Fault Analysis

- **Analysis types**
  - power flow - evaluate normal operating conditions
  - fault analysis - evaluate abnormal operating conditions
- **Fault types:**
  - balanced faults
    - three-phase
  - unbalanced faults
    - single-line to ground and double-line to ground
    - line-to-line faults
- **Results used for:**
  - specifying ratings for circuit breakers and fuses
  - protective relay settings
  - specifying the impedance of transformers and generators

Fault Analysis

- **Magnitude of fault currents depend on:**
  - the impedance of the network
  - the internal impedances of the generators
  - the resistance of the fault (arc resistance)
- **Network impedances are governed by**
  - transmission line impedances
  - transformer connections and impedances
  - grounding connections and resistances
- **Generator behavior is divided into three periods**
  - sub-transient period, lasting for the first few cycles
  - transient period, covering a relatively longer time
  - steady state period

Fault Studies

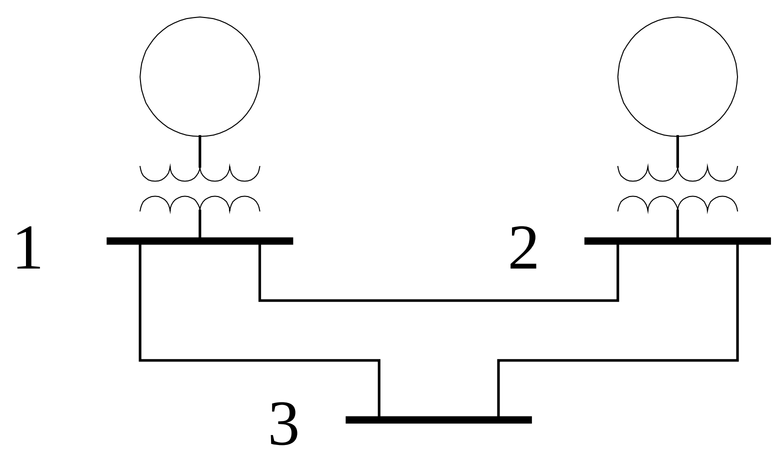
- **Sub-transient period, $X_G = X_d''$**
  - determine the interrupting capacity of HV circuit breakers
  - determine the operation timing of the protective relay system for high-voltage networks
- **Transient period, $X_G = X_d'$**
  - determine the interrupting capacity of MV circuit breakers
  - determine the operation timing of the protective relay system for medium-voltage networks
  - transient stability studies

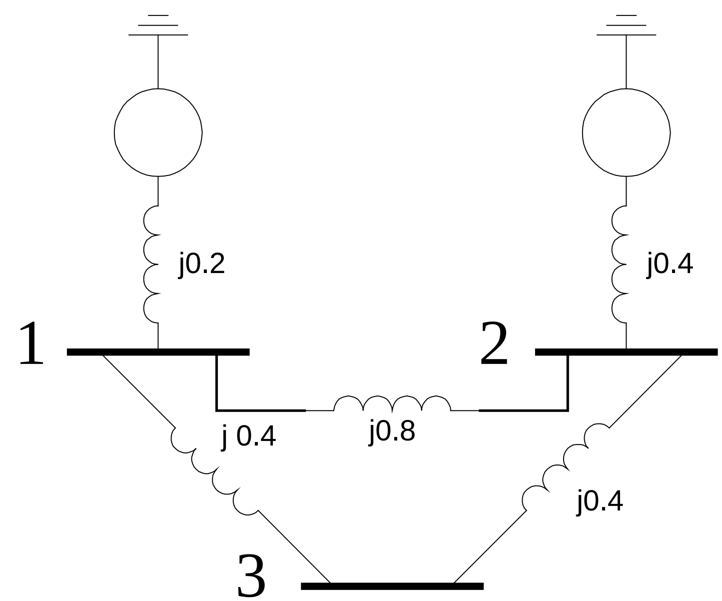
Fault Representation

- **A fault represents a structural network change**
  - equivalent to the addition of an impedance at the place of the fault
  - if the fault impedance is zero, the fault is referred to as a bolted fault or solid fault
- **First order method**
  - the faulted network can be solved conveniently by Thévenin’s method
  - network resistances are neglected
  - generators are modeled as an emf behind the sub-transient or transient reactance
  - shunt capacitances are neglected
  - system is considered as having no-load

