Transient Stability

- **The ability of the power system to remain in synchronism when subject to large disturbances**
  - Large power and voltage angle oscillations do not permit linearization of the generator swing equations
- **Lyapunov energy functions**
  - simplified energy method: the Equal Area Criterion
- **Time-domain methods**
  - numerical integration of the swing equations
  - Runga-Kutta numerical integration techniques

Equal Area Criterion

- **Quickly predicts the stability after a major disturbance**
  - graphical interpretation of the energy stored in the rotating masses
  - method only applicable to a few special cases:
    - one machine connected to an infinite bus
    - two machines connected together
- **Method provides physical insight to the dynamic behavior of machines**
  - relates the power angle with the acceleration power

Equal Area Criterion

- **For a synchronous machine connected to an infinite bus**

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e = P_{accel}$$

$$\frac{d^2\delta}{dt^2} = \frac{\pi f_0}{H}\left(P_m - P_e\right) = \frac{\pi f_0}{H}\cdot P_{accel}$$

- **The energy form of the swing equation is obtained by multiplying both sides by the system frequency (shaft rotational speed)**

$$\left(2\frac{d\delta}{dt}\right)\left(\frac{d^2\delta}{dt^2}\right) = \frac{\pi f_0}{H}\left(P_m - P_e\right)\left(2\frac{d\delta}{dt}\right)$$

Equal Area Criterion

$$2\left(\frac{d^2\delta}{dt^2}\right)\left(\frac{d\delta}{dt}\right)=\frac{\pi\ f_0}{H}\left(P_m-P_e\right)\left(2\frac{d\delta}{dt}\right)$$

- **The left hand side can be reworked as the derivative of the square of the system frequency (shaft speed)**

$$\frac{d}{dt}\left[\left(\frac{d\delta}{dt}\right)^2\right]=\frac{2\pi\ f_0}{H}\left(P_m-P_e\right)\frac{d\delta}{dt}$$

$$d\left[\left(\frac{d\delta}{dt}\right)^2\right]=\frac{2\pi\ f_0}{H}\left(P_m-P_e\right)d\delta$$

Equal Area Criterion

- **Integrating both sides with respect to time,**

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2\pi\ f_0}{H}\int_{\delta_0}^{\delta}\left(P_m - P_e\right)d\delta$$

$$\frac{d\delta}{dt} = \sqrt{\frac{2\pi\ f_0}{H}\int_{\delta_0}^{\delta}\left(P_m - P_e\right)d\delta}$$

- **The equation gives the relative speed of the machine. For stability, the speed must go to zero over time**

$$\frac{d\delta}{dt} = 0\Big|_{t\to\infty}$$

$$0 = \int_{\delta_0}^{\delta}\left(P_m - P_e\right)d\delta$$

Equal Area Criterion

- **Consider a machine operating at equilibrium**
  - the power angle, $\delta = \delta_0$
  - the electrical load, $P_{e0} = P_{m0}$
- **Consider a sudden increase in the mechanical power input**
  - $P_{m1} > P_{e0}$; the acceleration power is positive
  - excess energy is stored in the rotor and the power frequency increases, driving the relative power angle larger over time

$$U_{Potential} = \int_{\delta_0}^{\delta_1} (P_{m1} - P_e) d\delta > 0$$

$$\frac{d\delta}{dt} = \omega = \sqrt{\frac{2\pi f_0}{H} \int_{\delta_0}^{\delta} (P_m - P_e) d\delta} > 0$$

