Steady State Stability

- **The ability of the power system to remain in synchronism when subject to small disturbances**
- **Stability is assured if the system returns to its original operating state (voltage magnitude and angle profile)**
- **The behavior can be determined with a linear system model**
- **Assumption:**
  - the automatic controls are not active
  - the power shift is not large
  - the voltage angles changes are small

Steady State Stability

- **Swing Equation**

$$\frac{H_i}{\pi f_0}\frac{d^2\delta_i}{dt^2} = P_{mi} - P_{max}\sin\delta$$

- **Small disturbance modeling**

$\delta = \delta_0 + \Delta\delta$ Consider a small deviation

$$\frac{H}{\pi f_0}\frac{d^2(\delta_0 + \Delta\delta)}{dt^2} = P_m - P_{max}\sin(\delta_0 + \Delta\delta)$$

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2} + \frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} = P_m - P_{max}\left[\sin\delta_0\cos\Delta\delta + \cos\delta_0\sin\Delta\delta\right]$$

Steady State Stability

- **Simplification of the swing equation**

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2}+\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2}=P_m-P_{max}\left[\sin\delta_0\cos\Delta\delta+\cos\delta_0\sin\Delta\delta\right]$$

Substitute the following approximations

$$\Delta\delta << \delta \qquad \cos\Delta\delta\approx 1 \qquad \sin\Delta\delta\approx\Delta\delta$$

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2}+\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2}=P_m-P_{max}\sin\delta_0-P_{max}\cos\delta_0\cdot\Delta\delta$$

Group steady state and transient terms

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2}-P_m+P_{max}\sin\delta_0=-\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2}-P_{max}\cos\delta_0\cdot\Delta\delta$$

Steady State Stability

- **Simplification of the swing equation**

$$\frac{H}{\pi f_0}\frac{d^2\delta_0}{dt^2} - P_m + P_{max}\sin\delta_0 = -\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} - P_{max}\cos\delta_0\cdot\Delta\delta$$

$$0 = \frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} + P_{max}\cos\delta_0\cdot\Delta\delta$$

Steady state term is equal to zero

$$\left.\frac{dP_e}{d\delta}\right|_{\delta_0} = \left.\frac{d}{d\delta}P_{max}\sin\delta\right|_{\delta_0} = P_{max}\cos\delta_0 = P_s$$

$$\frac{H}{\pi f_0}\frac{d^2\Delta\delta}{dt^2} + P_s\cdot\Delta\delta = 0$$

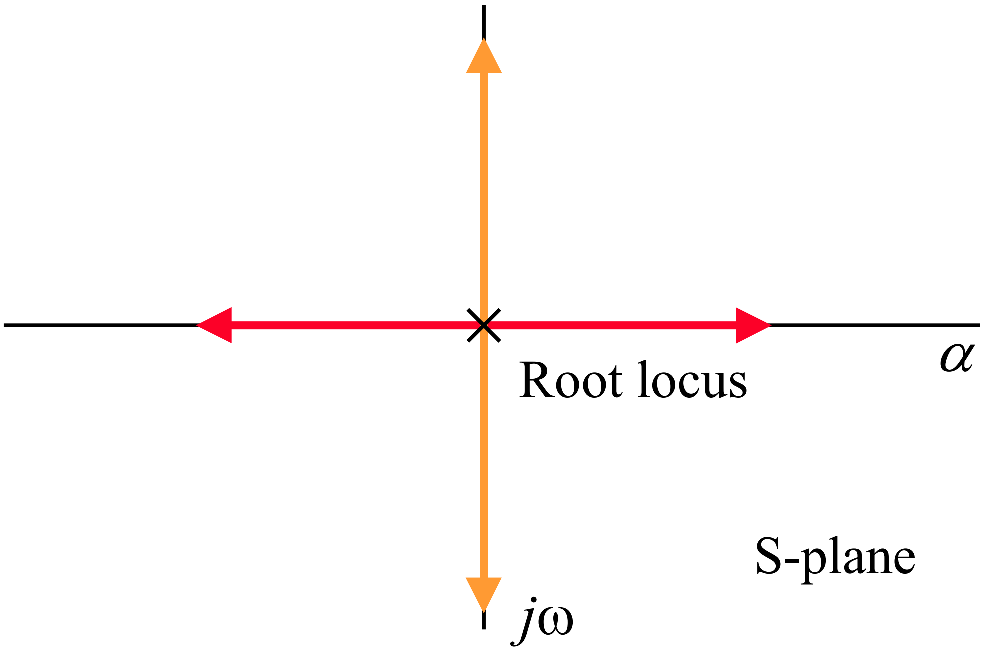
Second order equation.
The solution depends on the roots of the characteristic equation