Thévenin’s Method

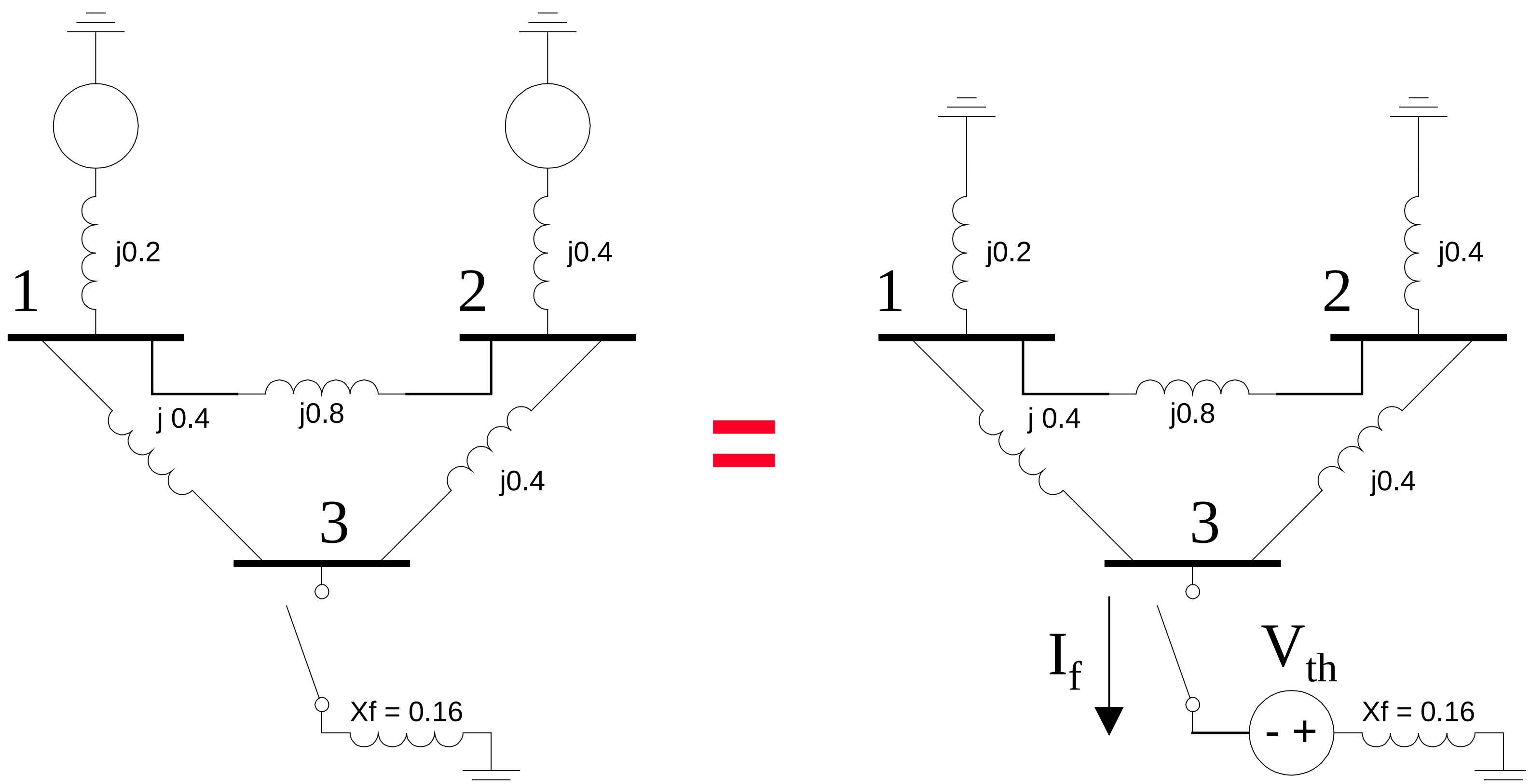
- The fault is simulated by switching a fault impedance at the faulted bus
- The change in the network voltages is equivalent to adding the prefault bus voltage with all other sources short curcuited



