598	1463	119.7	-93	139.1	80	112.9	98	121	-5
	<u>-33970</u>	<u>1128</u>	<u>3827</u>		<u>-288</u>		<u>277</u>		<u>190</u>		<u>-9</u>

Using cracked properties for the effects of variable actions

Coexistent effects at CG8

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial (steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Construction	-14776	500	2125		-148		148		75		-4
Gr 1 traffic	-6034	532	1400	103.1	-59	123.4	49	96.8	62	123	-4
	<u>-20810</u>	<u>1032</u>	<u>3525</u>		<u>-207</u>		<u>197</u>		<u>137</u>		<u>-8</u>

Using cracked properties for the effects of variable actions

Loading for maximum sagging bending

The maximum sagging moments on the composite beam occur at mid-span.

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial (steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Construction	6812	-417	-7		134		-113		-1.8		3
Traffic gr5	8585	-1880	-69	58.5	147	880.0	-10	1112	-7.7	305	6
Temp difference*	1257	34	0	58.5	21	880.0	-1	1112	-1.1	305	0
	<u>16654</u>	<u>-2263</u>	<u>-76</u>		<u>302</u>		<u>-124</u>		<u>-10.6</u>		<u>9</u>

Using short term composite properties for the effects of variable actions

* $\gamma_Q = 1.55$ and $\psi_0 = 0.6$ applied to characteristic values

Loading for maximum shear

Shear at pier position

The value of the maximum shear is needed to verify the shear resistance of the web and to determine the longitudinal shear on the stud connectors.

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial (steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Construction	-22857	529	2364		-192		189		83		-3
Gr 5 traffic	-7719	564	2169	123	-63	155	50	125	62	127	-4
	<u>-30576</u>	<u>1093</u>	<u>4533</u>		<u>-255</u>		<u>239</u>		<u>145</u>		<u>-7</u>

Using cracked properties for the effects of variable actions



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7.3 Effects of traffic loads plus construction loads (SLS)

The values of effects at SLS are needed to verify crack control in the slab at the pier.

Effects at pier position

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial (steel)	
				W (10^6 mm^3)	σ (10^6 mm^3)	W (10^6 mm^3)	σ (10^6 mm^3)	W (10^6 mm^3)	σ (10^6 mm^3)	A (10^3 mm^2)	σ
Stage 1	-3476	3	255	115.2	-30	102.0	34			105	0
Stage 2	968	-1	-57	115.2	8	102.0	-9			127	0
Stage 3	-6943	35	895	115.2	-60	102.0	68			127	0
Stage 4	-5693	257	769	119.7	-48	139.1	41	112.9	50	121	-2
Shrinkage	-3414	175	-15	119.7	-29	139.1	25	112.9	30	121	-1
Gr 1 traffic	-8231	444	1083	119.7	-69	139.1	59	112.9	73	121	-4
	<u>-26789</u>	<u>913</u>	<u>2930</u>		<u>-228</u>		<u>218</u>		<u>153</u>		<u>-7</u>

Using steel and cracked section properties

Effects at midspan

The values at midspan would be required if the ULS effects exceeded the elastic resistance but, by inspection, the stresses at ULS are less than the elastic design values.

7.4 Effects due to fatigue vehicle

The range of bending effects due to the passage of the fatigue vehicle in each lane is determined at the two locations already considered for static loading.

At pier

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial(steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Range, lane 1	-1803	161	226	121.5	-14.8	138.9	13.0	112.9	16.0	145.2	-1.1
Range, lane 2	-1279	-7		121.5	-10.5	138.9	9.2	112.9	11.3	145.2	0.0

Ratio lane 2/lane 1 moments = 0.709

At mid-span

	M_y	F_x	F_z	Bottom flange		Top flange		Top of slab		Axial(steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Range, lane 1	1457	-344	-9	58.5	24.9	880.0	-1.7	1112.0	-1.3	305.1	1.1
Range, lane 2	1017	-260	-13	58.5	17.4	880.0	-1.2	1112.0	-0.9	305.1	0.9

Ratio lane 2/lane 1 moments = 0.698

Maximum and minimum effects (for max/min stresses in reinforcement at pier)

	M_y	F_x	F_z	Bottom flange		Top flange		Top rebar		Axial(steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Max effect	-1560	146	205	121.5	-12.8	138.9	11.2	112.9	13.8	145.2	-1.0
min effect	243	-14	-21	121.5	2.0	138.9	-1.7	112.9	-2.2	145.2	0.1



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7.5 Effects in intermediate cross girders

The worst sagging under traffic actions occurs in the central cross girder.

Worst sagging on cross girder (ULS)

The following stresses are elastic stresses on the effective cross section (allowing for shear lag)

	M_y	F_x	F_z	Bottom flange		Top flange		Top of slab		Axial (steel)	
				W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	W (10^6 mm^3)	σ	A (10^3 mm^2)	σ
Stage 1	48	0	0	8.27	6	8.27	-6			27.7	0
Stage 2	0	0	0	8.27	0	8.27	0			27.7	0
Stage 3	523	2	0	8.27	63	8.27	-63			27.7	0
Stage 4	272	104	0	11.96	23	111	-2	453	-0.6	71.5	-1
Construction	843	106	0		92		-71		-0.6		-1
Gr1 traffic	1960	-93	43	12.89	152	-369	5	302	-6.5	150	1
Gr5 traffic	2192	-78	184	12.89	170	-369	6	302	-7.3	150	1
Total (gr5)	3035	28	184		262		-65		-7.9		0

Values for SLS would be needed if the total stresses exceeded the design elastic values but, by inspection, they are not exceeded.



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8 Verification of bare steel girder during construction

The two main girders are susceptible to lateral torsional buckling under the weight of the wet concrete (i.e. before it hardens and provides restraint to the top flanges).

The beams are partially restrained against buckling by the presence of the cross girders. The cross girders provide flexible torsional restraint to the beams.

Use the expressions in Appendix C of P356 to determine the non-dimensional slenderness and thus the buckling resistance. Use gross section properties for determination of LTB slenderness, even where the section is Class 4.

In this example, only the central span is considered. In general, the adequacy of each of the three spans would need to be considered, though the shorter spans would be deemed satisfactory by inspection by comparing with the larger span.

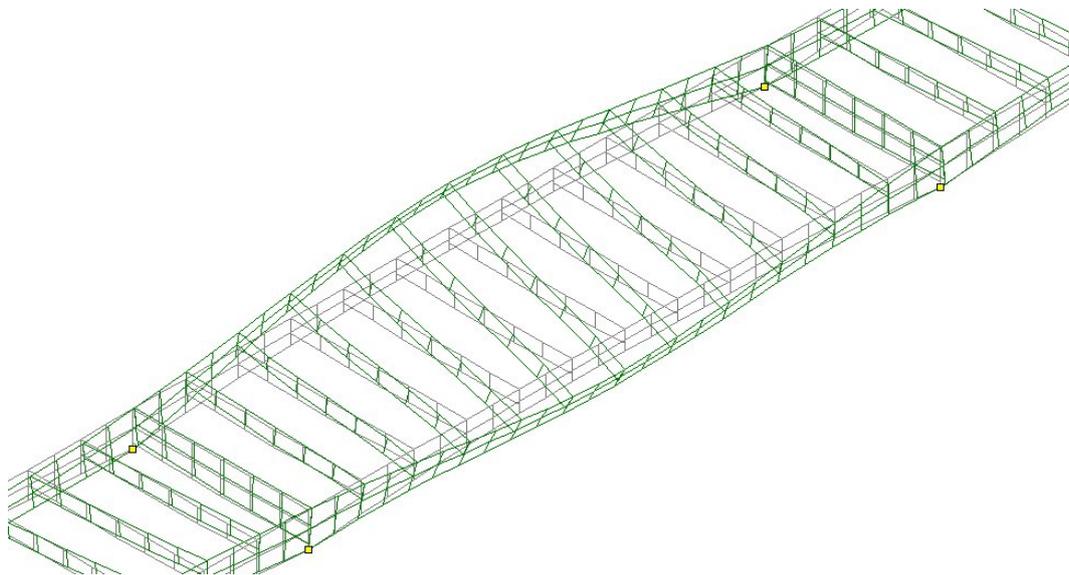
8.1 Torsional flexibility of paired main girders

In the global model, moments of 10 kNm about a longitudinal axis were applied at each end of the main span cross girders in construction stage 3.

The total torque applied to each beam is thus:

$$10 \times 11 = 110 \text{ kNm}$$

The deflected shape determined by the analysis is shown below.



The rotational displacements at the central gross girder, given by the analysis, were 1.600×10^{-4} rad

Thus, the torsional flexibility, for use in Appendix C, is:

$$\theta_R = 1.600 \times 10^{-4} / (110 \times 10^6) = 1.455 \times 10^{-12} \text{ rad/Nmm}$$

P356^[4]
3-1-1/6.3.2.2



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8.2 Evaluation of non-dimensional slenderness - main girders

Geometrical parameters:

$L_w = 42000$ mm and for the cross section at mid-span, the section properties are:

$$I_{z,c} = 1.707 \times 10^9$$

$$I_{z,t} = 2.133 \times 10^9$$

$$I_T = 5.104 \times 10^7$$

$$i_z = 212.1 \text{ mm}$$

$$h = 1200 \text{ mm}$$

$$t_f = 45 \text{ mm}$$

$$d_f = 1155 \text{ mm}$$

$$\lambda_F = \frac{L_w}{i_z} \cdot \frac{t_f}{h} = \frac{42000}{212.1} \cdot \frac{45}{1200} = 7.43$$

$$a = \frac{I_{z,c}}{I_{z,c} + I_{z,t}} = \frac{1.707}{1.707 + 2.133} = 0.44$$

$$\psi_a = 0.8(2a - 1) = 0.8 \times (2 \times 0.44 - 1) = -0.12$$

To determine V_{eq} , the following are needed:

$$\tau = 4a(1 - a) + \psi_a^2 = 4 \times 0.44 \times (1 - 0.44) + 0.12^2 = 1.0$$

$$\omega = \frac{\pi^2 d_f^2 E I_z}{G I_T L_w^2} = \frac{\pi^2 \times 1155^2 \times 3.84 \times 10^9 \times 2.6}{5.104 \times 10^7 \times 42000^2} = 1.460 \text{ (using } E/G = 2.6)$$

Thus:

$$V_{eq} = \left[\frac{2 a \omega}{\left[\sqrt{4 + \tau \omega} + \psi_a \sqrt{\omega} \right]^2} \right]^{0.25} = \left[\frac{2 \times 0.44 \times 1.46}{\left[\sqrt{4 + 1.46} + 0 \right]^2} \right]^{0.25} = 0.719$$

Thus the stiffness parameter $V_{eq}^4 L_w^3 / [E I_{z,c} \theta_R d_f^2 (1 - a)] = 50800$

And using the expression $k = \left[1 + \frac{V_{eq}^4 L_w^3}{\pi^4 E I_{z,c} d_f^2 \theta_R (1 - a)} \right]^{(-0.25)}$

The value of $k = 0.209$

The limiting (minimum) value of k is $(1.7 - 0.7 V_{eq}) L_r / L_w$

Taking $L_r = 3500$, the limit is:

$$(1.7 - 0.7 \times 0.719) \times 3500 / 42000 = 0.100, \text{ so use } k = 0.209$$

P356/C.4.3
and C.3.2

P356/C.4.5



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Assume $1/\sqrt{C_1} = 1.0$ (uniform moment - conservative assumption)

$U = 1.0$ (welded section)

$$V = \left\{ \left[4a(1-a) + 0.05\lambda_F^2 + \psi_a^2 \right]^{0.5} + \psi_a \right\}^{-0.5}$$

$$= \left\{ \left[4 \times 0.4(1-0.44) + 0.05 \times 7.43^2 + 0.12^2 \right]^{0.5} - 0.12 \right\}^{-0.5} = 0.741$$

Take $D = 1.2$ (destabilising loads)

$$\lambda_z = \frac{kL_w}{i_z} = \frac{0.209 \times 42000}{212.1} = 41.4$$

$$\lambda_1 = \pi \sqrt{\frac{E}{f_y}} = \pi \sqrt{\frac{210000}{345}} = 77.5$$

$$\beta_w = \frac{W_y}{W_{pl,y}} = \frac{3.970 \times 10^7}{15110 \times 10^6 / 345} = 0.906 \text{ (effective section modulus } W_y)$$

Thus:

$$\bar{\lambda}_{LT} = \frac{1}{\sqrt{C_1}} UVD \frac{\lambda_z}{\lambda_1} \sqrt{\beta_w} = 1 \times 1 \times 0.741 \times 1.2 \times \frac{41.4}{77.5} \sqrt{0.906} = 0.45$$

Slenderness derived from buckling analysis

Alternatively and less conservatively, slenderness could be derived from an elastic buckling analysis of the structure at the bare steel girder stage and then the value of $\bar{\lambda}_{LT}$

would be given by $\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}}$

where M_{cr} is given by the analysis.

