Power Flow Solution

- **The utility wants to know the voltage profile**
  - the <u>nodal voltages</u> for a given load and generation schedule
- **Types of network buses**
  - load bus
    - known real (P) and reactive (Q) power injections
  - generator bus
    - known real (P) power injection and the voltage magnitude (V)
  - slack bus (swing bus)
    - known voltage magnitude (V) and voltage angle (δ)
    - must have one generator as the slack bus
    - takes up the power slack due to losses in the network

Power Flow Equations

- **KCL**

$$\begin{aligned} I_i &= y_{i0}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + \cdots + y_{in}(V_i - V_n) \\ &= (y_{i0} + y_{i1} + y_{i2} + \cdots + y_{in})V_i - y_{i1}V_1 - y_{i2}V_2 - \cdots - y_{in}V_n \\ &= V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij}V_j \qquad j \neq i \end{aligned}$$

- **Power Law**

$$P_i + jQ_i = V_i I_i^* \quad \rightarrow \quad I_i = \frac{P_i - jQ_i}{V_i^*}$$

$$\frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij}V_j \qquad j \neq i$$

Gauss-Seidel Method

- **A non-linear algebraic equation solver**
  - method of successive displacements
  - iterative steps:
    - take a function and rearrange it into the form $x = \mathrm{g}(x)$ {there are several possible arrangements}
    - make an an initial estimate of the variable x: $x^{[0]} = \text{initial value}$
    - find an iterative improvement of $x^{[k]}$, that is: $x^{[k+1]} = \mathrm{g}(x^{[k]})$
    - a solution is reached when the difference between two iterations is less than a specified accuracy: $\left| x^{[k+1]} - x^{[k]} \right| \leq \varepsilon$
  - acceleration factors
    - can improve the rate of convergence: $\alpha > 1$
    - modified step: the improvement is found as $x^{[k+1]} = x^{[k]} + \alpha(\mathrm{g}(x^{[k]}) - x^{[k]})$

Gauss-Seidel Example

- **Find the root of the equation: $f(x) = x^3 - 6x^2 + 9x - 4 = 0$**
  - Step 1. Cast the equation into the $g(x)$ form.

$$9x = -x^3 + 6x^2 + 4$$

$$x = -\tfrac{1}{9}x^3 + \tfrac{6}{9}x^2 + \tfrac{4}{9} = g(x)$$

Gauss-Seidel Example

- Step 2. Starting with an initial guess of x[0] = 2, several iterations are performed.

$$x^{[1]} = g(x^{[0]} = 2) = -\tfrac{1}{9}(2)^3 + \tfrac{6}{9}(2)^2 + \tfrac{4}{9} = 2.2222$$

$$x^{[2]} = g(x^{[1]} = 2.2222) = -\tfrac{1}{9}(2.2222)^3 + \tfrac{6}{9}(2.2222)^2 + \tfrac{4}{9} = 2.5173$$

$$x^{[3]} = g(x^{[2]} = 2.5173) = -\tfrac{1}{9}(2.5173)^3 + \tfrac{6}{9}(2.5173)^2 + \tfrac{4}{9} = 2.8966$$