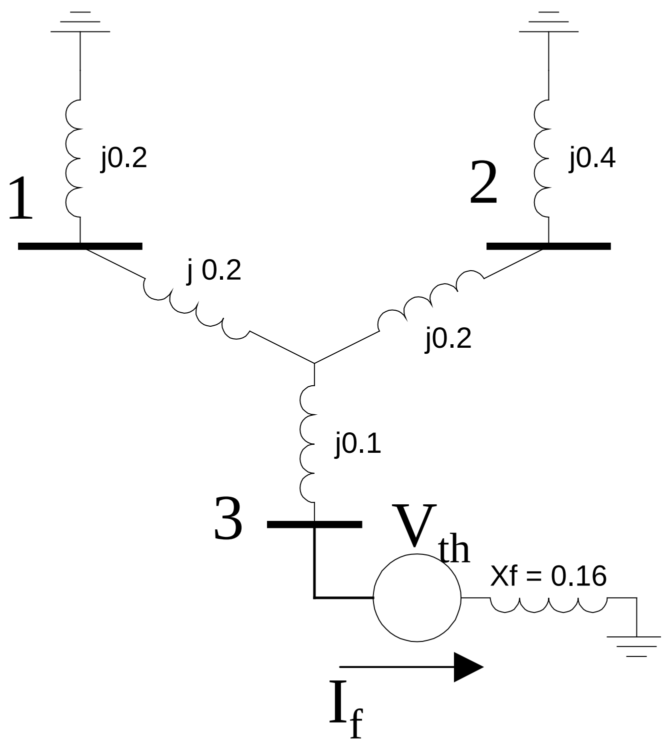


Thévenin's Method

- **3-phase fault with $Z_f = j0.16$ on bus 3**



Thévenin's Method



$$I_3^{[f]} = \frac{V_3^{[0]}}{Z_{33} + Z_f}$$

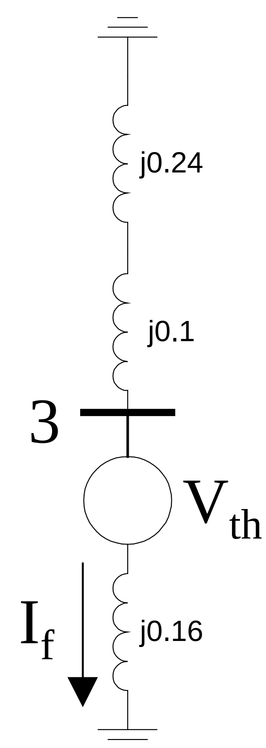
$$V_1^{[0]} = V_2^{[0]} = V_3^{[0]} = 1.0$$

$$Z_{1s} = Z_{2s} = \frac{(j0.4)(j0.8)}{(j1.6)} = j0.2$$

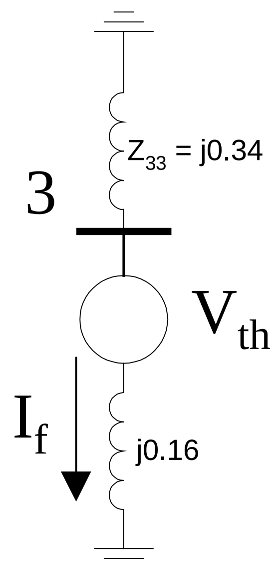
$$Z_{3s} = \frac{(j0.4)(j0.4)}{(j1.6)} = j0.1$$

$$Z_{33} = \frac{(j0.4)(j0.6)}{j0.4 + j0.6} + j0.1$$

$$Z_{33} = j0.34$$



Thévenin’s Method



$$Z_{33} = j0.34$$

$$I_3^{[f]} = \frac{V_3^{[0]}}{Z_{33} + Z_f} = \frac{1.0}{j0.34 + j0.16} = -j2.0$$