Equal Area Criterion

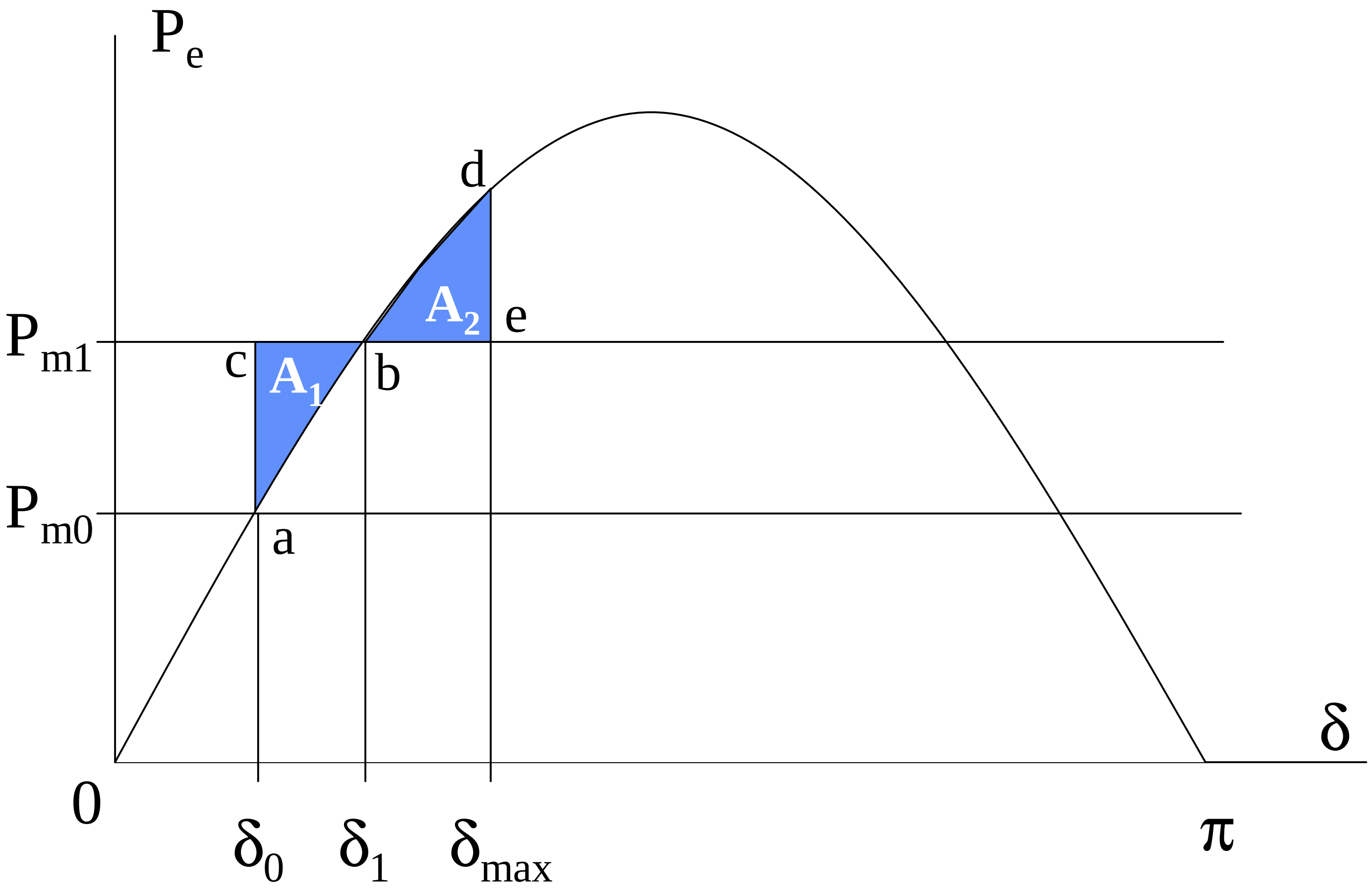
- with increase in the power angle, δ, the electrical power increases

$$P_e = P_{\max} \sin \delta$$

- when $\delta = \delta_1$, the electrical power equals the mechanical power, P_{m1}
- acceleration power is zero, but the rotor is running above synchronous speed, hence the power angle, δ, continues to increase
- now $P_{m1} < P_e$; the acceleration power is negative (deceleration), causing the rotor to decelerate to synchronous speed at $\delta = \delta_{max}$
- an equal amount of energy must be given up by the rotating masses

$$U_{Potential} = \int_{\delta_0}^{\delta_1} \left(P_{m1} - P_e\right) d\delta - \int_{\delta_1}^{\delta_{\max}} \left(P_{m1} - P_e\right) d\delta = 0$$

Equal Area Criterion



Equal Area Criterion

- **The result is that the rotor swings to a maximum angle**
  - at which point the acceleration energy area and the deceleration energy area are equal