$$x^{[4]} = 3.3376$$

$$x^{[5]} = 3.7398$$

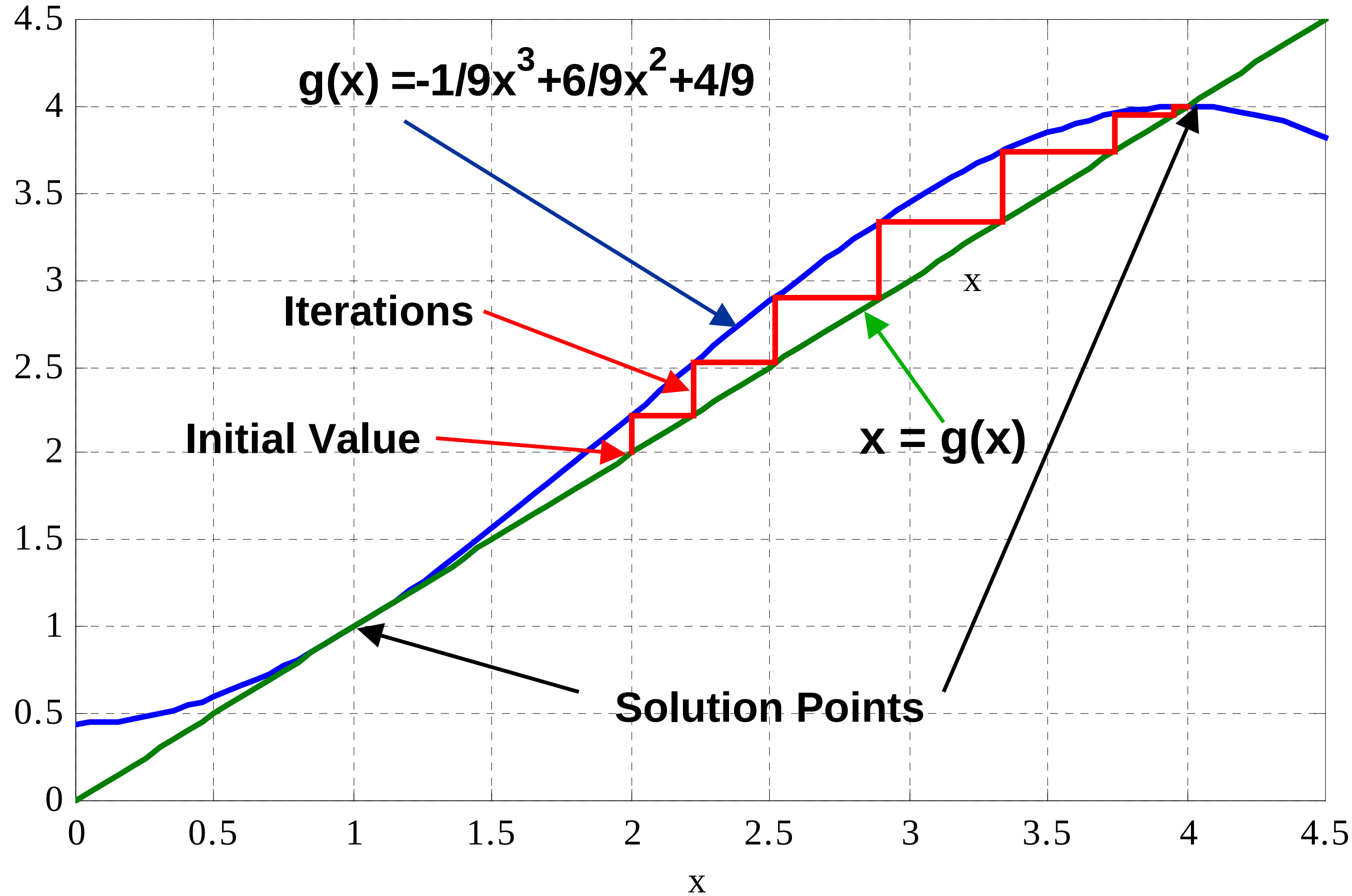
$$x^{[6]} = 3.9568$$

$$x^{[7]} = 3.9988$$

$$x^{[8]} = 4.0000$$

Gauss-Seidel Example

Matlab Results



Gauss-Seidel Example

- **Find the root of the equation: $f(x) = x^3 - 6x^2 + 9x - 4 = 0$ with an acceleration factor of 1.25**
  - Starting with an initial guess of $x^{[0]} = 2$.

$$x^{[0]} = 2$$

$$g(2) = -\tfrac{1}{9}(2)^3 + \tfrac{6}{9}(2)^2 + \tfrac{4}{9} = 2.2222$$

$$x^{[1]} = 2 + 1.25\left[2.2222 - 2\right] = 2.2778$$

$$g(2.2778) = -\tfrac{1}{9}(2.2778)^3 + \tfrac{6}{9}(2.2778)^2 + \tfrac{4}{9} = 2.5902$$

