Fault Analysis

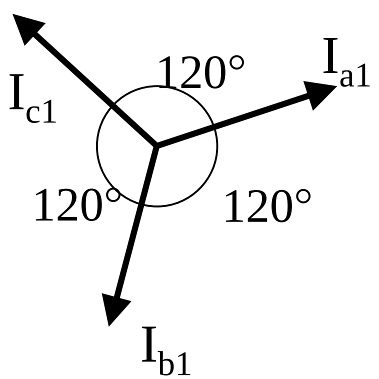
- **Fault types:**

	Percentage of total faults
balanced faults	
three-phase	<5%
unbalanced faults	
single-line to ground	60-75%
double-line to ground	15-25%
line-to-line faults	5-15%

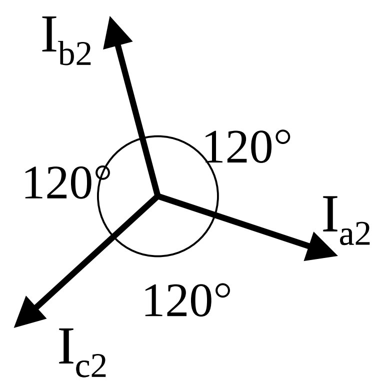
- **Unbalance fault analysis requires new tools**
  - symmetrical components
  - augmented component models

Symmetrical Components

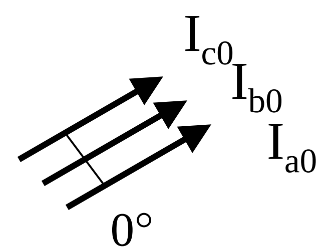
- **Allow unbalanced three-phase phasor quantities to be replaced by the sum of three separate but balanced symmetrical components**
  - applicable to current and voltages
  - permits modeling of unbalanced systems and networks
- **Representative symmetrical components**



abc sequence
positive sequence



acb sequence
negative sequence



zero sequence

Symmetrical Components

- **Positive sequence phasors**

$$I_{a1} = |I_{a1}|\angle(\delta + 0^\circ) \quad = I_{a1}$$

$$I_{b1} = |I_{a1}|\angle(\delta + 240^\circ) = a^2 I_{a1}$$

$$I_{c1} = |I_{a1}|\angle(\delta + 120^\circ) = a\, I_{a1}$$

- **Operator *a* identities**

$$a = 1\angle 120^\circ = -0.5 + j0.866$$

$$a^2 = 1\angle 240^\circ = -0.5 - j0.866$$

$$a^3 = 1\angle 0^\circ \quad = 1 + j0$$

$$1 + a + a^2 = 0$$

Symmetrical Components

- **Negative sequence phasors**

$$I_{a2} = |I_{a2}|\angle(\delta + 0°) = I_{a2}$$
$$I_{b2} = |I_{a2}|\angle(\delta + 120°) = a\, I_{a2}$$
$$I_{c2} = |I_{a2}|\angle(\delta + 240°) = a^2 I_{a2}$$

- **Zero sequence phasors**

$$I_{a0} = |I_{a0}|\angle(\delta + 0°) = I_{a0}$$
$$I_{b0} = |I_{a0}|\angle(\delta + 0°) = I_{a0}$$
$$I_{c0} = |I_{a0}|\angle(\delta + 0°) = I_{a0}$$

Symmetrical Components

- **Relating unbalanced phasors to symmetrical components**

$$I_a = I_{a0} + I_{a1} + I_{a2} \qquad = I_{a0} + I_{a1} + I_{a2}$$
$$I_b = I_{b0} + I_{b1} + I_{b2} \qquad = I_{a0} + a^2 I_{a1} + a\, I_{a2}$$
$$I_c = I_{c0} + I_{c1} + I_{c2} \qquad = I_{a0} + a\, I_{a1} + a^2 I_{a2}$$

- **In matrix notation**

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

Symmetrical Components

- [A] is known as the symmetrical components transformation matrix

$$\mathbf{I}_{abc} = \mathbf{A}\,\mathbf{I}_{012} \qquad \mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

- Solving for the symmetrical components leads to

$$\mathbf{I}_{012} = \mathbf{A}^{-1}\,\mathbf{I}_{abc}$$

$$\mathbf{A}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} = \frac{1}{3}\mathbf{A}^*$$