Stability

- **Stability Assessment**
  - When P_s is negative, one root is in the right-half s-plane, and the response is exponentially increasing and stability is lost
  - When P_s is positive, both roots are on the $j\omega$ axis, and the motion is oscillatory and undamped, the natural frequency is:

$$s^2 = -\frac{\pi f_0}{H} P_S$$

$$\omega_n = \sqrt{\frac{\pi f_0}{H} P_S}$$



Damping Torque

$$P_D = D\frac{d\delta}{dt}$$

Damping force is due to air-gap interaction

$$\frac{H}{\pi\ f_0}\frac{d^2\Delta\delta}{dt^2} + D\frac{d\Delta\delta}{dt} + P_S\Delta\delta = 0$$

$$\frac{d^2\Delta\delta}{dt^2} + \frac{\pi\ f_0}{H}D\frac{d\Delta\delta}{dt} + \frac{\pi\ f_0}{H}P_S\Delta\delta = 0$$

$$\frac{d^2\Delta\delta}{dt^2} + 2\zeta\,\omega_n\frac{d\Delta\delta}{dt} + \omega_n^2\Delta\delta = 0$$

$$\zeta = \frac{D}{2}\sqrt{\frac{\pi\ f_0}{H\,P_S}}$$

Characteristic Equation

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$\zeta = \frac{D}{2}\sqrt{\frac{\pi f_0}{H P_S}} < 1$$ for normal operation conditions

$$s_1, s_2 = -\zeta\omega_n \pm j\,\omega_n\sqrt{1-\zeta^2}$$ complex roots

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$ the damped frequency of oscillation

Laplace Transform Analysis

$$x_1 = \Delta\delta, \quad x_2 = \frac{d\Delta\delta}{dt}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

$$\mathcal{L}\left\{\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}\right\} \rightarrow s\mathbf{X}(s) - \mathbf{x}(0) = \mathbf{A}\mathbf{X}(s)$$

$$\mathbf{X}(s) = \left(s\mathbf{I} - \mathbf{A}\right)^{-1} \mathbf{x}(0)$$

$$\left(s\mathbf{I} - \mathbf{A}\right) = \begin{bmatrix} s & -1 \\ \omega_n^2 & s + 2\zeta\omega_n \end{bmatrix}$$

$$\mathbf{X}(s) = \frac{\begin{bmatrix} s + 2\zeta\omega_n & 1 \\ -\omega_n^2 & s \end{bmatrix}}{s^2 + 2\zeta\omega_n s + \omega_n^2} \mathbf{x}(0)$$

Laplace Transform Analysis

$$\Delta\delta(s) = \frac{(s + 2\zeta\omega_n)\Delta\delta_0}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\Delta\omega(s) = \frac{\omega_n^2 \Delta\delta_0}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\Delta\delta(t) = \frac{\Delta\delta_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t + \theta), \quad \theta = \cos^{-1}\zeta$$

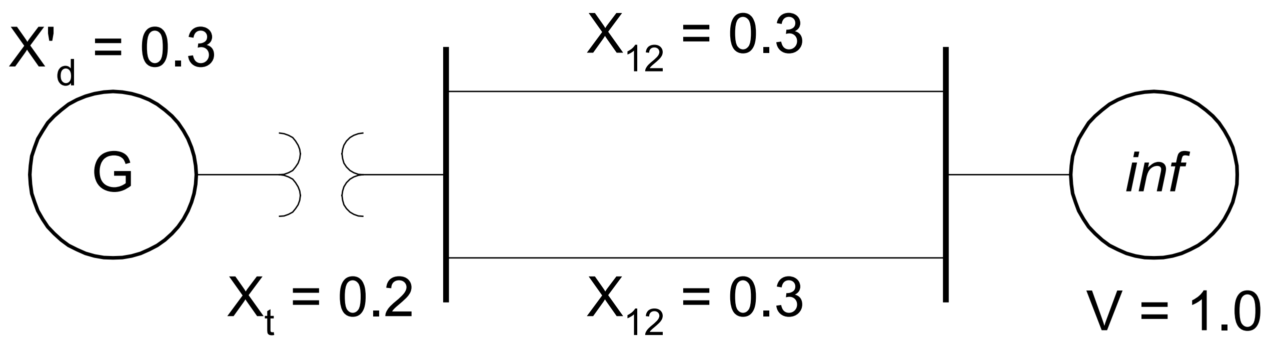
$$\Delta\omega(t) = -\frac{\omega_n \Delta\delta_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_d t)$$

$$\delta(t) = \delta_0 + \Delta\delta(t), \quad \omega(t) = \omega_0 + \Delta\omega(t)$$