8.3 Reduction factor

Since $h/b < 2$, use buckling curve c, $\alpha_{LT} = 0.49$

$$\phi_{LT} = 0.5 \left[1 + \alpha_{LT} (\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2 \right] = 0.5 \left[1 + 0.49(0.45 - 0.2) + 0.45^2 \right] = 0.66$$

Hence

$$\chi_{LT} = 1 / \left(\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2} \right) = 1 / \left(0.66 + \sqrt{0.66^2 - 0.45^2} \right) = 0.875$$

8.4 Verification

$$M_{b,Rd} = \frac{\chi W_{el} f_y}{\gamma_{M1}} = \frac{0.875 \times 3.970 \times 10^7 \times 345}{1.1} \times 10^{-6} = 10900 \text{ kNm}$$

$$M_{Ed} = 121 - 584 + 4371 = 3908 \text{ kNm (Sheet 17)} < M_{b,Rd} = 10900 \text{ kNm - OK}$$

P356/C.4.2
and C.4.3

P356/C.1

W_y and M_{pl}
from Sheets 9
and 10

3-2/ 6.3.2.2 3-
1-1/ 6.3.2.2
3-1-1/
NA.2.16

3-1-1/6.3.2.1



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8.5 LTB of cross girders

The adequacy of the LTB buckling resistance of the cross girders, spanning between the main beams, also needs to be verified. There are no intermediate restraints to the cross girders, unless they need to be paired together at their mid-span.

Geometrical parameters:

$L_w = 11700$ mm and for the central cross section, the section properties are:

$$I_{z,c} = 5.625 \times 10^7$$

$$I_{z,t} = 5.625 \times 10^7$$

$$I_T = 4.077 \times 10^6$$

$$i_z = 63.7 \text{ mm}$$

$$h = 896 \text{ mm (at middle of CG)}$$

$$t_f = 25 \text{ mm}$$

$$d_f = 871 \text{ mm}$$

$$\lambda_F = \frac{L_w}{i_z} \cdot \frac{t_f}{h} = \frac{11700}{63.7} \cdot \frac{25}{896} = 5.12$$

$$a = \frac{I_{z,c}}{I_{z,c} + I_{z,t}} = 0.5 \text{ (equal flanges)}$$

$$\psi_a = 0.8(2a - 1) = 0.8 \times (2 \times 0.5 - 1) = 0$$

For an unrestrained beam, $k = 1.0$

Assume $1/\sqrt{C_1} = 1.0$ (uniform moment - conservative assumption)

$$U = 1.0 \text{ (welded section)}$$

$$V = \left\{ \left[4a(1-a) + 0.05\lambda_F^2 + \psi_a^2 \right]^{0.5} + \psi_a \right\}^{-0.5}$$

$$= \left\{ \left[4 \times 0.5(1-0.5) + 0.05 \times 5.12^2 + 0 \right]^{0.5} - 0 \right\}^{-0.5} = 0.811$$

Take $D = 1.2$ (destabilising loads)

$$\lambda_z = \frac{kL_w}{i_z} = \frac{1.0 \times 11700}{63.7} = 183.7$$

$$\lambda_1 = \pi \sqrt{\frac{E}{f_y}} = \pi \sqrt{\frac{210000}{345}} = 77.5$$

$$\beta_w = \frac{W_y}{W_{pl,y}} = 8.273 \times 10^6 / (3207 \times 10^6 / 345) = 0.890$$

Thus:

$$\bar{\lambda}_{LT} = \frac{1}{\sqrt{C_1}} UVD \frac{\lambda_z}{\lambda_1} \sqrt{\beta_w} = 1 \times 1 \times 0.811 \times 1.2 \times \frac{183.7}{77.5} \sqrt{0.890} = 2.18$$

P356/C.4.3
and C.3.2

P356/C.1

W_y and M_{pl}
from
Sheet 14



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8.6 Reduction factor

Since $h/b > 2$, use buckling curve d, $\alpha_{LT} = 0.76$

$$\phi_{LT} = 0.5 \left[1 + \alpha_{LT} (\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2 \right] = 0.5 \left[1 + 0.76(2.18 - 0.2) + 2.18^2 \right] = 3.63$$

Hence

$$\chi_{LT} = 1 / \left(\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2} \right) = 1 / \left(3.63 + \sqrt{3.63^2 - 2.18^2} \right) = 0.153$$

3-2/ 6.3.2.2 3-
1-1/ 6.3.2.2
3-1-1/
NA.2.16

8.7 Verification

$$M_{b,Rd} = \frac{\chi W_{el} f_y}{\gamma_{M1}} = \frac{0.153 \times 8.273 \times 10^6 \times 345}{1.1} \times 10^{-6} = 397 \text{ kNm}$$

$M_{Ed} = 48 + 523 = 571 \text{ kNm}$ (Sheet 19) $> M_{b,Rd} = 397 \text{ kNm}$ - Not satisfactory

Consider pairing the cross girders together with channel bracing at their mid-span. This is then classed as a beam with a central torsional restraint.

3-1-1/6.3.2.1

8.8 Verification for paired cross girders

From global analysis, it is determined that the torsional flexibility of the central restraint is $\theta_R = 4.73 \times 10^{-11} \text{ rad/Nmm}$

Using the same values of λ_F and a as before, the value of V_{eq} is needed to calculate k .

For a bi-symmetric section, the value of V_{eq} may be taken as equal to V .

Thus:

$$V_{eq} = 0.811$$

Then the stiffness parameter $V_{eq}^4 L_w^3 / [EI_{z,c} \theta_R d_f^2 (1-a)] = 3270$ and thus $k = 0.494$

Consider whether the cross girder would then buckle into two half waves, which is indicated by the value of the 'arrow' on the appropriate central restraint curve (see Figure C.1 in P356)

The value of k at the position of the arrow is given by:

$$k = \left[\frac{1 + \pi^2 \alpha}{4(1 + 4\pi^2 \alpha)} \right]^{0.25}$$

Where $\alpha = \frac{V_{eq}^4}{\pi^2 (1 - V_{eq}^4)}$ for $V_{eq} \leq 0.999$

$$\alpha = \frac{0.811^4}{\pi^2 (1 - 0.811^4)} = 0.07725 \text{ and thus } k = \left[\frac{1 + \pi^2 \times 0.07725}{4(1 + 4\pi^2 \times 0.07725)} \right]^{0.25} = 0.574$$

P356/C.4.6

P356/C.4.5



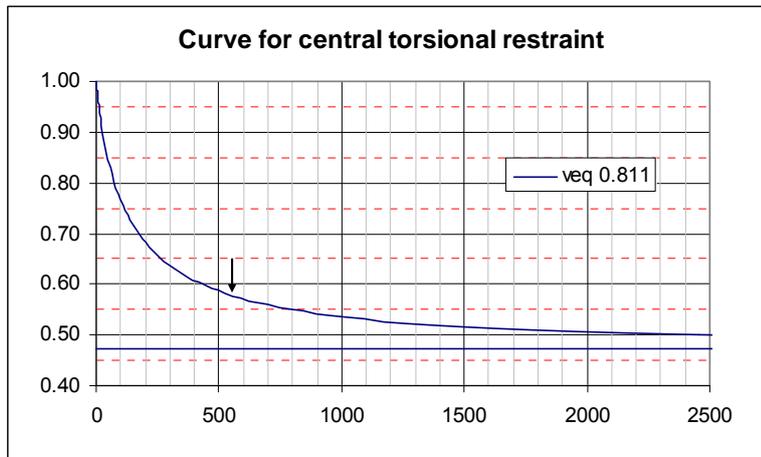
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Thus, since the value of k given by the stiffness parameter and the central restraint curve is less than the value of k at the position of the arrow on that curve, it is to the right of the arrow and the cross girder will buckle in two half waves.

(Note: Detailed evaluation of the expressions in P356 for this case show that the arrow is at a stiffness parameter of about 570.)



For two half waves (node at central restraint):

$\lambda_F = 2.56$ (half the previous value) and thus

$$V = \left\{ \left[4 \times 0.5(1 - 0.5) + 0.05 \times 2.56^2 + 0 \right]^{0.5} - 0 \right\}^{-0.5} = 0.932$$

$$\lambda_z = \frac{L_w}{i_z} = \frac{5850}{63.7} = 91.8$$

$$\bar{\lambda}_{LT} = \frac{1}{\sqrt{C_1}} UVD \frac{\lambda_z}{\lambda_1} \sqrt{\beta_w} = 1 \times 1 \times 0.932 \times 1.2 \times \frac{91.8}{77.5} \sqrt{0.890} = 1.25$$

$$\phi_{LT} = 0.5 \left[1 + \alpha_{LT} (\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2 \right] = 0.5 \left[1 + 0.76(1.25 - 0.2) + 1.25^2 \right] = 1.68$$

Hence

$$\chi_{LT} = 1 / \left(\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2} \right) = 1 / \left(1.68 + \sqrt{1.68^2 - 1.25^2} \right) = 0.36$$

$$M_{b,Rd} = \frac{\chi W_{el} f_y}{\gamma_{M1}} = \frac{0.36 \times 8.273 \times 10^6 \times 345}{1.1} \times 10^{-6} = 934 \text{ kNm} > M_{Ed} = 571 \text{ kNm OK}$$

P356/C.3.2



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9 Verification of composite girder

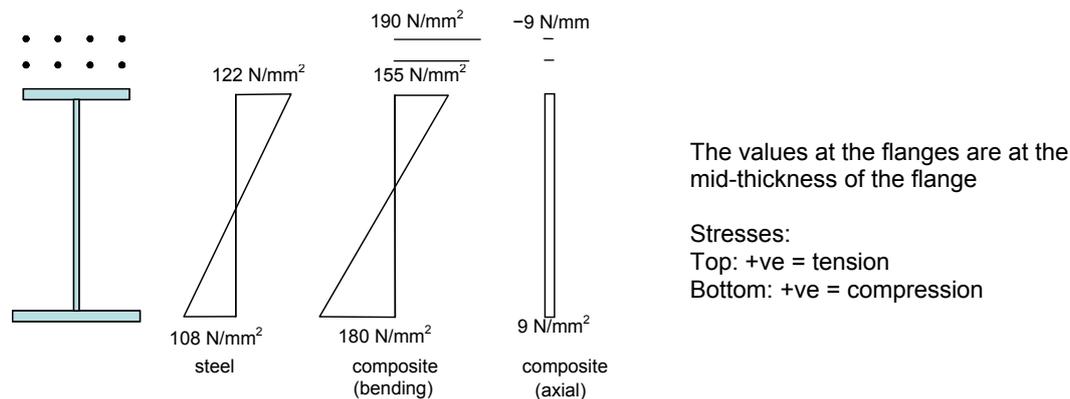
9.1 In hogging bending with axial force

The elastic design bending resistance for a beam constructed in stages depends on the design effects at the stages.