Symmetrical Components

- **In component form, the calculation for symmetrical components are**

$$I_{a0} = \tfrac{1}{3}\left(I_a + I_b + I_c\right)$$

$$I_{a1} = \tfrac{1}{3}\left(I_a + aI_b + a^2 I_c\right)$$

$$I_{a2} = \tfrac{1}{3}\left(I_a + a^2 I_b + aI_c\right)$$

Symmetrical Components

- **Similar expressions exist for voltages**

$$\mathbf{V}_{abc} = \mathbf{A}\ \mathbf{V}_{012}$$

$$\mathbf{V}_{012} = \mathbf{A}^{-1}\ \mathbf{V}_{abc}$$

- **The apparent power may also be expressed in terms of symmetrical components**

$$S_{3\phi} = \mathbf{V}_{abc}^{\mathbf{T}}\ \mathbf{I}_{abc}^{*}$$

$$S_{3\phi} = \left(\mathbf{A}\mathbf{V}_{012}\right)^{\mathbf{T}}\left(\mathbf{A}\mathbf{I}_{012}\right)^{*}$$

$$S_{3\phi} = \mathbf{V}_{012}^{\mathbf{T}}\mathbf{A}^{\mathbf{T}}\mathbf{A}^{*}\mathbf{I}_{012}^{*} \qquad \mathbf{A}^{\mathbf{T}}\mathbf{A}^{*} = 3$$

$$S_{3\phi} = 3\mathbf{V}_{012}^{\mathbf{T}}\mathbf{I}_{012}^{*} = 3V_{a0}I_{a0}^{*} + 3V_{a1}I_{a1}^{*} + 3V_{a2}I_{a2}^{*}$$