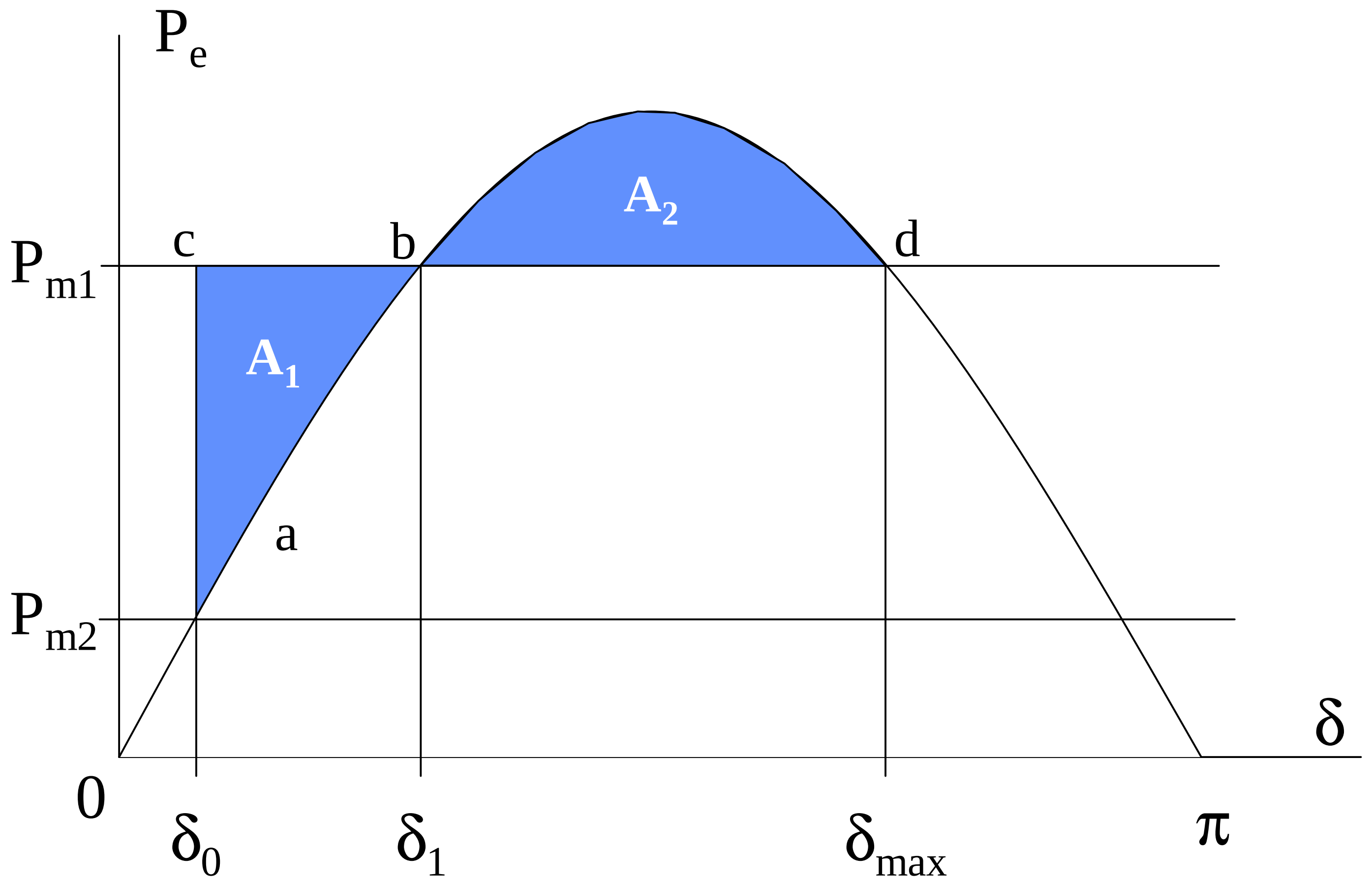
$$\int_{\delta_0}^{\delta_1} \left(P_{m1} - P_e\right) d\delta = \textbf{area}\, abc = \textbf{area}\, A_1$$

$$\int_{\delta_1}^{\delta_{\max}} \left(P_{m1} - P_e\right) d\delta = \textbf{area}\, bde = \textbf{area}\, A_2$$

$$\left|\textbf{area}\, A_1\right| = \left|\textbf{area}\, A_2\right|$$

  - this is known as the equal area criterion
  - the rotor angle will oscillate back and forth between δ and δ_{max} at its natural frequency