Thévenin’s Method

- **For more accurate solutions**
  - use the pre-fault bus voltages which can be obtained from the results of a power flow solution
  - include loads - to preserve linearity, convert loads to constant impedance model
  - Thevenin’s theorem allows the changes in the bus voltages to be obtained
  - bus voltages are obtained by superposition of the pre-fault voltages and the changes in the bus voltages
  - current in each branch can be solved

Short Circuit Capacity (SCC)

- **Measures the electrical strength of the bus**
- **Stated in MVA**
- **Determines the dimension of bus bars and the interrupting capacity of circuit breakers**
- **Definition:**

$$SCC = \sqrt{3}\, V_{L-L,k}^{[\text{pre-f}]}\ I_k^{[\text{f}]}$$

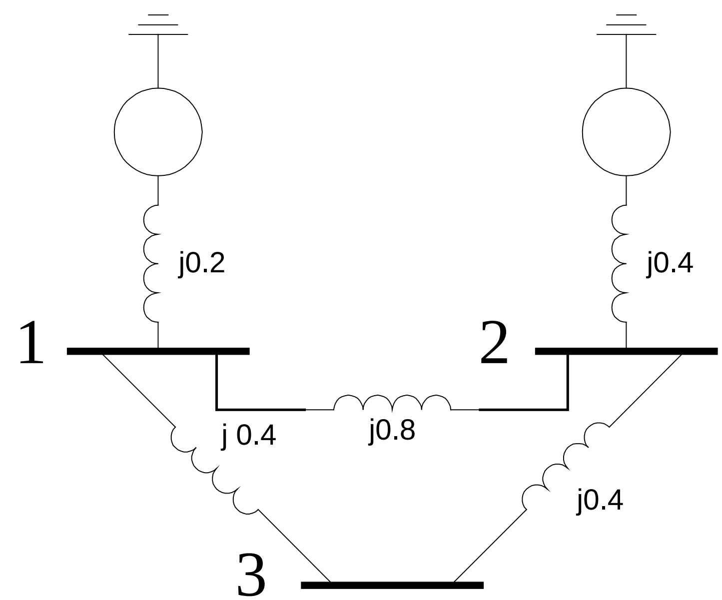
  - in per unit:

$$I_k^{[\text{f}]} = \frac{V_k^{[\text{pre-f}]}}{X_{kk}}$$

$$SCC = \frac{S_B}{X_{kk}}$$

Short Circuit Capacity (SCC)

- **Find the SCC for bus #3**



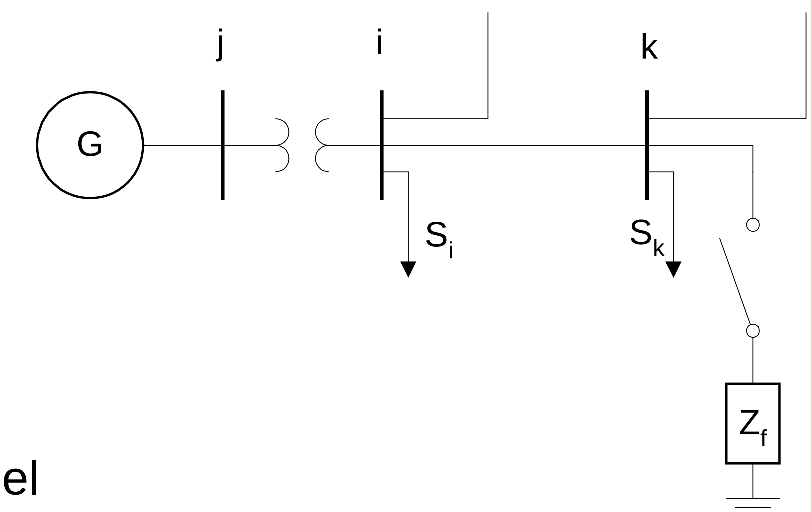
$$Z_{33} = j0.34$$

$$S_{base} = 100\text{ MVA}$$

$$SCC_3 = \frac{S_{base}}{|Z_{33}|} = \frac{100\text{ MVA}}{0.34} = 294\text{ MVA}$$