The bare steel section is Class 3 in bending and the composite cross section is Class 4.

The effects (stresses) in the cross section have been calculated on the basis of gross section properties for effects on the steel beam plus effective section properties on the composite section. *(This avoids the calculation of different section properties for each design situation, based on the zero stress position for combined effects.)*

From Sections 7.1 and 7.2, the design moment on the steel section is 12499 kNm and the total moment is 33970 kNm, which means that the moment on the composite (cracked) section is 21471 kNm. The stresses are as shown below.



The primary effects of shrinkage do not need to be included.

4-2/6.2.1.5(5)

Resistance of cross section

For verification of cross section resistance, the stresses should not exceed the limiting stresses f_{yd} and f_{sd} .

4-2/6.2.1.5

For this verification:

$$f_{yd} = f_y / \gamma_{M0} = 335 / 1.0 = 335 \text{ N/mm}^2 \text{ for the 60 mm bottom flange}$$

$$f_{sd} = f_{yk} / \gamma_s = 500 / 1.15 = 435 \text{ N/mm}^2 \text{ for the reinforcement}$$

By inspection, the stresses in both are OK

The reinforcement should also be checked for the combined global and local effects. The design of the slab is not covered in this example and local design stresses are not available. There is a margin between global stresses and stress limits that should be sufficient for inclusion of local effects.



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Buckling resistance

For verification of buckling resistance in bending, the design resistance of the cross section (on which $M_{b,Rd}$ is based) has to be determined using:

$$M_{el,Rd} = M_{a,Ed} + kM_{c,Ed}$$

4-2/(6.4)

Where k is a factor such that a stress limit is reached.

In this case the bottom flange will reach its limit first and the limit is:

$$f_{yd} = f_y / \gamma_{M1} = 335 / 1.1 = 305 \text{ N/mm}^2 \text{ (}\gamma_{M1} \text{ is used since buckling is being considered)}$$

Thus, considering bending stresses only:

$$M_{el,Rd} = 12499 + \frac{(305 - 108)}{180} \times 21471 = 35990 \text{ kNm}$$

To evaluate $M_{b,Rd}$, determine the slenderness

The slenderness of the length of beam in the hogging region could be evaluated considering the LTB of a composite section comprising the effective width of slab and the steel girder but it is much simpler and a little less conservative to use the simplified method of EN 1993-2, as recommended by EN 1994-2.

4-2/6.4.3.2

Consider the lateral buckling of an effective Tee section comprising the bottom flange and one third of the depth of the part of the web in compression. Take the depth in compression as that under total effects, including axial force.

3-2/6.3.4.2

Flange area is $800 \times 60 \text{ mm}$

Height to zero stress:

$$(297 / (297 + 268)) \times (2175 - 30) + 30 = 1158 \text{ mm}$$

Height of web in compression = 1098 mm

	Stress	Mid-height
Top flange	268	2175
Bottom flange	297	30

$$\text{Area of Tee} = 800 \times 60 + (1098 \times 20) / 3 = 55320 \text{ mm}^2$$

$$\text{Lateral 2}^{\text{nd}} \text{ moment of area} = 800^3 \times 60 / 12 = 2560 \times 10^6 \text{ mm}^4$$

$$\text{Radius of gyration} = \sqrt{2560 \times 10^6 / 55320} = 215 \text{ mm}$$

Initially, assume that the first cross girder provides effective lateral restraint to the flange, through U-frame action.

For a buckling length of 3500 mm (support to first cross girder, CG8):

$$N_E = \pi^2 \frac{EI}{L^2} = \pi^2 \frac{210000 \times 2560 \times 10^6}{3500^2} \times 10^{-3} = 433100 \text{ kN}$$

3-2/(6.12)

The lateral restraint is sufficiently stiff if its stiffness C_d satisfies:

$$C_d > \frac{4N_E}{L}$$

3-2/6.3.4.2
(6.13)

The stiffness of the U-frame is given by consideration of equal and opposite unit forces applied at the two bottom flanges. Following the guidance in Table D.3, this may be expressed as:

3-2/D.2.4

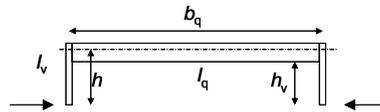


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$$\frac{1}{C} = \frac{h_v^3}{3EI_v} + \frac{h^2 b_q}{2EI_q}$$



At the first cross girder:

- $b_q = 11700$, $h_v = 1114$ mm $h = 1979$ mm, (to mean level of short-term NA)
- $I_q = 1.02 \times 10^{10}$ (short-term section, average value over tapered cross girder)
- $I_v = 1.89 \times 10^8$ (web plus flat stiffener)

$$\text{Hence } C_d = 210000 / \left(\frac{1114^3}{3 \times 1.89 \times 10^8} + \frac{1979^2 \times 11700}{2 \times 1.02 \times 10^{10}} \right) = 44800 \text{ N/mm}$$

The required C_d is:

$$\frac{4 \times 433100}{3500} \times 10^3 = 495000 \text{ N/mm}$$

Therefore the frame is not stiff enough to be considered as rigid

Now consider the stiffness of the second frame, for buckling over a length of 2 panels. Since the half wavelength has doubled,

$$N_E = 433100/4 = 108300 \text{ kN and required } C_d \text{ is then } 4 \times 108300/7.0 = 61900 \text{ kN/m}$$

For this frame, $h = 1729$ mm, $h_v = 864$ mm

$$C_d = 210000 / \left(\frac{864^3}{3 \times 1.89 \times 10^8} + \frac{1729^2 \times 11700}{2 \times 1.02 \times 10^{10}} \right) = 73600 \text{ N/mm } (\equiv \text{kN/m})$$

Which is sufficient to be considered as rigid

The verification may be carried out using

$$\bar{\lambda}_{LT} = \sqrt{\frac{A_{eff} f_y}{N_{crit}}}$$

3-2/(6.10)

Where $N_{crit} = mN_E$

N_E is the elastic critical buckling load for the equivalent column under uniform axial force and m is a parameter that allows for intermediate lateral spring restraints and for non-uniform axial force. Expressions are given in 6.3.4.2(7) depending on the ratio M_2/M_1 and V_2/V_1 , as well as on the intermediate spring stiffness.

In this case, the girder tapers over the buckling length and it would be complex to determine the value of m but the limiting (minimum) value of 1.0 may be used and is not overly conservative for this situation.

Thus

$$\bar{\lambda}_{LT} = \sqrt{\frac{A_{eff} f_y}{N_{crit}}} = \sqrt{\frac{55320 \times 335}{108300 \times 10^3}} = 0.414$$



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Since $h/b < 2$, use buckling curve c ($\alpha = 0.49$)

$$\phi_{LT} = 0.5 \left[1 + \alpha_{LT} (\bar{\lambda}_{LT} - 0.2) + \bar{\lambda}_{LT}^2 \right] = 0.5 \left[1 + 0.49(0.414 - 0.2) + 0.414^2 \right] = 0.638$$

Hence

$$\chi_{LT} = 1 / \left(\phi_{LT} + \sqrt{\phi_{LT}^2 - \bar{\lambda}_{LT}^2} \right) = 1 / \left(0.638 + \sqrt{0.638^2 - 0.414^2} \right) = 0.890$$

$$M_{b,Rd} = \chi M_{el,Rd} = 0.890 \times 35990 = 32030 \text{ kNm}$$

For verifying the contribution of axial resistance in the interaction criterion, consider the same Tee section (and thus the same slenderness and reduction factor).

No effective section for axial force is given in EN 1993-2 but it could be argued that the effective Tee that would buckle laterally should comprise half the area of the web: the slenderness with this amount of web is very little different from that of the effective Tee for bending. The same area is used here for both cases.

$$N_{b,Rd} = \chi A_{Tee} f_{yd} = 0.890 \times 55440 \times 305 = 15050 \text{ kNm}$$

$$N_{Ed} = A_{Tee} \times stress = 55320 \times 9 = 498 \text{ kN}$$

This verification of resistance to buckling may be carried out at a distance from the largest moment given by $0.25L_k$ (where $L_k = L/\sqrt{m}$)

Here, consider moment at 0.25×7000 from the support. Conservatively this can be interpreted linearly between the values at the two ends (7000 mm apart) or in this case linearly between values at the support and first cross girder.

At the support, $M_{Ed} = 33970 \text{ kNm}$

At the first cross girder $M_{Ed} = 20810 \text{ kNm}$

Hence $M_{Ed} = 27390 \text{ kNm}$ at $0.25L_k$

The section is subject to combined bending and axial force and a linear interaction will be assumed since the buckling mode is the same for both.

In the M/N interaction verification, use $N_{Ed} = 498 \text{ kN}$ (on the effective column) without reduction over the buckling length.

The interaction relationship is thus:

$$\frac{M_{Ed}}{M_{b,Rd}} + \frac{N_{Ed}}{N_{b,Rd}} = \frac{27390}{32030} + \frac{498}{15050} = 0.855 + 0.033 = 0.888 \text{ OK}$$

The buckling resistance is satisfactory.

Interaction with shear must also be considered (using cross section resistances).

3-2/6.3.2.2 3-1-1/6.3.2.2
3-1-1/NA.2.16

3-1-1/6.3.2.1

3-2/ 6.3.4.2(7)



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9.2 Maximum shear at support

The maximum shear in the girder at the intermediate support = 4533 kN

However, the bottom flange is inclined and contributes a vertical component of force The total stress in the bottom flange is 262 N/mm² and the inclination is 0.0952 rad

Hence vertical component = (262 × 800 × 60 / 1000) × 0.0952 = 1197 kN

$$V_{Ed} = 4533 - 1197 = 3336 \text{ kN}$$

Assume that no transverse web stiffeners are provided, other than those to attach cross girders (3500 mm spacing).

The web panel adjacent to the support is tapered; base the slenderness on the deeper end of the panel.

$$a_w = 3500 \text{ mm}$$

$$h_w = 2090 \text{ mm}$$

$$t = 20 \text{ mm}$$

$$f_y = 345 \text{ N/mm}^2$$

The factor $\eta = 1.0$ according to the NA

From equation (5.6):

$$\bar{\lambda}_w = \frac{h_w}{37.4t\epsilon\sqrt{k_t}} \text{ where } \epsilon = \sqrt{235/f_y} = \sqrt{235/345} = 0.83$$

Since $a_w > h_w$ and there are no longitudinal stiffeners:

$$k_t = 5.34 + 4.0(h_w/a)^2 = 5.34 + 4.0(2090/3500)^2 = 6.77$$

$$\bar{\lambda}_w = \frac{2090}{37.4 \times 20 \times 0.83\sqrt{6.77}} = 1.294$$

Since the girder is continuous, consider as a rigid endpost case. Thus, from Table 5.1:

$$\chi_w = 1.37 / (0.7 + \bar{\lambda}_w) = 1.37 / 1.994 = 0.687$$

$$V_{bw,Rd} = \frac{\chi_w f_{yw} h_w t}{\sqrt{3} \gamma_{M1}} = \frac{0.687 \times 345 \times 2090 \times 20}{\sqrt{3} \times 1.1} \times 10^{-3} = 5200 \text{ kN}$$

This resistance is adequate, even without a contribution from the flanges.

Using the same web slenderness, the shear resistance at the shallow end of the panel is 4813 kN. The force in the compression flange is less at that position and although the flange force is also less, $V_{Ed,net} = 2865 \text{ kN}$ (calculations not shown here). This is OK.