Equal Area Criterion - ΔP mechanical



Equal Area Criterion - ΔP mechanical

$$P_{m1}(\delta_1-\delta_0)-\int_{\delta_0}^{\delta_1}P_{\max}\sin\delta\,d\delta=\int_{\delta_1}^{\delta_{\max}}P_{\max}\sin\delta\,d\delta-P_{m1}(\delta_{\max}-\delta_1)$$

$$P_{m1}(\delta_{\max}-\delta_0)=P_{\max}(\cos\delta_0-\cos\delta_{\max})$$

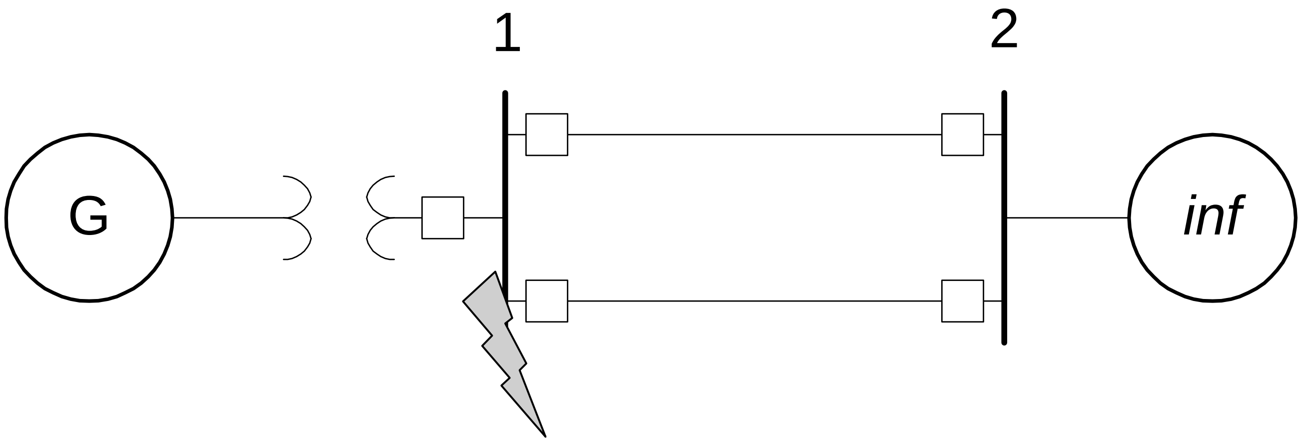
$$P_{m1}=P_{\max}\sin\delta_{\max}$$

$$(\delta_{\max}-\delta_0)\sin\delta_{\max}=\cos\delta_0-\cos\delta_{\max}$$

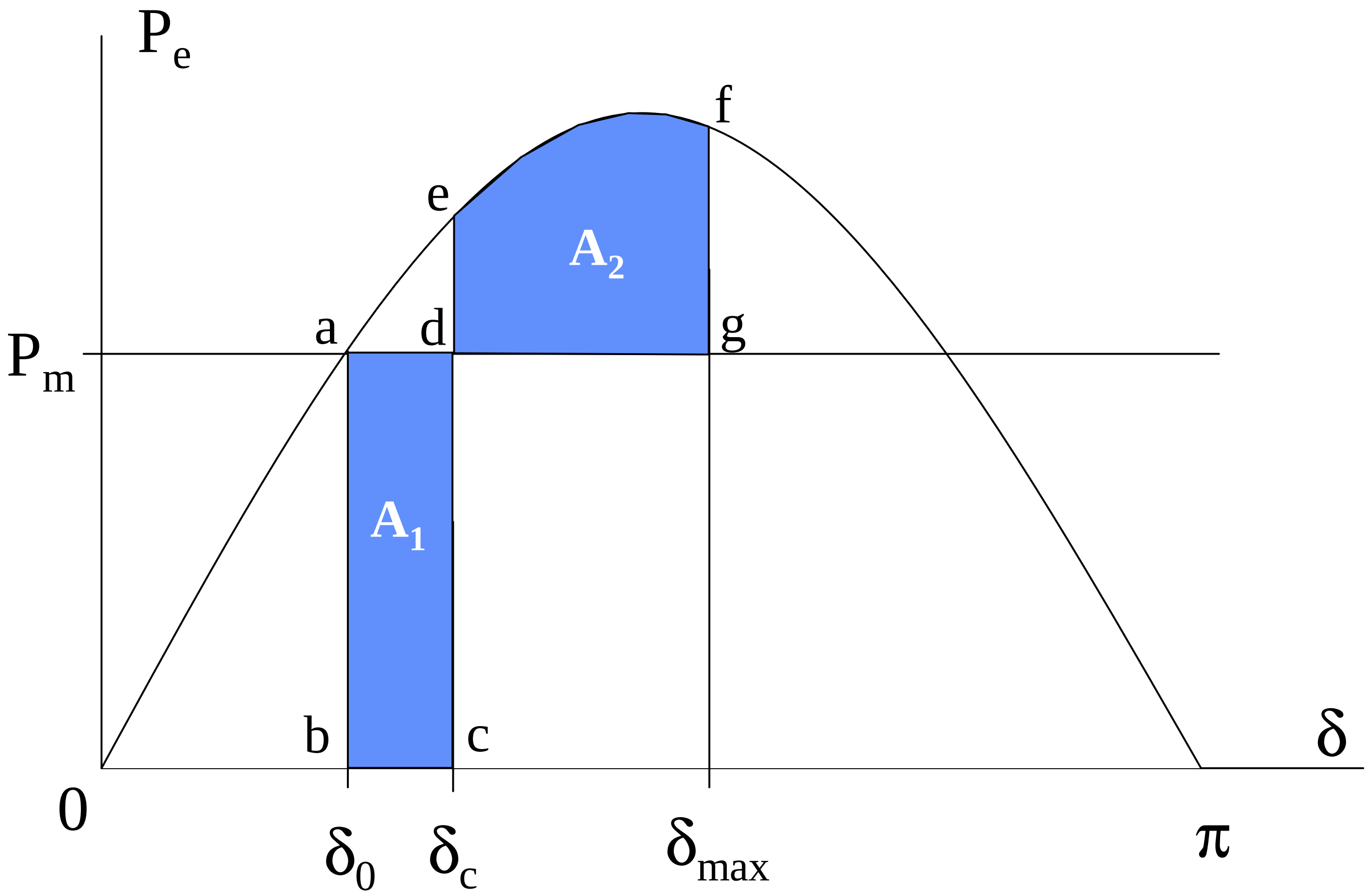
$$\rightarrow P_{m1}=P_{\max}\sin\delta_1$$