Fault Analysis Using Impedance Matrix

- **Network reduction by Thévenin's method is not efficient**
  - difficult to apply to large networks
- **Matrix algebra formation**
  - seek a matrix where the diagonal elements represent the source impedance for the buses
  - consider the following system
    - operating under balanced conditions
    - each generator represented by a constant emf behind a proper reactance (X_d, X_d', or X_d'')
    - lines represented by their equivalent π model



Fault Analysis Using Impedance Matrix

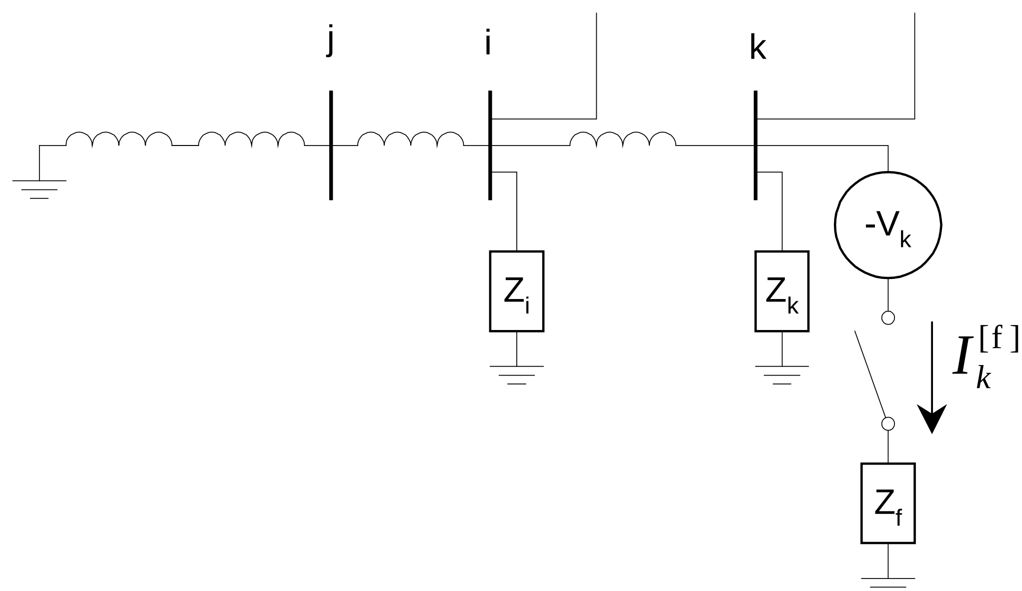
- **Place the prefault voltages into a vector**

$$\mathbf{V}_{bus}^{[\text{pre-f}]} = \begin{bmatrix} V_1^{[\text{pre-f}]} \\ \vdots \\ V_k^{[\text{pre-f}]} \\ \vdots \\ V_n^{[\text{pre-f}]} \end{bmatrix}$$

- **Replace the loads by a constant impedance model using the prefault bus voltages**

$$Z_{i-load} = \frac{\left|V_i^{[\text{pre-f}]}\right|^2}{S_{i-load}^*}$$

- **The change in the network voltage caused by the fault is equivalent to placing a fault voltage at the faulted bus with all the other sources short-circuited**



Fault Analysis Using Impedance Matrix

- Using superpositioning, the fault voltages are calculated from the prefault voltages by adding the change in bus voltages due to the fault

$$\mathbf{V}_{bus}^{[\mathrm{f}]} = \mathbf{V}_{bus}^{[\mathrm{pre-f}]} + \Delta\mathbf{V}_{bus}$$

$$\Delta\mathbf{V}_{bus} = \begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_n \end{bmatrix}$$