9.3 Combined bending and shear

The shear coexisting with maximum moment is 3827 kN less a contribution from the inclined flange of 1325 kN, giving a net value of $V_{Ed} = 2502 \text{ kN}$

$$\bar{\eta}_3 = \frac{V_{Ed}}{V_{bw,Rd}} = \frac{2502}{5200} = 0.481$$

Sheet 22

3-1-5/NA.2.4

3-1-5/5.3

3-1-5/A.3

3-1-5/5.2

3-1-5/7.1



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So interaction does not need to be considered for this design situation. ($\bar{\eta}_3 < 0.5$)

For the maximum shear design situation ($M_{Ed} = 30576$ kNm, $V_{Ed} = 4533$ kN, $N_{Ed} = 1092$ kN) the net shear is $V_{Ed,net} = 3336$ kN and thus:

$$\bar{\eta}_3 = \frac{V_{Ed}}{V_{bw,Rd}} = \frac{3336}{5200} = 0.642$$

So shear-moment interaction does need to be considered. ($\bar{\eta}_3 > 0.5$)

For interaction, consider the value of $M_{f,Rd}$. For this parameter, γ_{M0} applies and the value must take account of axial force.

The area of the bottom flange is smaller than that of the top plus the rebars and its compressive resistance is 16080 kN. Deduct half the compressive force (conservative, since that flange is smaller) and multiply by the lever arm between the two flanges (take $d = 2220$ mm). Thus $M_{f,Rd} = (16080 - 1092/2) \times 2.22 = 34500$ kNm

Since $M_{Ed} < M_{f,Rd}$ the interaction criterion of 3-1-5/7.1 does not apply and the combined effects are satisfactory.

As noted in Example 1, it is suggested in PD 6696-2 that M_{Ed} should be determined from accumulated stress, rather than as the sum of the moments. That calculation is not shown here but the value would be less than $M_{f,Rd}$.

9.4 In sagging bending

The composite cross section is Class 1 (pna in the top flange) so the plastic resistance can be utilised.

The plastic bending resistance of the short term composite section is 22519 kNm and the total design value of bending effects is 16654 kNm, with an axial tensile force of 2263 kN. The presence of tensile axial force on the plastic bending resistance is not covered by EN 1994-2; in this case, where the pna of the composite section is at the mid thickness of the top flange, the axial force only moves the pna within the top flange and there is negligible effect on the plastic bending resistance. The cross section is satisfactory by inspection.

It can also be seen that the stresses calculated elastically, taking account of construction in stages, are satisfactory, as follows:

Sheet 22

Sheet 10
Sheet 22

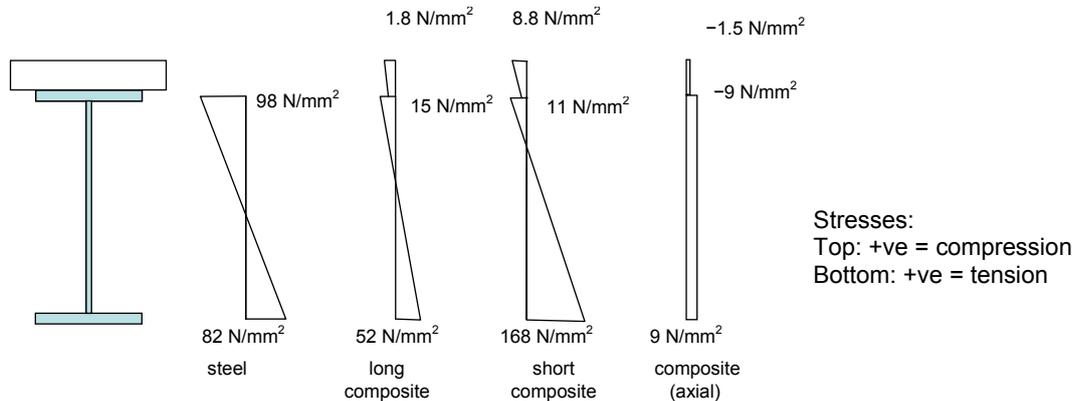


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From Sections 7.1 and 7.2, the design value of stresses are as shown below.



For verification of cross section resistance, the stresses should not exceed the limiting stresses f_{yd} and f_{cd} .

4-2/6.2.1.5

For this verification:

$$f_{yd} = f_y / \gamma_{M0} = 335 / 1.0 = 335 \text{ N/mm}^2 \text{ for the 50 mm bottom flange}$$

$$f_{cd} = f_{ck} / \gamma_c = 40 / 1.5 = 26.7 \text{ N/mm}^2 \text{ for the concrete}$$

By inspection, the stresses in both are OK ($\sigma_{bf} = 311 \text{ N/mm}^2$, $\sigma_c = 9.1 \text{ MPa}$)

If the elastic bending resistance were not adequate, to verify the shear connection the non-linear resistance to bending would have to be determined in accordance with 4-2/6.2.1.4 in order to determine the force N_c in the slab at ULS.

Combined global and local effects should be considered. Since the deck slab is not covered in this example, values of local effects are not available.

9.5 Verification of crack control at SLS (pier section)

Minimum reinforcement

The minimum required reinforcement is:

4-2/7.4.2

$$A_s = k_s k_c k_f f_{ct,eff} A_{ct} / \sigma_s$$

$$k_s = 0.9$$

$$k_c = \frac{1}{1 + h_c / 2z_0} + 0.3 \leq 1.0$$

Here $h_c = 250$ and $z_0 = (2200 + 125) - 1735 = 590 \text{ mm}$

$$k_c = \frac{1}{1 + 23255 / (2 \times 590)} + 0.3 = 1.125 \text{ but not more than } 1.0$$

$$k = 0.8$$

$$f_{ctm} = 3.5 \text{ (Table 3.1)}$$

$$w_{max} = 0.3 \text{ mm and thus for 20 mm bars, } \sigma_s = 200 \text{ N/mm}^2 \text{ (Table 7.1)}$$

$$A_s = 0.9 \times 1.0 \times 0.8 \times 3.5 \times (250 \times 3678) / 200 = 11600 \text{ mm}^2$$

height of NA of uncracked section from Sheet 10

2-1-1/3.1.2

2-2/7.3.1

4-2/7.4.2



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Area provided = $2 \times 314 \times 3678/150 = 15400 \text{ mm}^2$ - Satisfactory

Crack control

Requirements relate only to the quasi-permanent design situation and therefore local longitudinal stresses in the reinforcement are negligible.

Global stresses due to permanent loads, including shrinkage are 77 N/mm^2 in the top rebars. The tensile stress including the effect of tension stiffening are: $\sigma_s = \sigma_{s,0} + \frac{0.4 f_{ctm}}{\alpha_{st} \rho_s}$

For this calculation, the following parameters are needed:

$$A_s = 129800, I_s = 1.160 \times 10^{11} \text{ (for the bare steel section)}$$

$$A = 145200, I = 1.390 \times 10^{11} \text{ (for the cracked section)}$$

$$A_{ct} = 3678 \times 250 = 919500$$

$$\alpha_{st} = \frac{AI}{A_a I_a} = \frac{145200 \times 1390}{129800 \times 1160} = 1.340$$

$$\rho_s = A_s / A_{ct} = 15400 / 919500 = 0.0167 \text{ (i.e. 1.67\%)}$$

$$\sigma_s = 77 + \frac{0.4 \times 3.5}{1.340 \times 0.0167} = 140 \text{ N/mm}^2$$

From Table 7.2, maximum bar spacing = 300 mm > 150 mm provided - OK

Limiting stresses at SLS

Reinforcement stresses at SLS, including the effects of tension stiffening in cracked sections should also be considered. The elastic summations of stresses at ULS are less than the design values at the pier and at midspan and here it is judged that stresses are satisfactory at SLS, by inspection.

9.6 Verification of cross girders

As noted in Section 7.5 the stresses in the cross girders are within elastic limits, for the loads considered in the FE analysis. However, these transverse girders are required to prevent buckling of the slab where it is in compression. As noted in P356, Section 6.3.2, the cross girders are required to perform this function when the spacing of the main girders exceeds 30 times the slab thickness, which is the case in this example.

The cross girders need to be both stiff enough and strong enough to perform this function in addition to the resistance to the effects already calculated.

There are no explicitly worded Eurocode clauses for such design of composite girders but the following evaluation is based on guidance in Hendy & Murphy. The cross girder is considered as a transverse stiffener on the deck slab plate.

Sheet 23

Sheets 9 and 11

4-2/Table 7.2

References below are to Hendy and Murphy^[7]



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The stiffness requirement may be expressed as:

$$\delta + \Delta \leq \frac{b}{300}$$

Where Δ is the first order deflection due to transverse loads and δ is the second order deflection due to the longitudinal compression in the slab

From the analysis $w_{Ed} = 13.5$ mm at the centre of the middle cross girder under gr5 traffic loads. Strictly, Δ should be the observed relative deflection of one cross girder to those either side of it, but very conservatively (since the design situation would normally have similar loading on adjacent cross girders) it could be taken as the deflection of the most heavily loaded cross girder. i.e. $\Delta = w_{Ed}$

For the case where there is no axial force in the cross girder, the expression for δ reduces to:

$$\delta = w'_0 \left(\frac{\sigma_m b^4}{\pi^4 EI_{st} - \sigma_m b^4} \right)$$

In which:

$$w'_0 = w_0 + w_{Ed}$$

w_0 is the initial imperfection, which may be taken as $L/400$ according to EN 1090-2, (Essential tolerance D.1.6(5), with $L = 2 \times 3500$ and thus $w_0 = 17.5$ mm)

Hence $w'_0 = 17.5 + 13.5 = 31$ mm

The value of σ_m depends on the longitudinal force in the slab. From the results in Section 7.2 the maximum stress at the top of the slab is 9.1 N/mm² (bending plus axial effects) and the level of zero stress is approximately at the slab top flange interface. The effective width of slab acting with each main girder is 3975 mm and thus the force in the slab is:

$$N_{Ed} = (9.1/2) \times 7950 \times 250 \times 10^{-3} = 9040 \text{ kN}$$

Note: if plastic bending resistance had been utilised, the force in the slab would have been much greater.

Assuming that the ratio of column-like buckling stress to plate-like buckling stress is unity (conservative) and with equal spacing of the cross girders:

$$\sigma_m = \frac{2N_{Ed}}{ab} = \frac{2 \times 9040 \times 10^3}{3500 \times 11700} = 0.44 \text{ N/mm}^2$$

$$\text{Thus } \delta = 31 \times \left(\frac{0.44 \times 11700^4}{\pi^4 \times 210000 \times 1.16 \times 10^{10} - 0.44 \times 11700^4} \right) = 1.08 \text{ mm}$$

$\delta + \Delta = 1.08 + 13.5 = 14.58$ mm. which is less than $b/300$ (39 mm) – OK

$$w'_0 = w_0 + w_{Ed}$$

D6.6-22

Based on D6.6-21

Based on D6.6-5



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The strength of the cross girders also needs to be checked for this effect. The destabilising effect of the slab applies an additional moment of:

$$(w'_0 + \delta) \frac{\sigma_m b^2}{\pi^2} = (31 + 1.08) \times \frac{0.44 \times 11700^2}{\pi^2} \times 10^{-6} = 196 \text{ kNm}$$

This results in an additional tensile stress in the bottom flange of:

$$196 \times 10^6 / 12.89 \times 10^6 = 15 \text{ N/mm}^2$$

This gives a total stress of $262 + 15 = 277 \text{ N/mm}^2$ – OK.

Note that, although the cross girders are stiff enough to act as restraints to the slab against buckling, verification of the slab still needs to consider second order effects, since the slenderness a/t ($=3500/250 = 14$) may exceed the limit below which such effects can be ignored (see 2-1-1/5.8.3). Detailed design of the deck slab is not covered in this example.

Based on
D6.6-20

Sheet 24



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10 Longitudinal shear

The resistance to longitudinal shear is verified for the web/flange weld, the shear connectors and the transverse reinforcement at the pier and at mid-span. (*In practice, intermediate values would also be verified, to optimise the provision of shear connectors.*)

Since the composite beam model for which moments and forces have been extracted from the 3D FE model does not include the full width of slab, the shear flow will depend on both the cross section properties in bending (applied to the shear force on the section) and on the variation of axial force along the beam, due to the shear transferred from the portions of slab not included in the beam section.