Function is nonlinear in δ_{max}
Solve using Newton-Raphson

3-Phase Fault



Equal Area Criterion - 3 phase fault



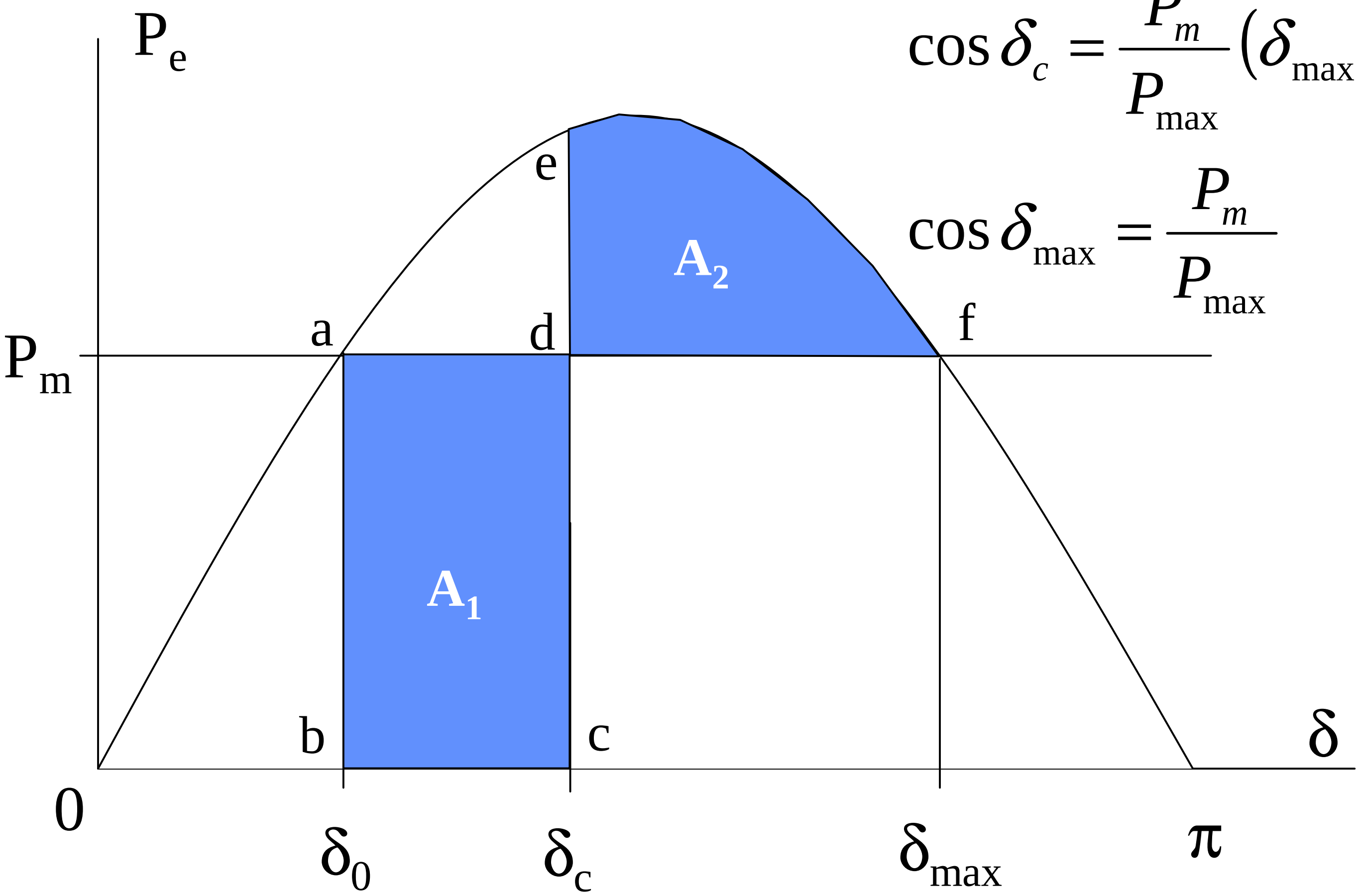
Equal Area Criterion - 3 phase fault

$$\int_{\delta_0}^{\delta_c} P_m \, d\delta = \int_{\delta_c}^{\delta_{\max}} \left(P_{\max} \sin\delta - P_m \right) d\delta$$

$$P_m \left(\delta_c - \delta_0 \right) = P_{\max} \left(\cos\delta_c - \cos\delta_{\max} \right) - P_m \left(\delta_{\max} - \delta_c \right)$$