$$x^{[2]} = 2.2778 + 1.25\left[2.5902 - 2.2778\right] = 2.6683$$

Gauss-Seidel Example

- Additional iterations

$$x^{[3]} = 3.0801$$

$$x^{[4]} = 3.1831$$

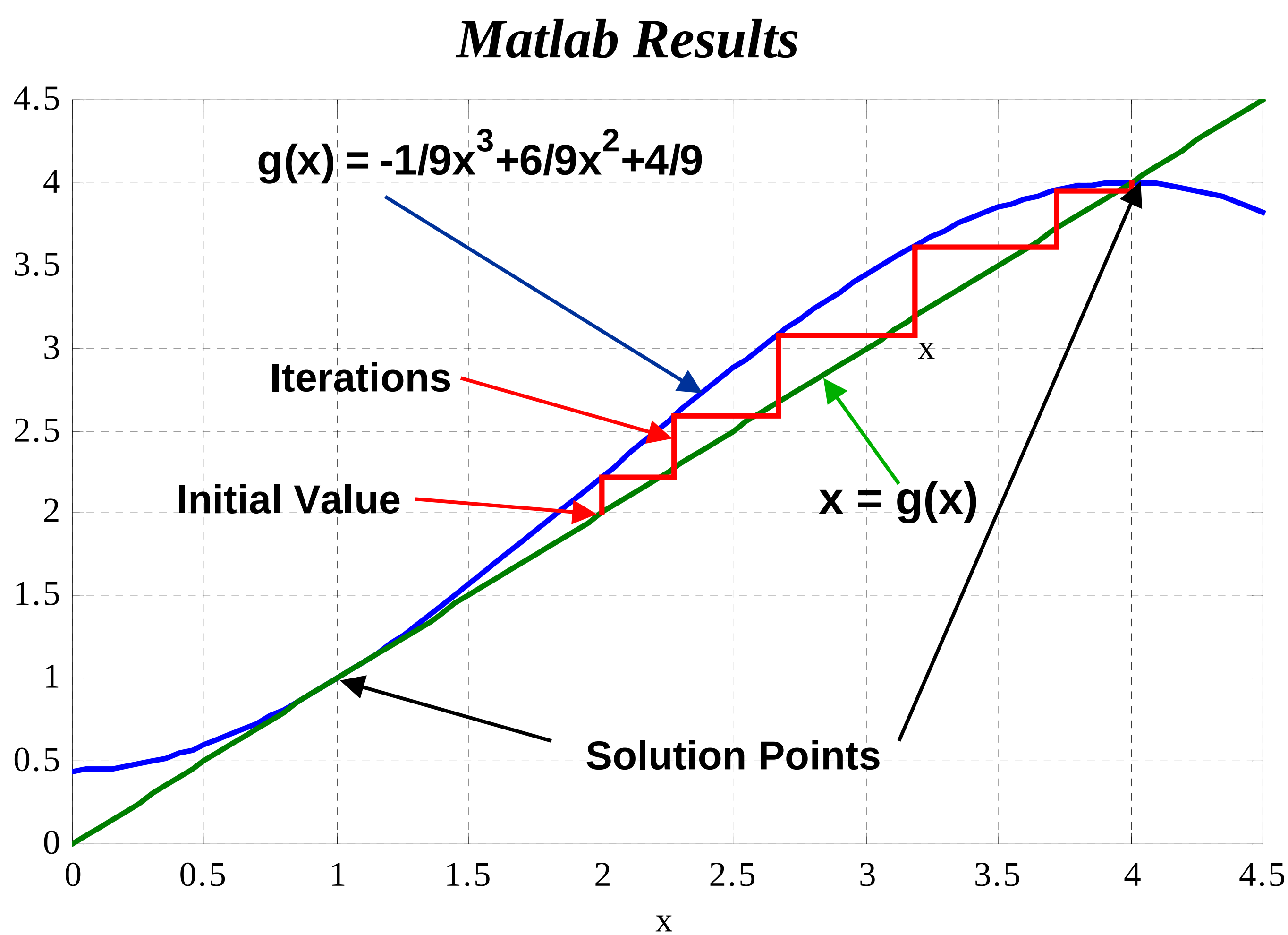
$$x^{[5]} = 3.7238$$

$$x^{[6]} = 4.0084$$

$$x^{[7]} = 3.9978$$

$$x^{[8]} = 4.0005$$

Gauss-Seidel Example



with acceleration factor: 1.25

Gauss-Seidel for a System of Equations

- **Consider a system of *n* equations**

$$
\begin{aligned}
f_1(x_1, x_2, \cdots, x_n) &= c_1 \\
f_2(x_1, x_2, \cdots, x_n) &= c_2 \\
\vdots \qquad\qquad &\quad \vdots \\
f_n(x_1, x_2, \cdots, x_n) &= c_n
\end{aligned}
$$

- **Rearrange each equation for one of the variables**

$$
\begin{aligned}
x_1 &= c_1 + g_1(x_1, x_2, \cdots, x_n) \\
x_2 &= c_2 + g_2(x_1, x_2, \cdots, x_n) \\
\vdots & \qquad\qquad \vdots \\
x_n &= c_n + g_n(x_1, x_2, \cdots, x_n)
\end{aligned}
$$