Example

- **Obtain the symmetrical components of a set of unbalanced currents**

$$I_a = 1.6\angle 25^\circ$$

$$I_b = 1.0\angle 180^\circ$$

$$I_c = 0.9\angle 132^\circ$$

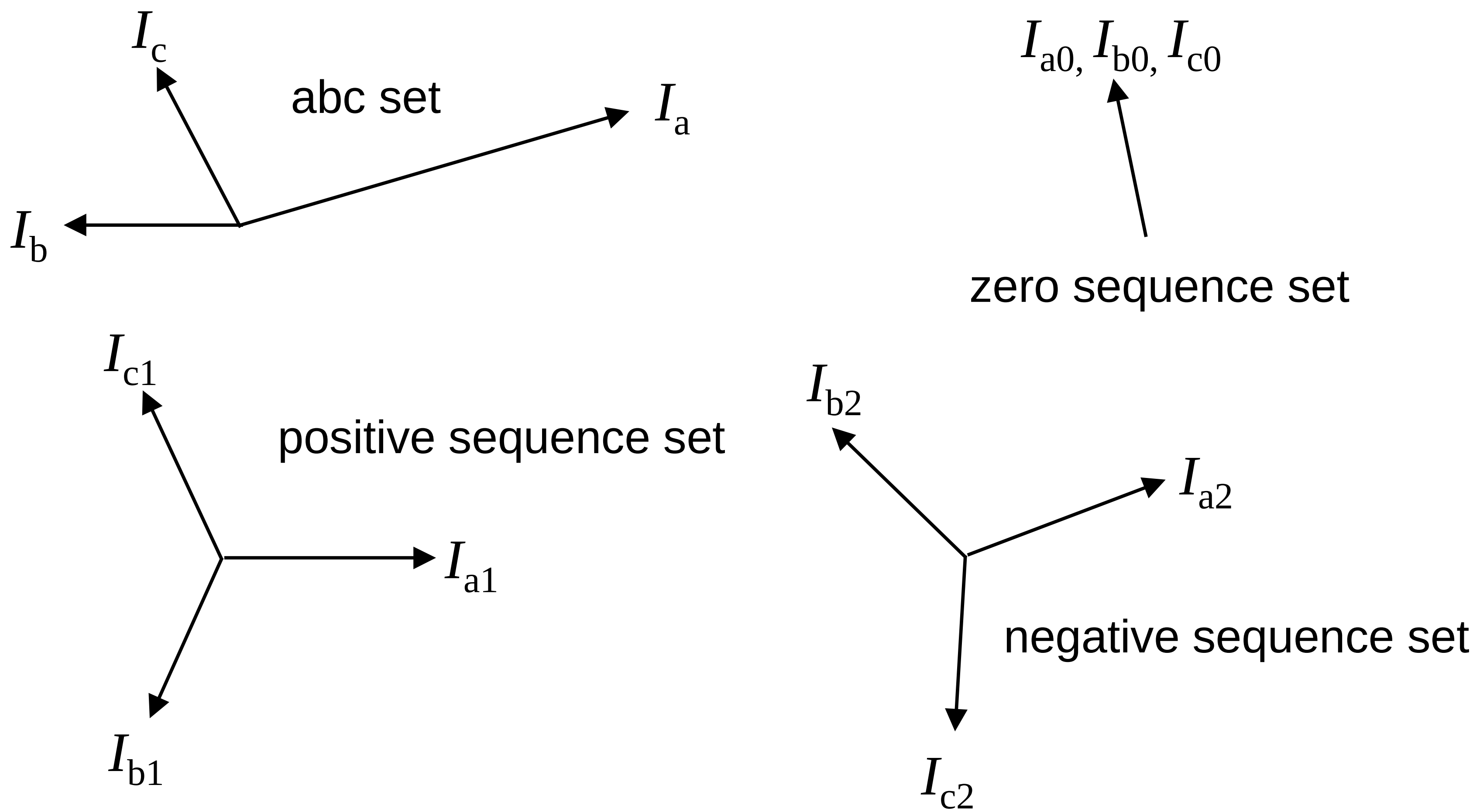
- **Solution**

$$I_{a0} = \frac{(1.6\angle 25^\circ) + (1.0\angle 180^\circ) + (0.9\angle 132^\circ)}{3} = 0.45\angle 96.5^\circ$$

$$I_{a1} = \frac{(1.6\angle 25^\circ) + a(1.0\angle 180^\circ) + a^2(0.9\angle 132^\circ)}{3} = 0.94\angle -0.1^\circ$$

$$I_{a2} = \frac{(1.6\angle 25^\circ) + a^2(1.0\angle 180^\circ) + a(0.9\angle 132^\circ)}{3} = 0.60\angle 22.3^\circ$$

Example



I_c
abc set
I_a
I_b
I_{a0}, I_{b0}, I_{c0}
zero sequence set
I_{c1}
positive sequence set
I_{a1}
I_{b1}
I_{b2}
I_{a2}
negative sequence set
I_{c2}

Example

- **The symmetrical components of a set of unbalanced voltages are**

$$V_{a0} = 0.6\angle 90^\circ$$

$$V_{a1} = 1.0\angle 30^\circ$$

$$V_{a2} = 0.8\angle -30^\circ$$

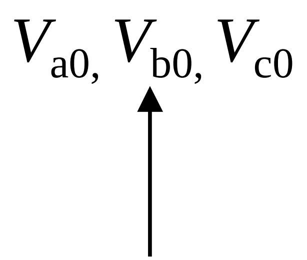
Obtain the original unbalanced voltages:

$$V_a = (0.6\angle 90^\circ) + (1.0\angle 30^\circ) + (0.8\angle -30^\circ) = 1.7088\angle 24.2^\circ$$

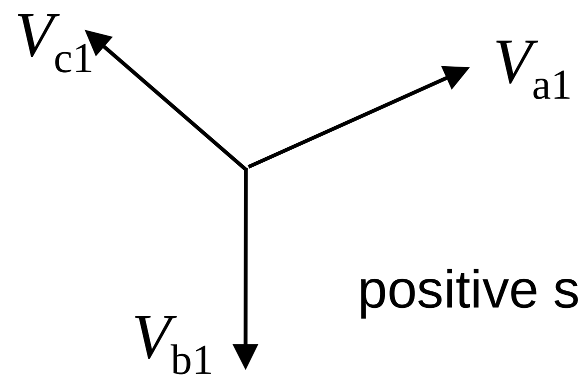
$$V_b = (0.6\angle 90^\circ) + a^2(1.0\angle 30^\circ) + a(0.8\angle -30^\circ) = 0.4\angle 90^\circ$$

$$V_c = (0.6\angle 90^\circ) + a\,(1.0\angle 30^\circ) + a^2(0.8\angle -30^\circ) = 1.7088\angle 155.8^\circ$$