$$\cos\delta_c = \frac{P_m}{P_{\max}} \left(\delta_{\max} - \delta_c \right) + \cos\delta_{\max}$$

Critical Clearing Time



$$\cos\delta_c = \frac{P_m}{P_{max}}(\delta_{max} - \delta_c) + \cos\delta_{max}$$

$$\cos\delta_{max} = \frac{P_m}{P_{max}}$$

Critical Clearing Time

$$\frac{H}{\pi\ f_0}\frac{d^2\delta}{dt^2} = P_m - P_e = P_m \leftarrow P_e = 0$$

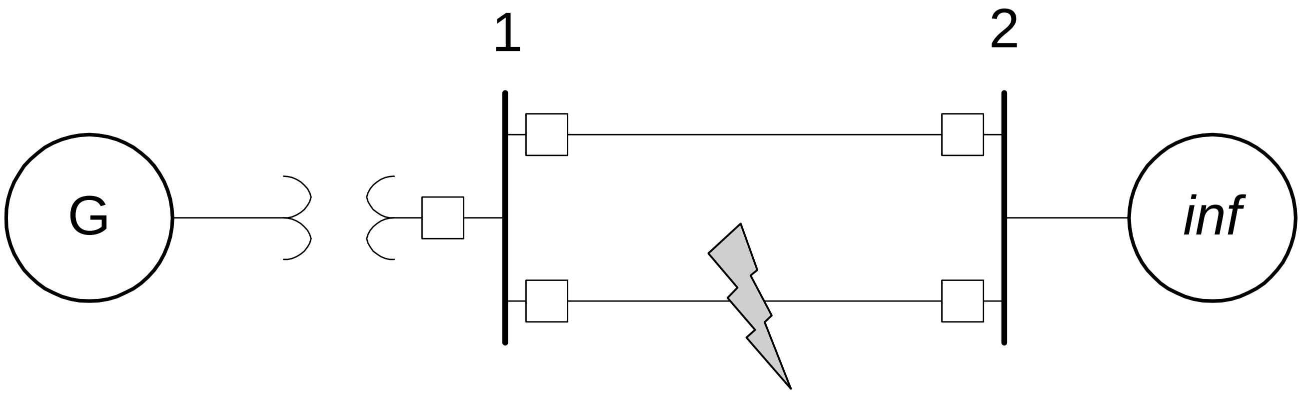
$$\frac{d^2\delta}{dt^2} = \frac{\pi\ f_0}{H} P_m$$

$$\frac{d\delta}{dt} = \frac{\pi\ f_0}{H} P_m \int_0^t dt = \frac{\pi\ f_0}{H} P_m\, t$$