10.1 Effects for maximum shear

ULS values at pier

	Shear force	Axial force	
		Pier	CG8
Shear on steel section (stages 1-3)	1443	51	38
Shear on long-term composite section (cracked)	936	303	227
Shear on short-term composite section (cracked)	2169	563	351

ULS values at midspan (CG13)

	Shear force	Axial force	
		CG13	CG12
Shear on steel section (stages 1-3)	-7	40	40
Shear on long-term composite section	0	-457	-407
Shear on short-term composite section	755	$\Delta F_x = 530$	

Values from Sections 7.1 and 7.2

SLS values

(*Only values for composite section noted*)

	Pier	Span (CG13)
Shear on long-term composite section	769	0
Shear on short-term composite section (worst effects)	1612	559

10.2 Section properties

To determine shear flows elastically, the parameter \bar{Az}/I_y is needed for each section and stage.

For composite sections, uncracked unreinforced composite section properties can be used to determine shear flow. 4-2/6.6.2.1(2)

	Section at pier		Section at mid-span	
	Web/flange \bar{Az}/I_y (m ⁻¹)	Flange/slab \bar{Az}/I_y (m ⁻¹)	Web/flange \bar{Az}/I_y (m ⁻¹)	Flange/slab \bar{Az}/I_y (m ⁻¹)
Steel section	0.392		0.803	
Long term section	0.443	0.276	0.797	0.633
Short term section	0.464	0.388	0.790	0.753



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Note that the use of the $\bar{A}z/I_y$ parameter applied to the shear on the cross section is conservative at the pier because the inclined bottom flange also provides a component of the shear resistance (but, since the flange force reduces away from the support, it would not be appropriate to apply the parameter to the net shear carried by the web).

If the plastic resistance had been utilized, the shear flow would need to have been determined by consideration of the forces in the slab, taking account of the non-linear resistance in accordance with 4-2/6.2.1.4.

Additionally, the axial force in the section varies along the girder and this must be accompanied by a shear flow.

10.3 Shear flow at ULS

Force at web/flange junction

At pier $1443 \times 0.392 + 936 \times 0.443 + 2169 \times 0.464 = 1867 \text{ kN/m}$

Near mid-span $7 \times 0.803 + 0 \times 0.797 + 755 \times 0.794 = 591 \text{ kN/m}$

Force at flange/slab junction

At pier $936 \times 0.276 + 2169 \times 0.388 = 1100 \text{ kN/m}$

Near mid-span $0 \times 0.633 + 755 \times 0.753 = 569 \text{ kN/m}$

At the pier, the difference in axial force between pier and cross girder is:

$866 - 578 = 288 \text{ kN}$

This is equivalent to a shear flow of $288/3.5 = 82 \text{ kN/m}$

The shear due to this variation of axial force at the slab/girder interface depends on the ratio of steel area / cracked area (cracked area used to be consistent with FE model):

Shear flow $= A_s/A \times 82 = 129800/145200 \times 82 = 73 \text{ kN/m}$

The shear at the web/flange junction depends on the area of web plus bottom flange:

Shear flow $= A_{w+f}/A \times 82 = 51 \text{ kN/m}$

At midspan, the difference in axial force between CG12 and CG13 is:

50 kN long term and

530 kN short term

This is equivalent to a shear flows of $50/3.5 = 14 \text{ kN/m}$ and $530/3.5 = 151 \text{ kN/m}$

The shear at the slab/girder interface depends on the ratio of steel area to composite areas:

Shear flow $= 85320/164300 \times 14 = 7 \text{ kN/m (long)}$

$= 85320/305100 \times 151 = 42 \text{ kN/m (short)}$

The shear at the web/flange junction depends on the area of web plus bottom flange:

Shear flow $= 53320/164300 \times 14 = 5 \text{ kN/m (long)}$

$= 53320/305100 \times 151 = 26 \text{ kN/m (short)}$



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The total shear flows are thus:

At the pier: slab/flange shear = 1173 kN/m, web/flange shear = 1918 kN/m
At midspan: slab/flange shear = 618 kN/m, web/flange shear = 622 kN/m

10.4 Shear flow at SLS

Force at flange/slab junction

At pier $769 \times 0.276 + 1612 \times 0.385 = 837$ kN/m
At mid-span $0 \times 0.633 + 559 \times 0.753 = 421$ kN/m

The shear flow at SLS is required for verification of the shear connectors

These values are 76% and 74% respectively of the ULS values and the ratio reflects the different partial factors at ULS and SLS. It can be assumed that similar ratios apply to the shear flow due to variation of axial force.

10.5 Web/flange welds

Design weld resistance given by the simplified method of EN 1993-1-8, 4.5.3.3 is:

$$F_{w,Rd} = f_{vw,d} a \text{ where } f_{vw,d} = \frac{f_u / \sqrt{3}}{\beta \gamma_{M2}}$$

For 6 mm throat fillet weld (8.4 mm leg length) $a = 6$ mm

For web and flange grade 355 in thickness range 3 -100 mm, $f_u = 470$ N/mm²

EN 10025-2

From Table 3-1-8/4.1 $\beta = 0.9$

$$\text{Thus } F_{w,Rd} = \frac{6 \times 470 / \sqrt{3}}{0.9 \times 1.25} = 1447 \text{ N/mm (kN/m)}$$

Resistance of 2 welds = 2890 kN/m > 1918 kN/m shear flow in pier girder - OK

10.6 Shear connectors

Stud shear connectors 19 mm diameter 150 mm long (type SD1 to EN ISO 13918) are assumed, with $f_u = 450$ N/mm²

The resistance of a single stud is given by 4-2/6.6.3.1 as the lesser of:

$$P_{Rd} = \frac{0.8 \times f_u \times \pi \times d^2 / 4}{\gamma_V} \tag{6.6.3.1(1) Eq (6.18)}$$

$$P_{Rd} = \frac{0.29 \times \alpha \times d^2 \sqrt{f_{ck} \times E_{cm}}}{\gamma_V} \tag{Eq (6.19)}$$

$$\alpha = 1.0 \text{ as } \frac{h_{sc}}{d} = \frac{150}{19} > 4 \tag{Eq (6.21)}$$

$$P_{Rd} = \frac{0.8 \times 450 \times \pi \times (19^2 / 4)}{1.25} \times 10^{-3} = 81.7 \text{ kN} \tag{Eq (6.18)}$$

$$P_{Rd} = \frac{0.29 \times 1.0 \times 19^2 \times \sqrt{40 \times 35 \times 10^3}}{1.25} \times 10^{-3} = 99.1 \text{ kN} \tag{Eq (6.19)}$$



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Therefore the design resistance of a single headed shear connector is

$$P_{Rd} = 81.7 \text{ kN}$$

If studs are grouped and spaced at 150 mm spacing along the beam (to suit transverse reinforcement), then a row of 3 studs has a design resistance of:

$$F_{Rd} = 81.7 \times 3 / 0.150 = 1634 \text{ kN/m}$$

This is adequate at the pier ($F_{Rd} = 1634 > F_{Ed} = 1173 \text{ kN/m}$)

Alternatively, rows of 5 studs at 300 mm spacing would provide 1362 kN/m.

The requirement at mid-span is about 53% that at the support and thus 3 studs at 300 spacing or 2 studs at 150 mm spacing would be adequate..

Consideration of fatigue resistance of the shear connection is covered in Section 11.3.

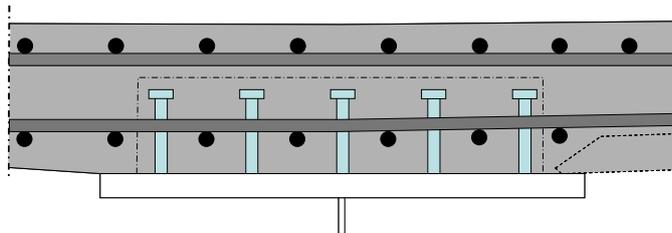
Resistance at SLS

At SLS the shear connector resistance is limited to $k_s P_{Rd}$ with $k_s = 0.75$.

The SLS shear flows are 76% of the ULS value at the pier and 75% at mid-span. The requirement is thus only slightly more onerous and the provision is satisfactory by inspection.

10.7 Transverse reinforcement

Consider the transverse reinforcement required to transfer the full shear resistance of 5 studs at 300 mm spacing, i.e. 1362 kN/m.



For a critical shear plane around the studs (type b-b in 4-2/Figure 6.15 and shown chain dotted above) the shear resistance is provided by twice the area of the bottom bars.

The shear force to be resisted is given by 4-2/(6.21) as $1362 / \cot \theta$

Take $\cot \theta = 1$, hence required resistance = 1362 kN/m

Assume B20 bars at 150 mm spacing:

$$\text{Resistance} = A_s f_y d / s_f = (2 \times 314) \times (500 / 1.15) / 150 \times 10^{-3} = 1821 \text{ kN/m}$$

The transverse bars are adequate.

The underside of the heads of the studs need to be at least 40 mm above the transverse bars. In this case an overall stud height of 150 mm should be sufficient.

4-2/NA.2.11

4-2/6.6.6.1

2-1-1/6.2.4

2-1-1/6.2.4

4-2/6.6.5.4



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11 Fatigue assessment

11.1 Assessment of structural steel details

The design value of the stress range in structural steel is:

$$\gamma_{Ff} \Delta \sigma_{E2} = \gamma_{Ff} \lambda \phi_2 \Delta \sigma_p$$

where $\phi_2 = 1.0$ and γ_{Ff} is given by the NA as 1.0

The value of $\lambda = \lambda_1 \lambda_2 \lambda_3 \lambda_4$ (but not more than λ_{max})

For intermediate supports where L , the length of the critical influence line (in m) is more than 30 m $\lambda_1 = (1.70 + 0.5 \times (L - 30) / 50)$

Here, $L = (24.5 + 42) / 2 = 33.25$ m and thus $\lambda_1 = 1.73$

For span regions, $\lambda_1 = (2.55 - 0.7 \times (L - 10) / 70)$ and here $L = 42$ m and thus $\lambda_1 = 2.23$

The value of λ_2 is given by $\lambda_2 = \left(\frac{Q_{m1}}{Q_0} \right) \times \left(\frac{N_{Obs}}{N_0} \right)^{1/5}$

Where $Q_0 = 480$ kN and $N_0 = 0.5 \times 10^6$

From 3-2/NA.2.39, $Q_{m1} = 260$ kN

From 1-2/Table NA.4, $N_{Obs} = 1 \times 10^6$

$$\text{Hence } \lambda_2 = \left(\frac{260}{480} \right) \times \left(\frac{1.0}{0.5} \right)^{0.2} = 0.62$$

For a 120 year design life the value of λ_3 given by 3-2/Table 9.2 is 1.037:

The value of λ_4 depends on the relative magnitude of the stress range due to the passage of FLM3 in the second lane and is given by:

$$\lambda_4 = \left(1 + \frac{\text{effect in lane 2}}{\text{effect in lane 1}} \right)^{0.2}$$

Design stress ranges at pier

At the pier, the stress range $\Delta \sigma_p$ in top and bottom flanges (at their mid thickness) is:

Bottom flange: 15.9 N/mm²

Top flange: 11.9 N/mm²

The ratio of lane 2/lane 1 effects = 0.709 and thus $\lambda_4 = 1.13$

$$\lambda = 1.73 \times 0.62 \times 1.037 \times 1.13 = 1.242$$

For support regions where $L > 30$ m $\lambda_{max} = 1.80 + 0.90 \times (L - 30) / 50$

Thus, for $L = 33.25$, $\lambda_{max} = 1.86$

Hence $\lambda = 1.242$

3-2/9.4.1 and
 9.5.1
 3-2/NA.2.35

Sheet 23



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The design stress ranges are thus:

Bottom flange: $1.0 \times 1.242 \times 15.9 = 20 \text{ N/mm}^2$

Top flange: $1.0 \times 1.242 \times 11.9 = 15 \text{ N/mm}^2$

The worst detail category that might apply is for a bearing plate welded to the underside of the bottom flange, which, for a flange plate over 50 mm thick, is category 36 (3-1-9/Table 8.5, detail 6).