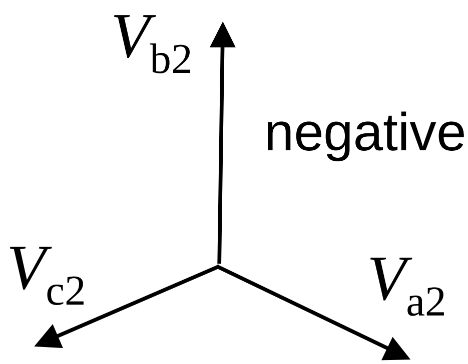
Example



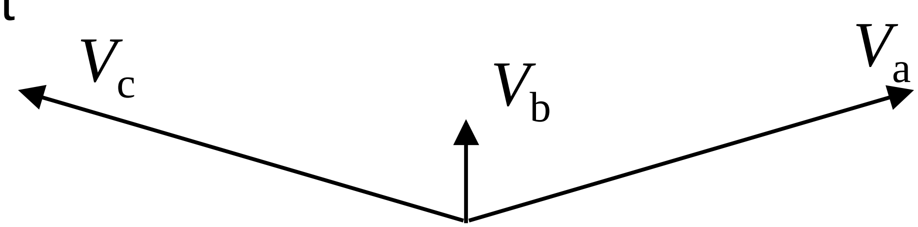
zero sequence set



positive sequence set



negative sequence set

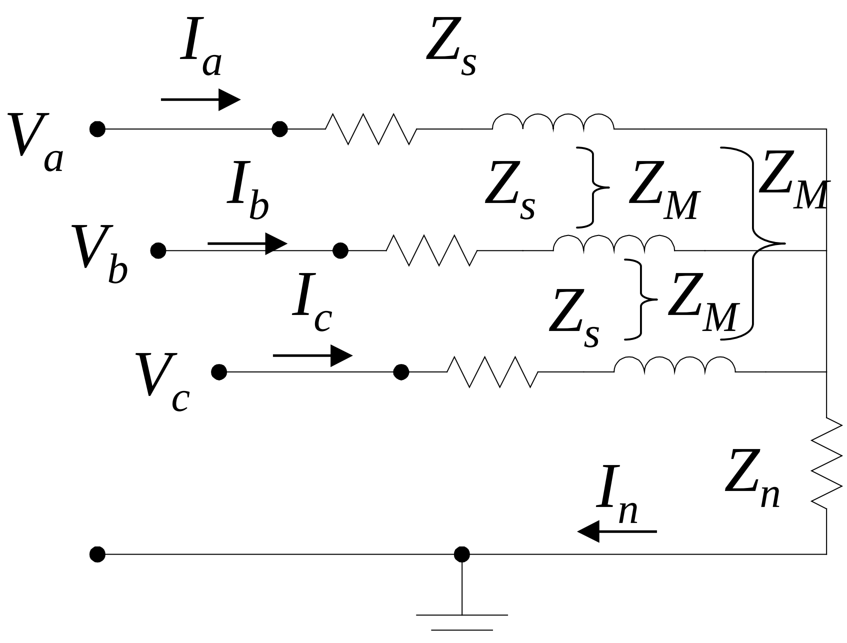


abc set

Sequence Impedances

- **The impedance offered to the flow of a sequence current creating sequence voltages**
  - positive, negative, and zero sequence impedances
- **Augmented network models**
  - wye-connected balanced loads
  - transmission line
  - 3-phase transformers
  - generators

Balanced Loads



Model and governing equations

$$V_a = Z_S I_a + Z_M I_b + Z_M I_c + Z_n I_n$$
$$V_b = Z_M I_a + Z_S I_b + Z_M I_c + Z_n I_n$$
$$V_c = Z_M I_a + Z_M I_b + Z_S I_c + Z_n I_n$$
$$I_n = I_a + I_b + I_c$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_S + Z_n & Z_M + Z_n & Z_M + Z_n \\ Z_M + Z_n & Z_S + Z_n & Z_M + Z_n \\ Z_M + Z_n & Z_M + Z_n & Z_S + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\mathbf{V}_{abc} = \mathbf{Z}_{abc} \mathbf{I}_{abc}$$

Balanced Loads

$$\mathbf{V}_{abc} = \mathbf{Z}_{abc}\mathbf{I}_{abc} \rightarrow \left(\mathbf{A}\,\mathbf{V}_{012}\right) = \mathbf{Z}_{abc}\left(\mathbf{A}\,\mathbf{I}_{012}\right)$$