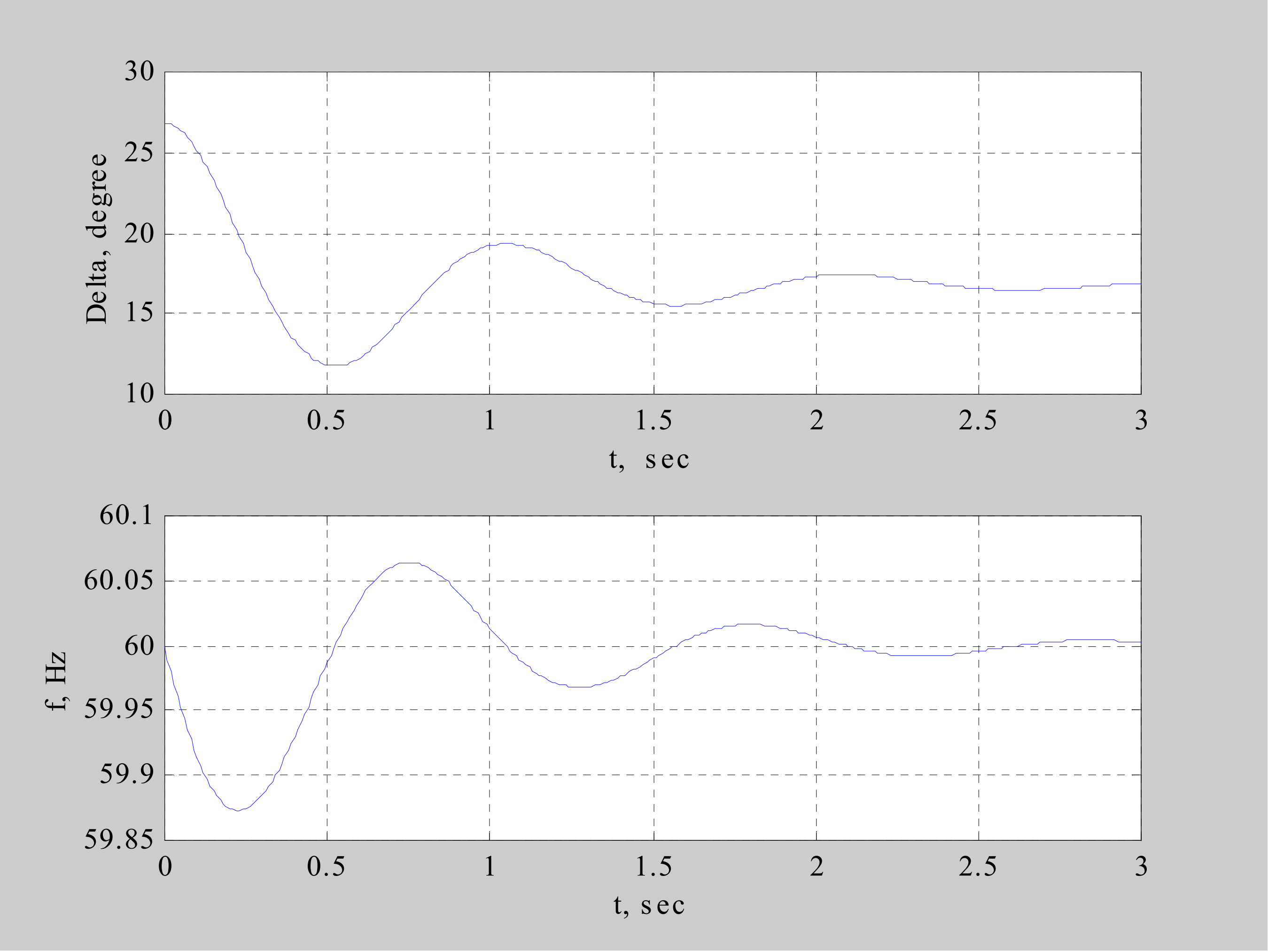
Example

- A 60 Hz synchronous generator having inertia constant H = 9.94 MJ/MVA and a transient reactance $X'_d = 0.3$ pu is connected to an infinite bus through the following network. The generator is delivering 0.6 *pu* real power at 0.8 power factor lagging to the infinite bus at a voltage of 1 *pu*. Assume the damping power coefficient is *D* = 0.138 *pu*. Consider a small disturbance of 10° or 0.1745 radians. Obtain equations of rotor angle and generator frequency motion.

Example



Example


30
25
20
15
10
Delta, degree
0
0.5
1
1.5
2
2.5
3
t, sec
60.1
60.05
60
59.95
59.9
59.85
f, Hz
0
0.5
1
1.5
2
2.5
3
t, sec