Gauss-Seidel for a System of Equations

- **steps**
  - assume an approximate solution for the independent variables

    $$\left(x_1^{[0]}, x_2^{[0]}, \cdots, x_n^{[0]}\right)$$

  - find the results in a new approximate solution

    $$\left(x_1^{[k+1]}, x_2^{[k+1]}, \cdots, x_n^{[k+1]}\right)$$

  - in the Gauss-Seidel method, the updated values of the variables calculated in the preceding equations are used immediately in the solution of the subsequent equations

The Power Flow Equation

- **The equation**

$$P_i + jQ_i = V_i I_i^* \quad \rightarrow \quad I_i = \frac{P_i - jQ_i}{V_i^*}$$

$$\frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j \qquad j \neq i$$

- **The Gauss-Siedel form**

$$V_i = \frac{\dfrac{P_i - jQ_i}{V_i^*} + \sum_{j=1}^{n} y_{ij} V_j}{\sum_{j=0}^{n} y_{ij}} \quad j \neq i \quad \Rightarrow \quad V_i^{[k+1]} = \frac{\dfrac{P_i - jQ_i}{V_i^{*[k]}} + \sum_{j=1}^{n} y_{ij} V_j^{[k]}}{\sum_{j=0}^{n} y_{ij}} \quad j \neq i$$

Power Injections

- **Rewriting the power equation to find P and Q**

$$P_i^{[k+1]} = \Re\left\{ V_i^{*[k]} \left[V_i^{[k]} \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j^{[k]} \right] \right\} \quad j \neq i$$

$$Q_i^{[k+1]} = -\Im\left\{ V_i^{*[k]} \left[V_i^{[k]} \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j^{[k]} \right] \right\} \quad j \neq i$$

  - the real and reactive powers are scheduled for the load buses that is, they remain fixed
  - the currents and powers are expressed as going into the bus
    - for generation the powers are positive
    - for loads the powers are negative
    - the scheduled power is the sum of the generation and load powers

Solution by Gauss-Seidel

- **The complete set of equations become:**

$$V_i^{[k+1]} = \frac{\dfrac{P_i^{[sch]} - jQ_i^{[sch]}}{V_i^{*[k]}} + \sum_{j=1}^{n} y_{ij} V_j^{[k]}}{\sum_{j=0}^{n} y_{ij}} \qquad j \neq i$$

$$P_i^{[k+1]} = \Re\left\{ V_i^{*[k]} \left[V_i^{[k]} \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j^{[k]} \right] \right\} \qquad j \neq i$$

$$Q_i^{[k+1]} = -\Im\left\{ V_i^{*[k]} \left[V_i^{[k]} \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j^{[k]} \right] \right\} \qquad j \neq i$$

Solution by Gauss-Seidel

- **Rewriting the equations in terms of the Y-Bus**

$$V_i^{[k+1]} = \frac{\dfrac{P_i^{[sch]} - jQ_i^{[sch]}}{V_i^{*[k]}} - \sum_{j=1,\, j\neq i}^{n} Y_{ij} V_j^{[k]}}{Y_{ii}}$$

$$P_i^{[k+1]} = \Re\left\{ V_i^{*[k]} \left[V_i^{[k]} Y_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{n} Y_{ij} V_j^{[k]} \right] \right\}$$

$$Q_i^{[k+1]} = -\Im\left\{ V_i^{*[k]} \left[V_i^{[k]} Y_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{n} Y_{ij} V_j^{[k]} \right] \right\}$$

Solution by Gauss-Seidel

- **System characteristics**
  - Since both components (V & δ) are specified for the slack bus, there are $2(n - 1)$ equations which must be solved iteratively
  - For the load buses, the real and reactive powers are known: scheduled
    - the voltage magnitude and angle must be estimated
    - in per unit, the nominal voltage magnitude is 1 pu
    - the angles are generally close together, so an initial value of 0 degrees is appropriate

Solution by Gauss-Seidel

- For the generator buses, the real power and voltage magnitude are known
  - the real power is scheduled
  - the reactive power is computed based on the estimated voltage values
  - the voltage is computed by Gauss-Seidel, only the imaginary part is kept
  - the complex voltage is found from the magnitude and the iterative imaginary part

$$e_i^{[k+1]} = \sqrt{\left|V_i^{[sch]}\right|^2 - \left(f_i^{[k+1]}\right)^2} \qquad V_i = e_i + j\, f_i$$

Example

- **Using the Gauss-Seidel method, determine the phasor values of the voltage at the load buses 2 and 3, accurate to 2 decimal places**