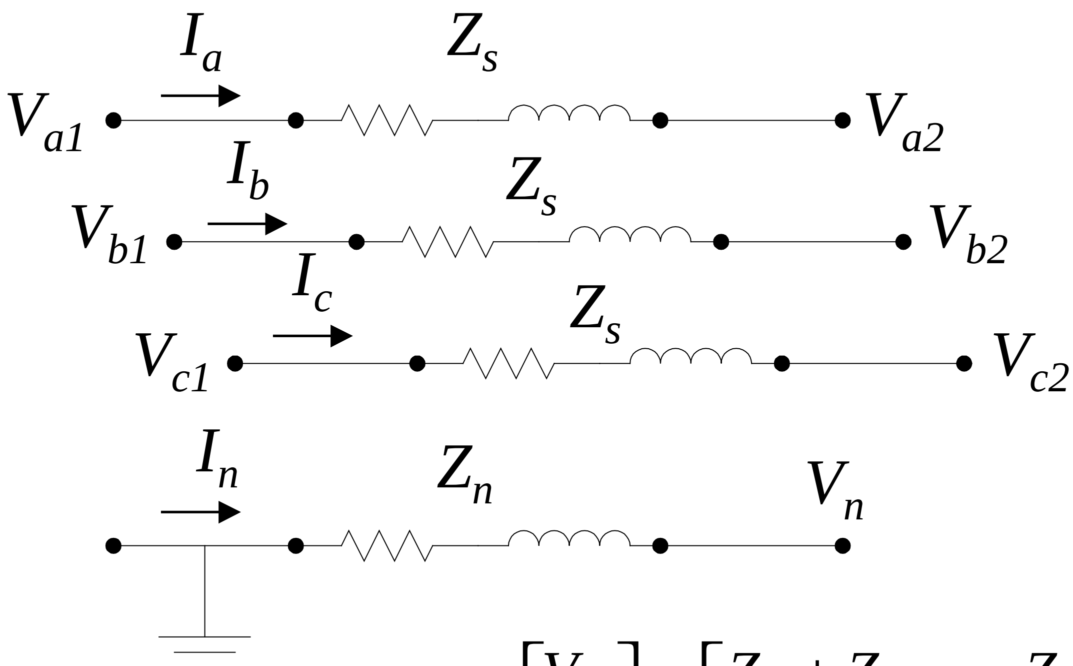
$$\mathbf{V}_{012} = \left[\mathbf{A}^{-1}\mathbf{Z}_{abc}\mathbf{A}\right]\mathbf{I}_{012} \rightarrow \mathbf{V}_{012} = \mathbf{Z}_{012}\,\mathbf{I}_{012}$$

$$\mathbf{Z}_{012} = \left[\mathbf{A}^{-1}\mathbf{Z}_{abc}\mathbf{A}\right]$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} Z_S + Z_n & Z_M + Z_n & Z_M + Z_n \\ Z_M + Z_n & Z_S + Z_n & Z_M + Z_n \\ Z_M + Z_n & Z_M + Z_n & Z_S + Z_n \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= \begin{bmatrix} Z_S + 3Z_n + 2Z_M & 0 & 0 \\ 0 & Z_S - Z_M & 0 \\ 0 & 0 & Z_S - Z_M \end{bmatrix}$$

Transmission Line



$$V_{a1} = Z_S I_a - Z_n I_n + V_{a2}$$
$$V_{b1} = Z_S I_b - Z_n I_n + V_{b2}$$
$$V_{c1} = Z_S I_c - Z_n I_n + V_{c2}$$
$$V_n = 0 + Z_n I_n$$
$$I_n + I_a + I_b + I_c = 0$$

$$\begin{bmatrix} V_{a1} \\ V_{b1} \\ V_{c1} \end{bmatrix} = \begin{bmatrix} Z_S + Z_n & Z_n & Z_n \\ Z_n & Z_S + Z_n & Z_n \\ Z_n & Z_n & Z_S + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + \begin{bmatrix} V_{a2} \\ V_{b2} \\ V_{c2} \end{bmatrix}$$