3-1-9/
NA.2.5.3

Design value of fatigue strength = $36/\gamma_{Mf} = 36/1.1 = 33 \text{ N/mm}^2$ OK

Design stress ranges in mid-span

At mid-span, there is negligible stress range in the top flange. The range in the bottom flange is 26.0 N/mm^2 . The ratio of lane 2/lane 1 effects = 0.698 and thus $\lambda_4 = 1.112$

$\lambda = 2.23 \times 0.62 \times 1.037 \times 1.112 = 1.599$

The design stress range is thus:

Bottom flange: $1.0 \times 1.599 \times 26.0 = 42 \text{ N/mm}^2$

3-1-9/
Table 8.4

The most onerous detail would be a transverse web stiffener, which is detail category 80 and the fatigue strength = $80/1.1 = 73 \text{ N/mm}^2$. This is OK even for stiffeners welded to the bottom flange.

11.2 Assessment of reinforcing steel

The design value of the stress range in reinforcing steel is $\gamma_{F,fat} \Delta\sigma_{S,equ}$ where the value of $\gamma_{F,fat}$ is given by Table NA.1 as $\gamma_{F,fat} = 1.0$

2-1/6.8.5

$\Delta\sigma_{S,equ}$ is referred to in EN 1994-2 as $\Delta\sigma_E$, given by:

4-2/6.8.6.1

$$\Delta\sigma_E = \lambda \phi |\sigma_{max,f} - \sigma_{min,f}|$$

The value of $\lambda = \lambda_s$

2-2/NN.2.1

and $\lambda_s = \phi_{fat} \lambda_{s,1} \lambda_{s,2} \lambda_{s,3} \lambda_{s,4}$

Where ϕ_{fat} is a damage equivalent impact factor

The value ϕ effectively duplicates ϕ_{fat} but since $\phi = 1.0$, this is not significant

4-2/6.8.6.1

The value of the stress range due to FLM3 needs to be increased by a factor of 1.75 (in regions of intermediate supports) in accordance with NN.2.1(101). Stresses also need to be increased for the effect of tension stiffening in accordance with 4-2/7.4.3

2-2/NN.2.1

Based on cracked section properties, the stress in the top rebars due to permanent actions is 77 N/mm^2 (see SLS values in Section 7.3).

EN 1991-2
Annex B

The maximum tensile stress due to the FLM3 fatigue vehicle in lane 1 is 11.5 N/mm^2 (see Section 7.4) and this is increased by the 1.75 factor, giving a value of 22.4 N/mm^2 . Thus, ignoring tension stiffening, $\sigma_{max,f} = 99 \text{ N/mm}^2$.

The minimum stress (compressive) due to the FLM3 fatigue vehicle in lane 1 is 2.1 N/mm^2 (see Section 7.4) and this is increased by the 1.75 factor, giving a value of 3.6 N/mm^2 . Thus, ignoring tension stiffening, $\sigma_{min,f} = 73 \text{ N/mm}^2$.



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To determine the effect of tension stiffening, the following parameters were determined on Sheet 38:

$$\rho_s = 0.0167$$

$$\alpha_{st} = 1.34$$

$$f_{ctm} = 3.5 \text{ MPa (for C40/50 concrete)}$$

$$\Delta\sigma_s = \frac{0.2f_{ctm}}{\alpha_{st}\rho_s} = \frac{0.2 \times 3.5}{1.34 \times 0.0167} = 31 \text{ N/mm}^2$$

Thus, the maximum and minimum stresses including tension stiffening are:

$$\sigma_{s,max,f} = \sigma_{max,f} + \Delta\sigma_s = 99 + 31 = 130 \text{ N/mm}^2$$

$$\text{And } \sigma_{s,min,f} = \sigma_{s,max,f} \frac{M_{E,d,min,f}}{M_{E,d,max,f}}$$

Using the ratio of stresses, rather than directly using moments:

$$\sigma_{s,min,f} = 130 \times \frac{73}{99} = 96 \text{ N/mm}^2$$

For intermediate support region and span of 33.25 m, $\lambda_{s,1} = 0.988$

(The curve can be expressed approximately, for $L < 50$ m by the expression $0.3(L/100)^2 + 0.25(L/100) + 0.872$.)

For $N_{Obs} = 1 \times 10^6$, medium distance traffic and straight bars ($k_2 = 9$): $\bar{Q} = 0.94$ and

$$\lambda_{s,2} = \bar{Q}^{k_2} \sqrt{\frac{N_{Obs}}{2.0}} = 0.94^9 \sqrt{\frac{1.0}{2.0}} = 0.870$$

For 120 year design life:

$$\lambda_{s,3} = k_2 \sqrt{\frac{N_{Years}}{100}} = 9 \sqrt{\frac{120}{100}} = 1.020$$

For 2 slow lanes:

$$\lambda_{s,4} = k_2 \sqrt{\frac{\sum N_{Obs,i}}{N_{Obs,1}}} = 9 \sqrt{\frac{2.0}{1.0}} = 1.080$$

For road surface of good roughness $\phi_{fat} = 1.2$

Thus $\lambda = 1.2 \times 0.988 \times 0.87 \times 1.02 \times 1.08 = 1.136$

$$\Delta\sigma_E = 1.136 \times 1.0 \times |130 - 96| = 1.136 \times 34 = 39 \text{ N/mm}^2$$

$$\gamma_{F,fat} \Delta\sigma_{S,equ} = 1.0 \times 39 = 39 \text{ N/mm}^2$$

$$\frac{\Delta\sigma_{Rsk}}{\gamma_{s,fat}} = \frac{162.5}{1.15} = 141 \text{ N/mm}^2 > 39 \text{ mm}^2 \text{ OK}$$

4-2/7.4.3

2-1-1/Table 3.1
See 4-2/6.8.5.4
for 0.2 factor

4-2/6.8.5.4

2-2/Figure
NN.1

2-2/Table
NN.1

1-2/Annex B

2-1-1/6.8.5



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Note: The above verification considers only global stresses in the reinforcement, which is appropriate for regions close to the main girder. Fatigue under local loading should also be considered and then combined global and local effects must be evaluated; with a ladder deck the worst local effects are midway between main girders, where global effects are less. Design of the deck slab is outside the scope of this example.

11.3 Assessment of shear connection

The design value of the stress range in shear studs is given as $\gamma_{Fr} \Delta\tau_{E,2}$ where

$$\Delta\tau_{E,2} = \lambda_v \Delta\tau$$

In which $\Delta\tau$ is the range of shear stress in the cross section of the stud.

EN 1994-2 refers to EN 1993-2 for the value of γ_{Fr} , which is given as 1.0

The value of $\lambda_v = \lambda_{v,1} \lambda_{v,2} \lambda_{v,3} \lambda_{v,4}$

Since the span is less than 100 m, $\lambda_{v,1} = 1.55$

The values of λ_2 , λ_3 and λ_4 are calculated in the same manner as for structural steel but with an exponent of 1/8 rather than 1/5

$$\text{Hence } \lambda_2 = \left(\frac{260}{480}\right) \times \left(\frac{1.0}{0.5}\right)^{0.125} = 0.591$$

$$\lambda_3 = \left(\frac{120}{100}\right)^{0.125} = 1.023$$

The value of λ_4 depends on the relative magnitude of the stress range due to the passage of FLM3 in the second lane and is given by:

$$\lambda_4 = \left(1 + \frac{\text{effect in lane 2}}{\text{effect in lane 1}}\right)^{0.125}$$

Shear at pier

The range of vertical shear force at the pier is 423 kN and the ratio of lane 2/lane 1 effects is 241/423 = 0.570. The variation of axial force over 3.5 m is 41 kN. Sheet 19

As noted earlier, the axial force in the cross section is consequent upon both the separate modelling of the edge beam and of the unequal loading on the two main girders (the FLM3 is in lane 1). These cause longitudinal shears along both edges of the portion of slab acting with the steel girder; some of that shear has to be transferred to the steel beam (pro rata to area) and thus needs to be included in the design shear flow on the studs.

At the pier, the studs are 19 mm diameter, in rows of 3 at 150 mm spacing (or 6 at 300 mm spacing)

$$A_z / I_y = 0.388 \text{ m}^{-1} \text{ hence shear flow due to bending/shear} = 423 \times 0.388 = 164 \text{ kN}$$

$$\text{Shear flow due to axial force} = 41/3.5 \times A_s/A = 10 \text{ kN/m}$$

(cracked properties used, to be consistent with analysis model)

$$\text{Total shear flow} = 164 + 10 = 174 \text{ kN/m}$$



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The stress range per stud = $174 \times 0.150 / (3 \times \pi d^2 / 4) = 30.6 \text{ N/mm}^2$

$$\lambda_4 = (1 + 0.570)^{0.125} = 1.058$$

$$\lambda_v = 1.55 \times 0.591 \times 1.023 \times 1.058 = 0.991$$

$$\Delta\tau_{E,2} = 0.991 \times 30.6 = 30 \text{ N/mm}^2$$

The reference value of fatigue strength for a shear stud is $\Delta\tau_c = 90$

The partial factor on fatigue strength $\gamma_{Mf} = 1.1$.

The design strength is thus $90/1.1 = 81 \text{ N/mm}^2 > 30.6 \text{ N/mm}^2$ OK

Additionally, since the flange is in tension, the interaction with normal stress in the steel flange must be verified, using:

$$\frac{\gamma_{Ff} \Delta\sigma_{E,2}}{\Delta\sigma_c / \gamma_{Mf}} + \frac{\gamma_{Ff} \Delta\tau_{E,2}}{\Delta\tau_c / \gamma_{Mf,s}} \leq 1.3$$

With $\Delta\sigma_c = 80$.

Coexistent stresses should be used but conservatively one can consider the most onerous values for each of $\Delta\sigma_c$ and $\Delta\tau_c$

$$\frac{1.0 \times 15}{80/1.1} + \frac{1.0 \times 30}{90/1.1} = 0.57 \text{ OK}$$

Shear at mid-span

The range of vertical shear force at mid-span is 219 kN and the ratio of lane 2/lane 1 effects is $143/219 = 0.653$. The variation of axial force is 144 kN over 3.5 m

$$\bar{A}_z / I_y = 0.753 \text{ m}^{-1} \text{ hence shear flow due to bending/shear} = 219 \times 0.753 = 165 \text{ kN}$$

Shear flow due to axial force = $144/3.5 \times A_s/A = 12 \text{ kN}$

Total shear flow = $165 + 12 = 177 \text{ kN/m}$

At mid-span, if the 19 mm studs are in rows of 3 at 300 mm spacing

Stress range = $177 \times 0.300 / (3 \times 284) = 62 \text{ N/mm}^2$

$$\lambda_4 = (1 + 0.653)^{0.125} = 1.065$$

$$\lambda_v = 1.55 \times 0.591 \times 1.023 \times 1.065 = 0.998$$

$$\Delta\tau_{E,2} = 0.998 \times 62 = 62 \text{ N/mm}^2 < 81 \text{ N/mm}^2 \text{ OK}$$

4-2/6.8.3

3-1-9/
NA.2.5.3

4-2/6.8.7.2

Sheet 19



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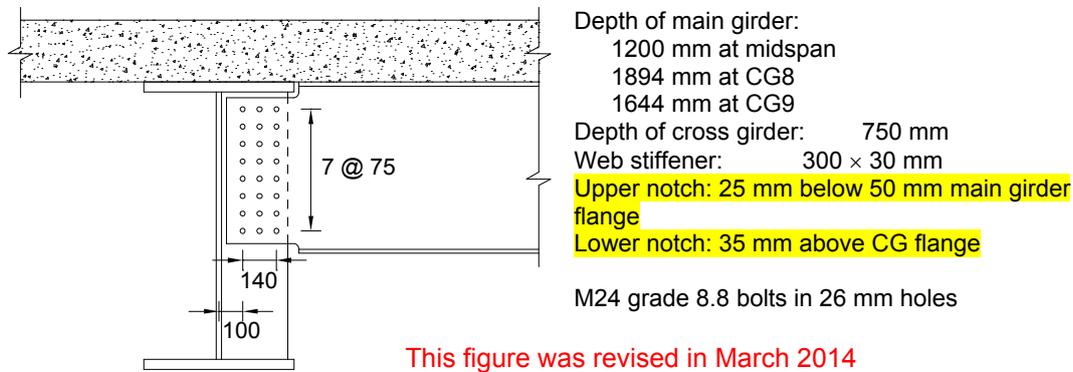
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12 Cross girder to main girder connection

Consider the centre span cross girders at midspan and adjacent to the intermediate support.