- The change in bus voltages can be calculated from the network matrix

$$\mathbf{I}_{bus} = \mathbf{Y}_{bus} \; \mathbf{V}_{bus}$$

$$\mathbf{I}_{bus}^{[\mathrm{Fault}]} = \mathbf{Y}_{bus} \; \Delta\mathbf{V}_{bus}$$

$$\mathbf{I}_{bus}^{[\mathrm{Fault}]} = \begin{bmatrix} 0 \\ \vdots \\ -I_k^{[\mathrm{Fault}]} \\ \vdots \\ 0 \end{bmatrix}$$

Fault Analysis Using Impedance Matrix

$$\mathbf{I}_{bus}^{[\text{Fault}]} = \mathbf{Y}_{bus}\ \Delta\mathbf{V}_{bus}$$

$$\begin{bmatrix} 0 \\ \vdots \\ -I_k^{[f]} \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} y_{11} & \cdots & y_{1k} & \cdots & y_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ y_{k1} & \cdots & y_{kk} & \cdots & y_{kn} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ y_{n1} & \cdots & y_{nk} & \cdots & y_{nn} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_n \end{bmatrix}$$

$$\Delta\mathbf{V}_{bus} = \mathbf{Y}_{bus}^{-1}\ \mathbf{I}_{bus}^{[\text{Fault}]}$$

$$\mathbf{Z}_{bus} = \mathbf{Y}_{bus}^{-1}$$

$$\Delta\mathbf{V}_{bus} = \mathbf{Z}_{bus}\ \mathbf{I}_{bus}^{[\text{Fault}]}$$

Fault Analysis Using Impedance Matrix

$$\begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_n \end{bmatrix} = \begin{bmatrix} z_{11} & \cdots & z_{1k} & \cdots & z_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ z_{k1} & \cdots & z_{kk} & \cdots & z_{kn} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ z_{n1} & \cdots & z_{nk} & \cdots & z_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ -I_k^{[f]} \\ \vdots \\ 0 \end{bmatrix}$$

$$\mathbf{V}_{bus}^{[\mathrm{f}]} = \mathbf{V}_{bus}^{[\mathrm{pre-f}]} + \Delta\mathbf{V}_{bus}$$

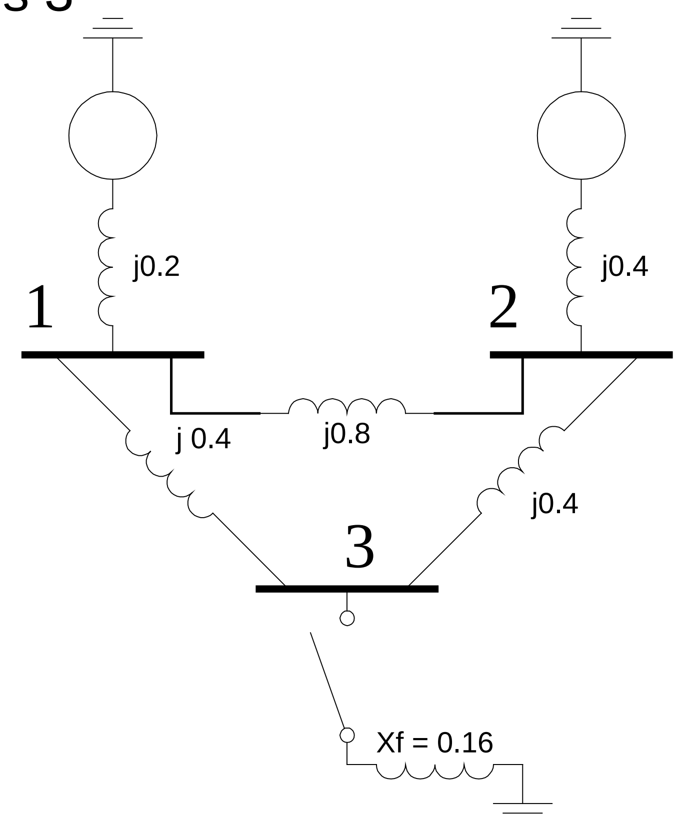
$$\mathbf{V}_{bus}^{[\mathrm{f}]} = \mathbf{V}_{bus}^{[\mathrm{pre-f}]} + \mathbf{Z}_{bus}\ \mathbf{I}_{bus}^{[\mathrm{f}]}$$