$$\mathbf{V}_{abc1} = \mathbf{Z}_{abc} \mathbf{I}_{abc} + \mathbf{V}_{abc2}$$

Transmission Line

$$\mathbf{V}_{abc1} = \mathbf{Z}_{abc}\mathbf{I}_{abc} + \mathbf{V}_{abc2} \rightarrow \mathbf{A}\,\mathbf{V}_{012-1} = \mathbf{Z}_{abc}\mathbf{A}\,\mathbf{I}_{012} + \mathbf{A}\,\mathbf{V}_{012-2}$$

$$\mathbf{V}_{012-1} = \mathbf{A}^{-1}\mathbf{Z}_{abc}\mathbf{A}\,\mathbf{I}_{012} + \mathbf{V}_{012-2} = \mathbf{Z}_{012}\,\mathbf{I}_{012} + \mathbf{V}_{012-2}$$

$$\mathbf{Z}_{012} = \mathbf{A}^{-1}\mathbf{Z}_{abc}\mathbf{A}$$

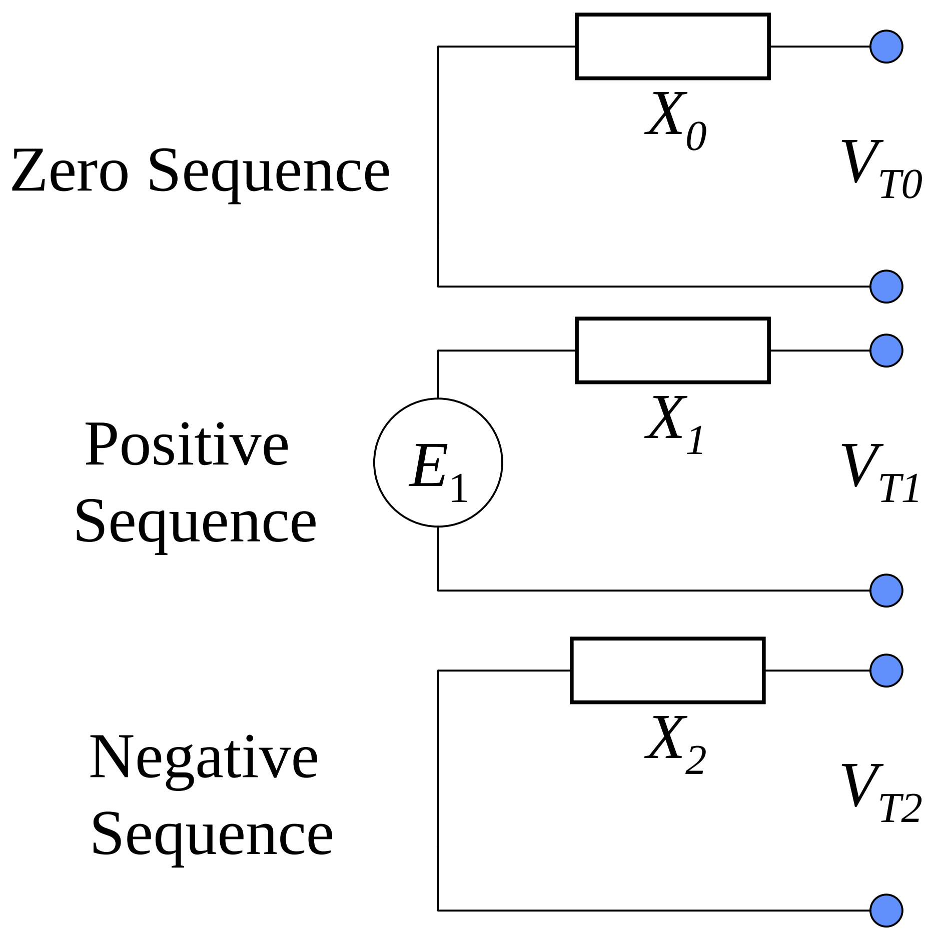
$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} Z_S + Z_n & Z_n & Z_n \\ Z_n & Z_S + Z_n & Z_n \\ Z_n & Z_n & Z_S + Z_n \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= \begin{bmatrix} Z_S + 3Z_n & 0 & 0 \\ 0 & Z_S & 0 \\ 0 & 0 & Z_S \end{bmatrix}$$