12.1 Structural arrangement of connection

The structural arrangement of the connection between an intermediate cross girder and a main girder is shown below. Only the web of the cross girder is connected to the stiffener on the main girder; the flanges are not connected.



12.2 Design basis

The cross girders behave essentially as simply supported beams, spanning between the main girders but there are small end moments due to permanent and variable loads.

Until the concrete slab has hardened, there is very little end moment, apart from that corresponding to the self weight of the cantilever. The magnitude of this moment is approximately equal for the cross girders at midspan and a little greater at CG8 and CG9, because of the lateral restraint provided by the deep pier crosshead. At the centroid of the bolt group this small hogging moment is partly offset by the sagging due to the end shear times the distance from the main girder web. At this stage, the bolt group alone has to resist the combination of shear and moment. The connection should be designed not to slip at this stage but this is easily achieved because the magnitudes of the effects are relatively modest.

Once the slab has been cast, the connection will act compositely with the deck slab. A small hogging moment will arise due to the self weight of surfacing on the cantilever. Under traffic loading, end moments at the midspan cross girder will be small and either hogging or sagging (the moment arises due to unequal loading on adjacent cross girders and warping stiffness of the main girders). End moments on CG8 adjacent to the intermediate support are larger and predominantly hogging, due to the proximity of the deep crosshead girder at the support. The values at CG9 are between those for CG8 and for the midspan cross girders.

For the actions on the composite structure, it is assumed that vertical shear at the connection is resisted by steel web of the cross girder and moments are resisted by a composite section comprising a width of slab and the web of the cross girder.

Text highlighted in yellow indicates a revision made in March 2014



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It is assumed that the vertical shear is shared equally by all the bolts and that the moment (at line of the centroid of the bolt group) is determined from an elastic stress distribution on an effective Tee section comprising slab and beam web. The horizontal force in the lowest part of the web due to the moment at the middle of the connection is calculated and the force is shared between the bolts in the bottom row.

The bolted connections provide restraint to the main girder against LTB in regions adjacent to the intermediate support and are therefore designed for no slip at ULS and the effects on the connection must include an allowance for a lateral restraint force at the level of the main girder bottom flange. In midspan regions, slip could be tolerated but it is better to use the same connection detail and design basis for all the intermediate cross girders.

Strictly, the forces in the bolts due to bending moments should be derived by considering the three stages of bare steel, long-term composite Tee and short-term composite Tee, and adding the results from all three stages. However, since the connection is designed to be slip resistant at ULS and the effects at the bare steel stage are modest it is considered adequate to design the connection as a slip resistant connection for the total effects, applied to a short-term composite effective section.

12.3 Design situations

From a review of a range of different loading arrangements, it was concluded that the most onerous design situation for the connection is with gr5 loading (the SV100 vehicle) arranged such that the end shear is greatest. Situations where the end moments are greatest were found to have a lesser end shear and thus to be less onerous for the design of the connection.

The total ULS effects due to dead loads are.

At the middle cross girder (CG13): $M = 85 \text{ kNm}$ (hogging) and $V = 304 \text{ kN}$

At the end cross girder (CG8): $M = 212 \text{ kNm}$ (hogging) and $V = 295 \text{ kN}$

At the second cross girder (CG9) $M = 128 \text{ kNm}$ (hogging) and $V = 308 \text{ kN}$
 (Moments are on the line of the main girder web)

For CG13, the effects due to gr5 traffic load at ULS are:

$M = 184 \text{ kNm}$ (hogging) and $V = 859 \text{ kN}$

For CG8, the effects due to gr5 traffic load at ULS are:

$M = 447 \text{ kNm}$ (hogging) and $V = 933 \text{ kN}$

For CG9, the effects due to gr5 traffic load at ULS are:

$M = 276 \text{ kNm}$ (hogging) and $V = 879 \text{ kN}$

Effects
 tabulated on
 Sheet 20



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12.4 Design resistance of individual bolt connections

The connection is made with M24 grade 8.8 preloaded bolts in 26 mm holes.

Cross girder web thickness = 15 mm ($\sigma_y = 355 \text{ N/mm}^2$)

Main girder stiffener thickness = 30 mm ($\sigma_y = 345 \text{ N/mm}^2$)

Slip resistance

$$F_{s,Rd} = \frac{k_s n \mu}{\gamma_{M3}} F_{p,C}$$

$$F_{p,C} = 0.7 f_{ub} A_s = 0.7 \times 800 \times 353 = 198 \text{ kN}$$

$\mu = 0.5$ (class A friction surface)

$k_s = 1.0$ (normal clearance holes)

$$F_{s,Rd} = \frac{1.0 \times 198 \times 1 \times 0.5}{1.25} = 79.2 \text{ kN}$$

As the connection will be designed against slip at ULS, the forces in the bolts must be determined by an 'elastic' distribution of stresses in the web of the cross girder

For information, the following calculation of bearing/shear resistance at ULS is included. It shows that the resistance in bearing/shear is greater than against slip. This higher value could be used where restraint of the main girders against LTB is not needed but, as explained earlier, the same connection detail would normally be used on all cross girders.

ULS shear resistance of a single bolt

$$\text{Resistance} = F_{v,Rd} = \frac{\alpha_v f_{ub} A}{\gamma_{M2}}$$

$A = A_s = 353 \text{ mm}^2$

$f_{ub} = 800 \text{ N/mm}^2$

$\alpha_v = 0.6$ for grade 8.8 bolts

$$\text{Resistance} = \frac{0.6 \times 353 \times 800}{1.25} \times 10^{-3} = 136 \text{ kN}$$

ULS bearing resistance of a single bolt on the cross girder web

$$\text{Resistance} = F_{b,Rd} = \frac{k_1 \alpha_b f_u d t}{\gamma_{M2}} \text{ kN}$$

$$k_1 = \min \left(2.8 \frac{e_2}{d_0} - 1.7; 2.5 \right) \text{ for edge bolts}$$

Here, the force under combined shear and bending is not perpendicular to an edge, so use the lesser 'edge' distance

$$k_1 = \min \left(2.8 \frac{45}{26} - 1.7; 2.5 \right) = 2.5$$

3-1-8/3.9.1

(3.6)

(3.7)

3-1-8/
Table 3.4

bolt strength:
3-1-8/3.1.1

3-1-8/
Table 3.4



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$$k_1 = \min\left(1.4 \frac{p_2}{d_0} - 1.7; 2.5\right) \text{ for inner bolts}$$

Again, the force under combined shear and bending is not perpendicular to the lines of bolts, so use the lesser spacing

$$k_1 = \min\left(1.4 \frac{70}{26} - 1.7; 2.5\right) = 2.07$$

α_b is the smallest of α_d , f_{ub}/f_u and 1.0

$$\alpha_d = \frac{e_1}{3d_0} \text{ for end bolts and } \alpha_d = \frac{p_1}{3d_0} - \frac{1}{4} \text{ for inner bolts}$$

$$\alpha_d = \frac{45}{3 \times 26} = 0.58 \text{ for end bolts and } \alpha_d = \frac{70}{3 \times 26} - \frac{1}{4} = 0.65 \text{ for inner bolts}$$

The bearing resistances in different locations in the splice will be different. Consider the bottom corner bolt as a end-edge bolt) although the component due to shear is away from the end, not toward it)

$$F_{b,Rd} = \frac{2.5 \times 0.58 \times 355 \times 24 \times 15}{1.25} = 148 \text{ kN}$$

So the ULS bearing/shear resistance is determined by the shear capacity of the bolt and this value must be used for all the bolts in the group.

3-1-8/3.7

12.5 Design forces on bolt group

Midspan cross girder (CG13)

Moment at centroid of bolt group

$$M = -(85 + 184) + (304 + 859) \times (100 + 70) \times 10^{-3} = -71 \text{ kNm (hogging)}$$

$$\text{Shear} = (304 + 859) = 1163 \text{ kN}$$

Cross girder adjacent to pier (CG8)

Moment at centroid of bolt group

$$M = -(212 + 447) + (295 + 933) \times (100 + 70) \times 10^{-3} = -450 \text{ kNm (hogging)}$$

$$\text{Shear} = (295 + 933) = 1228 \text{ kN}$$

Second cross girder from pier (CG9)

This cross girder provides the effective lateral restraint to the bottom flange that is assumed in determining the buckling resistance in Section 9.1. For this cross girder, in addition to moments from analysis, include an allowance of 1% of the maximum force in the bottom flange over the braced length (here $L_k = \ell$).

3-2/6.3.4.2(5)

$$\text{Flange force} = 297 \times 48000 \times 10^{-3} = 14300 \text{ kN}$$

$$\text{Lateral force} = 143 \text{ kN}$$

Moment depends on height from flange to CG of the effective Tee section (see below)



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Moment at centroid of bolt group

$$M = -(128 + 276) + (308 + 879) \times (100 + 70) \times 10^{-3} = -202 \text{ kNm (hogging)}$$

$$\text{Shear} = (308 + 879) = 1187 \text{ kN}$$

12.6 Effective section at connection

Number of vertical 'columns' of bolts: 3

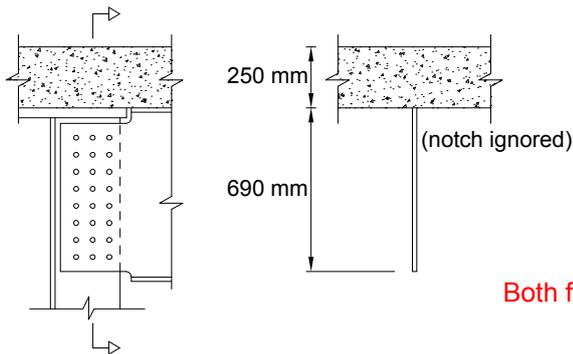
Number of horizontal rows of bolts: 8

$$\text{Total number of bolts} = 3 \times 8 = 24$$

Vertical spacing: 75 mm

Horizontal spacing: 70 mm

To determine forces in the bolts, consider the stresses in a Tee section comprising the slab and the web of the cross girder. Ignore the effect of the notch at the top of the web and the presence of the flanges of the main girder and cross girder (they are not connected).



Both figures were revised in March 2014

The rules in EN 1994-2 for determining the effective width of the top flange do not cover this situation, where there is an end moment from the warping restraint of the girder, rather than continuity. For the design of the connection it was judged that a width of slab equal to the width of the main girder top flange should be used.

Short term section properties for composite Tee section:

$$\text{Slab area} = 800 \times 250 / 6.0 = 33,300 \text{ mm}^2 \text{ (steel units)}$$

$$\text{Steel 'web' area} = 690 \times 15 = 10350 \text{ mm}^2$$

$$\text{Neutral axis of composite section} = 236 \text{ mm below top of slab}$$

$$\text{For composite section, } I = 2.33 \times 10^9 \text{ mm}^4$$

Offsets and section moduli (steel units) for key positions in the section:

	H below NA (mm)	Modulus (steel units) (mm ³)	Modulus (conc. units) (mm ³)
Top of slab	-236		-59.3 × 10 ⁶
Mid way rows 7 & 8	621.5	3.75 × 10 ⁶	
Bottom of web	704	3.31 × 10 ⁶	

Text highlighted in yellow indicates a revision made in March 2014



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While this simple model is reasonable for determining the forces in the bolts (if the effective width of slab width were doubled, to 1600 mm, this would reduce bolt forces by only a few percent), it is not reliable for verification of stresses in the slab. Slab stresses are influenced by interaction with the surrounding regions, including the presence of the cantilever, and any verification of local transverse stresses in the slab is better served by examination of the FE model. The local slab stresses were considered acceptable in this example.