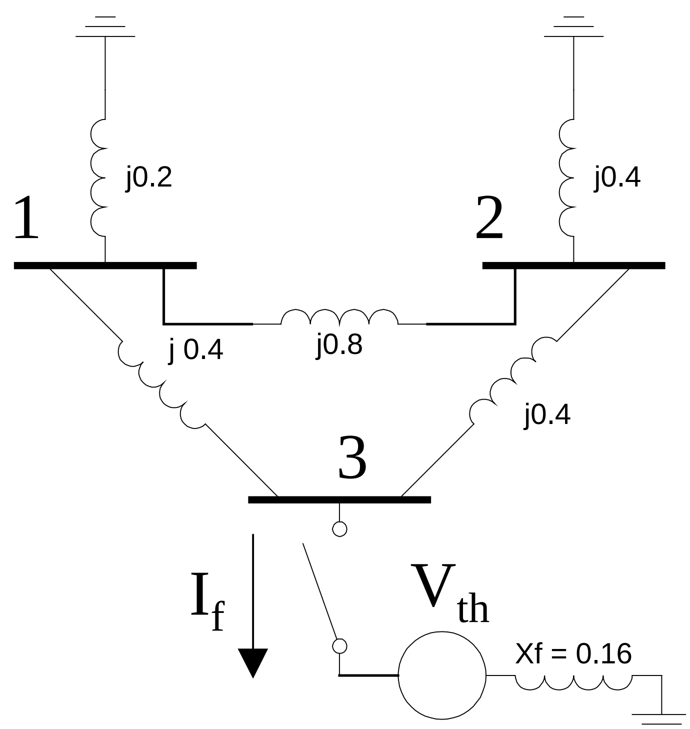
$$V_k^{[\mathrm{f}]} = V_k^{[\mathrm{pre-f}]} + Z_{kk}\ I_k^{[\mathrm{f}]} \quad \& \quad V_k^{[\mathrm{f}]} = Z_f\ I_k^{[\mathrm{f}]}$$

$$I_k^{[\mathrm{f}]} = \frac{V_k^{[\mathrm{pre-f}]}}{Z_{kk} + Z_f}$$

Example

- **3-phase fault with Z_f = j0.16 on**
  - bus 3





Example

$$Y_{bus} = \begin{bmatrix} -j8.75 & j1.25 & j2.50 \\ j1.25 & -j6.25 & j2.50 \\ j2.50 & j2.50 & -j5.00 \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.16 & j0.08 & j0.12 \\ j0.08 & j0.24 & j0.16 \\ j0.12 & j0.16 & j0.34 \end{bmatrix}$$

$$I_3^{[f]} = \frac{V_3^{[pre-f]}}{Z_{33} + Z_f} = \frac{1.0\,pu}{j0.34 + j0.16} = -j2.0\,pu$$

Example

$$V_1^{[f]} = V_1^{[pre-f]} - Z_{13} I_3^{[f]} = 1.0\,pu - (j0.12)(-j2.0) = 0.76\,pu$$

$$V_2^{[f]} = V_2^{[pre-f]} - Z_{23} I_3^{[f]} = 1.0\,pu - (j0.16)(-j2.0) = 0.68\,pu$$

$$V_3^{[f]} = V_3^{[pre-f]} - Z_{33} I_3^{[f]} = 1.0\,pu - (j0.34)(-j2.0) = 0.32\,pu$$