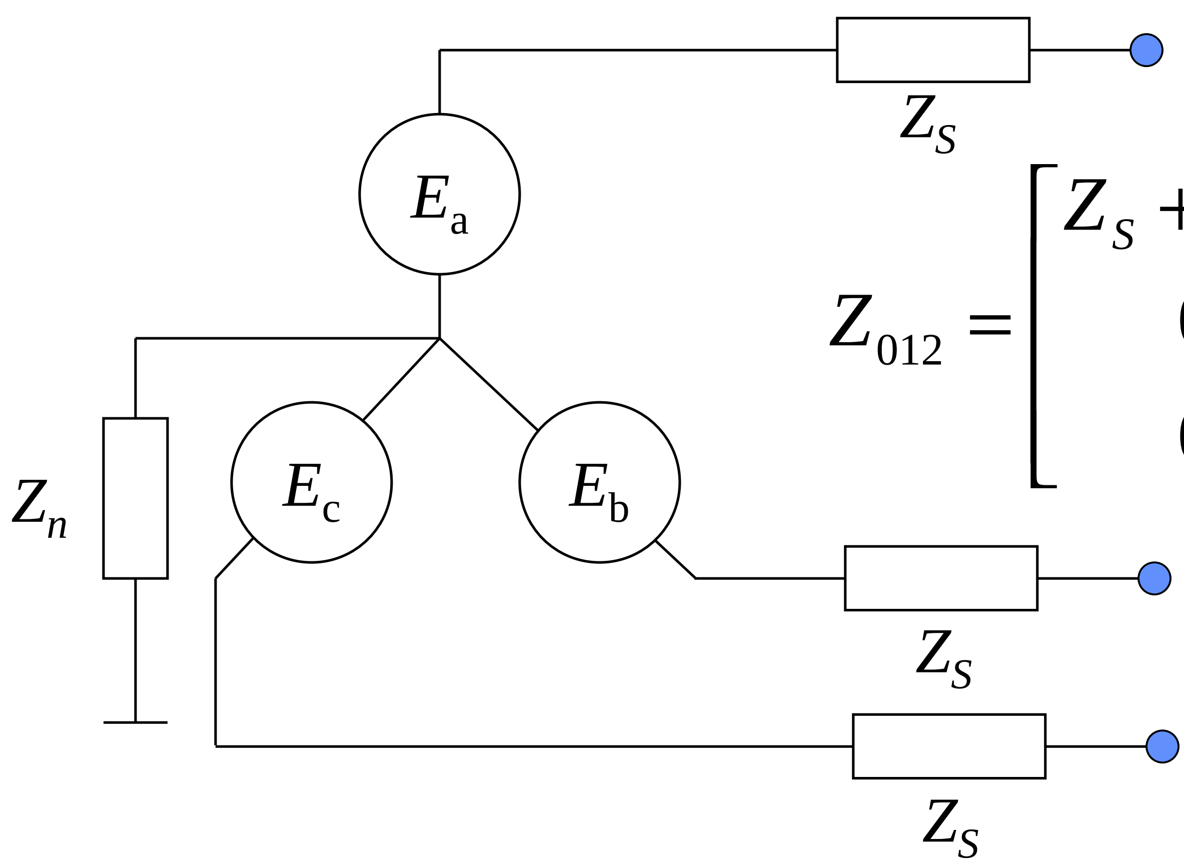
Generators

- **Similar modeling of impedances to sequence impedances**
- **Typical values for common generators**
  - remember that the transient fault impedance is a function of time
  - positive sequence values are the same as X_d, X_d', and X_d''
  - negative sequence values are affected by the rotation of the rotor
    - $X_2 \sim X_d''$
  - zero sequence values are isolated from the airgap of the machine
    - the zero sequence reactance is approximated to the leakage reactance
    - $X_0 \sim X_L$

Generator Model



Impedance Grounded Generators



$$Z_{012} = \begin{bmatrix} Z_S + 3Z_n & 0 & 0 \\ 0 & Z_S & 0 \\ 0 & 0 & Z_S \end{bmatrix}$$

Transformers

- **Series Leakage Impedance**
  - the magnetization current and core losses represented by the shunt branch are neglected (they represent only 1% of the total load current)
  - the transformer is modeled with the equivalent series leakage impedance
- **Three single-phase units & five-legged core three-phase units**
  - the series leakage impedance is the same for all the sequences $$Z_0 = Z_1 = Z_2 = Z_\ell$$
- **Three-legged core three-phase units**
  - the series leakage impedance is the same for the positive and negative sequence only $$Z_1 = Z_2 = Z_\ell$$