Central cross girder (CG13)

Share the vertical shear equally between all the bolts.

$$\text{Force/bolt} = 1163/24 = 48.5 \text{ kN}$$

Determine force (in web portion) at the level of the bottom row of bolts.

Stress midway between lowest two rows, for $M = 71 \text{ kNm}$ (Sheet 53):

$$\frac{71 \times 10^6}{3.75 \times 10^6} = 19 \text{ N/mm}^2$$

Stress at bottom of web:

$$\frac{71 \times 10^6}{3.31 \times 10^6} = 21 \text{ N/mm}^2$$

$$\text{Force} = \text{average stress} \times \text{depth} = (19 + 21)/2 \times 15 \times (690 - 607.5) \times 10^{-3} = 25.0 \text{ kN}$$

$$\text{Force/bolt} = 25.0/3 = 8.3 \text{ kN}$$

The total resultant force is thus:

$$F = \sqrt{48.5^2 + 8.3^2} = 49.2 \text{ kN} < 79.2 \text{ kN OK}$$

Note that the above simple method for deriving the force on the lowest bolt is adequate for evenly spaced bolts and where the bottom portion of web is approximately symmetrically disposed about the bolt row. In other cases the stresses in the web should be resolved into pure bending and axial components and the moment and force then applied separately to the bolt group to derive the greatest bolt force.

The stress in the concrete due to the moment is:

$$\text{Top of slab: } \frac{71 \times 10^6}{59.3 \times 10^6} = 1.2 \text{ N/mm}^2 \text{ (tension)}$$

As noted above, this value is not a reliable value but is small and well within the tensile strength of the concrete f_{ctm} .

Cross girder adjacent to pier (CG8)

Share the vertical shear equally between all the bolts

$$\text{Force/bolt} = 1228/24 = 51.2 \text{ kN}$$

Text highlighted in yellow indicates a revision made in March 2014



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CALCULATION SHEET

Job No.	BCR113	Sheet	56 of 57	Rev	A
Job Title	Composite highway bridges: Worked examples				
Subject	Example 2: Ladder deck three-span bridge Section 12: Cross girder to main girder connection				
Client SCI	Made by	DCI	Date	July 2009	
	Checked by	RJ	Date	Sep 2009	

Determine force (in web portion) at the level of the bottom row of bolts
Stress midway between lowest two rows for $M = 450$ kNm (Sheet 53):

$$\frac{450 \times 10^6}{3.75 \times 10^6} = 120 \text{ N/mm}^2$$

Stress at bottom of web:

$$\frac{450 \times 10^6}{3.31 \times 10^6} = 136 \text{ N/mm}^2$$

$$\text{Force} = \text{average stress} \times \text{depth} = (120 + 136)/2 \times 15 \times (690 - 607.5) \times 10^{-3} = 158.3 \text{ kN}$$

$$\text{Force/bolt} = 158.3/3 = 52.8 \text{ kN}$$

The total resultant force is thus:

$$F = \sqrt{51.2^2 + 52.8^2} = 73.5 \text{ kN} < 79.2 \text{ kN OK}$$

The stress in the concrete due to the moment is:

$$\text{Top of slab: } \frac{450 \times 10^6}{59.3 \times 10^6} = 7.59 \text{ N/mm}^2 \text{ (tension)}$$

This stress is about double the tensile strength (which is the limit for using uncracked properties for determining flexural effects, according to 4-2/5.4.2.3). Although the surface stress given by the connection model is not considered reliable for verification of the slab, it is indicative that the local stress is not excessive. The tensile force for the stress distribution given by the model could be resisted by the transverse reinforcement (20 mm bars at 150 mm centres).

The force in the slab (in this model) is introduced through the shear stud connectors on the flange of the main girder. Over a length of 800 mm the force transferred to the slab will probably be shared between at least 15 studs (3 rows of 5) and the resultant (from combined longitudinal and transverse shears) will be within the design capacity (81.7 kN, see Section 10.6). The verification of the connectors for the combined effects is not shown here.

Separate checks will be needed for fatigue assessment of the stud connectors. The determination of appropriate loading and load effects is not covered here.

Second cross girder from pier (CG9)

Share the vertical shear equally between all the bolts

$$\text{Force/bolt} = 1187/24 = 49.5 \text{ kN}$$

Extra moment due to lateral restraint provided to bottom flange

(take lever arm as mid-slab to mid bottom flange)

$$M = 143 \times (1894 - 60/2 - 125) / 1000 = 214 \text{ kNm (take as hogging)}$$

$$\text{Total moment} = 202 + 214 = 416 \text{ kNm (hogging)}$$

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CALCULATION SHEET

Job No.	BCR113	Sheet	57 of 57	Rev	A
Job Title	Composite highway bridges: Worked examples				
Subject	Example 2: Ladder deck three-span bridge Section 12: Cross girder to main girder connection				
Client SCI	Made by	DCI	Date	July 2009	
	Checked by	RJ	Date	Sep 2009	

Determine force (in web portion) at the level of the bottom row of bolts

Stress midway between lowest two rows:

$$\frac{416 \times 10^6}{3.75 \times 10^6} = 111 \text{ N/mm}^2$$

Stress at bottom of web:

$$\frac{416 \times 10^6}{3.31 \times 10^6} = 126 \text{ N/mm}^2$$

$$\text{Force} = \text{average stress} \times \text{depth} = (111 + 126)/2 \times 15 \times (690 - 607.5) \times 10^{-3} = 147 \text{ kN}$$

$$\text{Force/bolt} = 147/3 = 49.0 \text{ kN}$$

The total resultant force is thus:

$$F = \sqrt{49.5^2 + 49.0^2} = 69.7 \text{ kN} < 79.2 \text{ kN OK}$$

The stress in the concrete due to the moment is:

$$\text{Top of slab: } \frac{416 \times 10^6}{59.3 \times 10^6} = 7.03 \text{ N/mm}^2 \text{ (tension)}$$

As noted for CG8, this is not a reliable value for verification of the slab.

As for CG8, the shear connectors need to be verified for the combined longitudinal effects but that verification is not given here.

Text highlighted in yellow indicates a revision made in March 2014

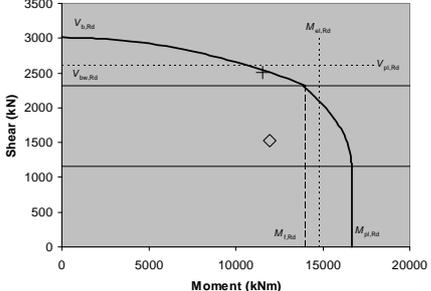
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Thomas Telford, 2007

Corrigenda noted up to 10/3/2014

Page	Editorial/ Technical	Comment/correction
8	T	It should have been noted that taking the notional size, h_o as the slab thickness is a simplification. The actual value according to 2-1-1/B.1 is $h_o = 2A_c/u$ and the length of perimeter u should include the top surface (it is counted as a drying surface) and the bottom surface other than that in contact with the steel flanges. The more accurate value of h_o is thus slightly greater than the slab thickness and the modular ratio is slightly smaller. The effect of the difference is generally negligible.
9	E	In the evaluation of $\varepsilon_{cd,0}$ (four lines from the bottom of the page), the value of f_{cm} is shown as 40 (it should be 48) but the calculated value of $\varepsilon_{cd,0}$ is correct.
10	E	In lines 7 and 8, the total shrinkage should be given as ε_{cs} .not as ε_{cd} .
17	T	The width between shear studs (400mm) results in an edge distance (to the face of the stud) of 40 mm. This is satisfactory for the flange but the haunch should be slightly wider, to ensure compliance with 4-2/Figure 6.14.
17	E	In the penultimate paragraph, 1400/8 should be 14000/8
18	T	The elastic modulus calculated for the centroid of the flanges. This modulus has been used for calculation of ULS stresses (as permitted by 3-1-1/6.2.1(9) and also for SLS stresses (no rule is given for determining steel stresses at SLS) and for fatigue stresses (strictly different moduli apply when checking the upper and lower surfaces).
19	E	It should have been noted that the span girder properties in the second table (“Cracked composite sections – hogging”) relate to the section reinforced as at the pier girder, not the lesser reinforcement for the span girder. The stresses calculated on page 27 use these properties.
21	E	In the second paragraph, first line, ε_{cd} should be ε_{cs} .
29	E	In the summation of effects for the pier side of the splice, the shear force F_z due to soil pressure should be negative (–55 kN) not positive and thus the total should be 834 kN, not 944 kN. The correct value of 834 is used on page 54.
30	T	It should have been noted that, for the SLS effects at the splice position, the moment due to the restraint of thermal expansion is negligible but that the axial force causes a stress of -10 N/mm^2 .
32	E	In the second paragraph below the diagram, it is the total torque <i>applied to each beam</i> (text was omitted) and in the equation below, the “ $\times 2$ ” should be deleted (the answer is correct).
37	T	In the selection of buckling curve, which requires the value of h/b , it should be noted that the value of b is taken as that of the smaller flange. 3-1-1/6.3.2.3 and NA.2.17 do not advise which value should be used when the flanges are of unequal width.

Continued on next page.

Page	Editorial/Technical	Comment/correction
41	T	<p>The interaction curve does not show the upper limit to the value of $V_{b,Rd}$ given by 3-1-5/5.2(1) (which is effectively $V_{pl,Rd}$ calculated using γ_{M1} rather than γ_{M0}). The corrected curve is shown below.</p> 
47	E	The reference to 4-2/(6.21) should be to 2-1-1/(6.21)
47	T	A check for concrete crushing in the flange should also have been carried out here using 2-1-1/(6.22).
50	E	In the penultimate line, the value “124” should be “146”. The result is correct.
51	E	In the evaluation of λ the value “0.94” should be “0.97”. This affects the result (1.12 not 1.08) and the value of $\Delta\sigma_E$ (20 not 19)
54	E	The values for web moment should be 155/127/–33/–11 (as quoted later, on page 59). The corrected values are based on a simplification that assumes that the stresses at the top and bottom of the web are the same as the flange stresses.
58	E	In verifying the slip resistance at SLS, the design effect is 3168 kN, not 3312 kN.
59	E	(Just above Section 12.7) The maximum resultant force at ULS is 169 kN (as in the Table), not 164 kN.
62	T	The example does not apply the NA rule (NA.2.7.2 to BS EN 1993-1-5) for force in the stiffener.
64-66	E	A few minor editorial errors, see Sheets 60, 61 and 62 of Worked Example 1.
73	T	In the table, “Drying shrinkage” should be “Total shrinkage”. The font size for its value is incorrect.
80	E	In the second paragraph, second line, “for” should be inserted after “will be used”.
104	T	For the selection of buckling curve, which requires the value of h/b , neither h nor b are defined for a composite section of varying depth. The value of main girder depth at the first cross girder ($h = 1894$) and flange width ($b = 800$) were used to select curve c but it is arguable that curve d should be used instead.
104	T	A further check should have been made mid-way between the pier and CG1. The utilization $M_{Ed}/M_{b,Rd}$ is slightly greater at that location.
106	T	The value of the plastic bending resistance should have been reduced by a factor of 0.9 because the cross section at the support is Class 3/4, as required by 4-2/6.2.1.3(2).
120	T	Figure revised to show notch at lower flange and different bolt spacing. Text changed, indicated using yellow highlighting.
124	T	Figure revised to show notch at lower flange, different vertical bolt spacing and reduced depth of web (by 35 mm, for notch). Text changed, indicated using yellow highlighting.
125	T	Text changed, indicated using yellow highlighting.
126	T	Text changed, indicated using yellow highlighting.
127	T	Text changed, indicated using yellow highlighting.