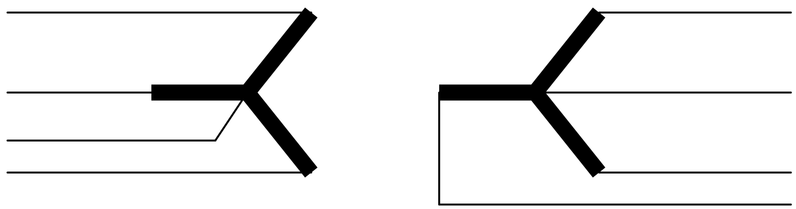
Transformers

- **Wye-delta transformers create a phase shifting pattern for the various sequences**
  - the positive sequence quantities rotate by +30 degrees
  - the negative sequence quantities rotate by -30 degrees
  - the zero sequence quantities can not pass through the transformer
- **USA standard**
  - independent of the winding order (Δ-Y or Y- Δ)
  - the positive sequence line voltage on the HV side leads the corresponding line voltage on the LV side by 30°
  - consequently, for the negative sequence voltages the corresponding phase shift is -30°

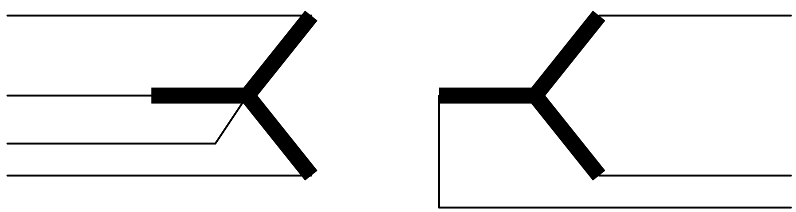
Transformers

- **Zero-sequence network connections of the transformer depends on the winding connection**
  - primary winding - wye / wye-grounded / delta
  - secondary winding - wye / wye-grounded / delta

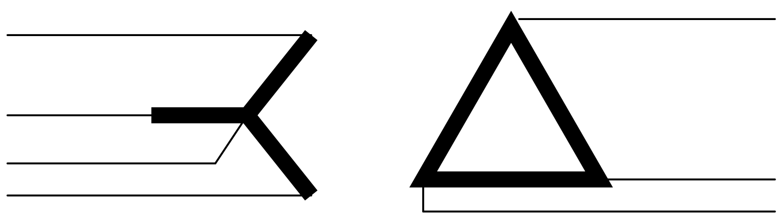
Transformers



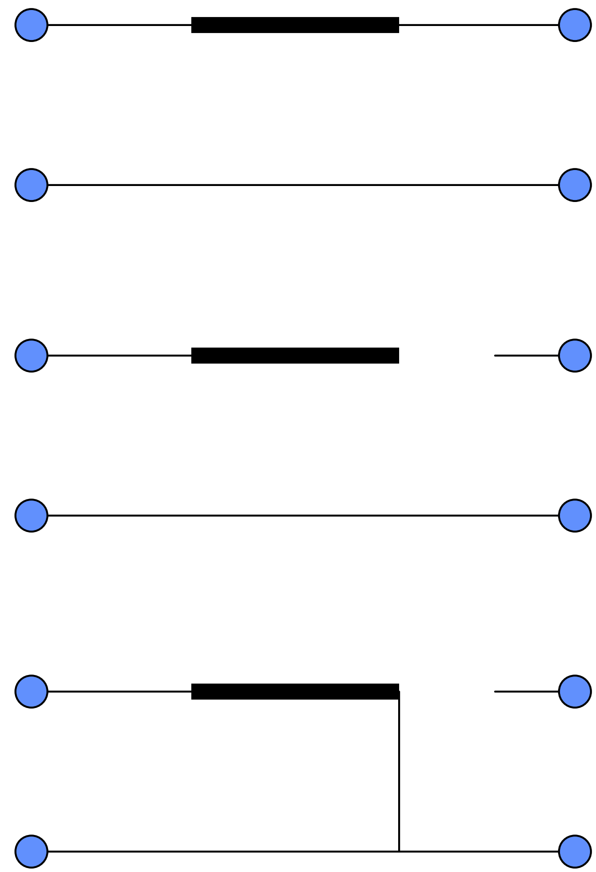
wye-grounded wye-grounded



wye-grounded wye



wye-grounded delta



Transformers