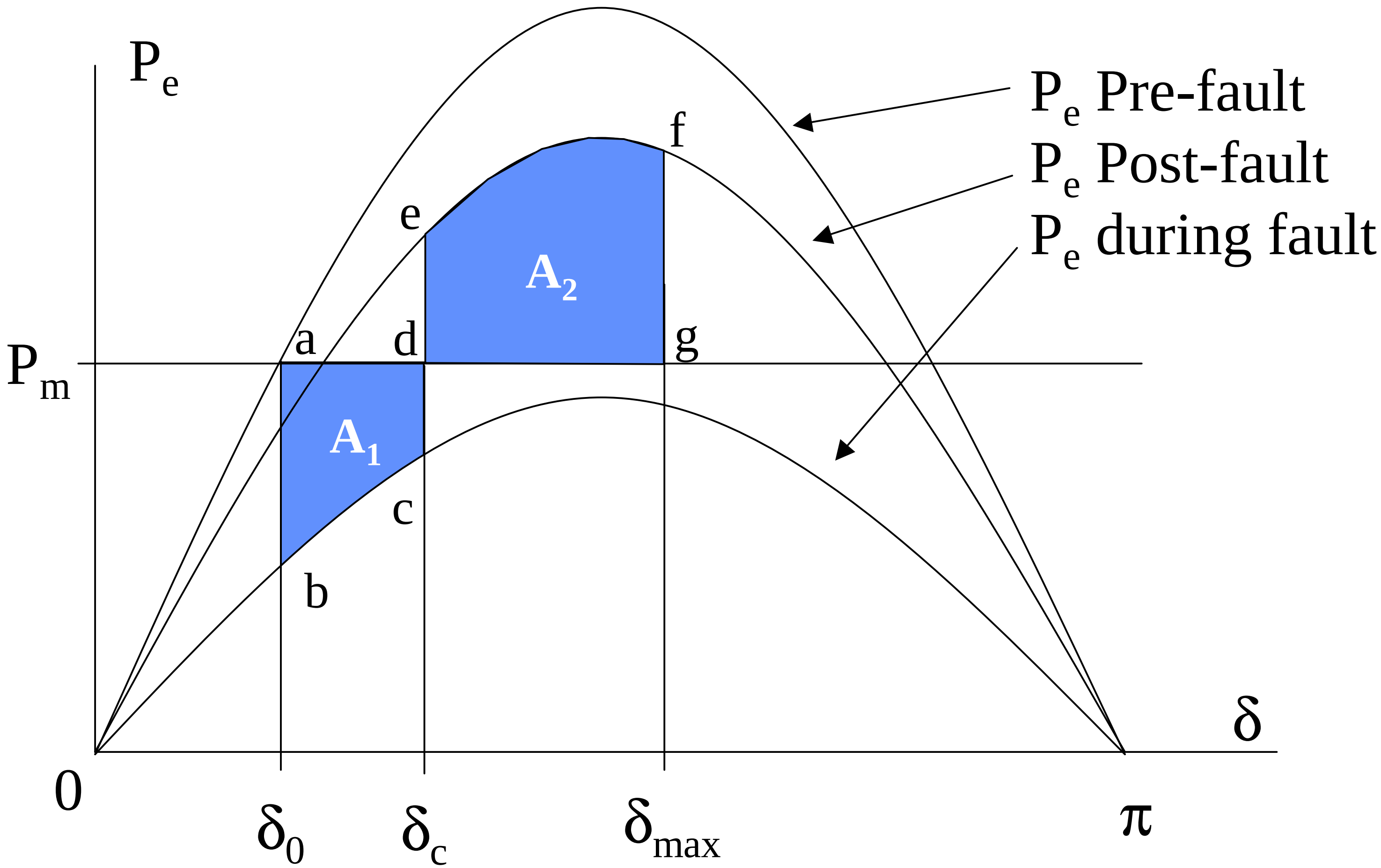
$$\delta = \frac{\pi\ f_0}{2H} P_m\, t^2 + \delta_0$$

$$t_c = \sqrt{\frac{2H\left(\delta_c - \delta_0\right)}{\pi\ f_0\ P_m}}$$

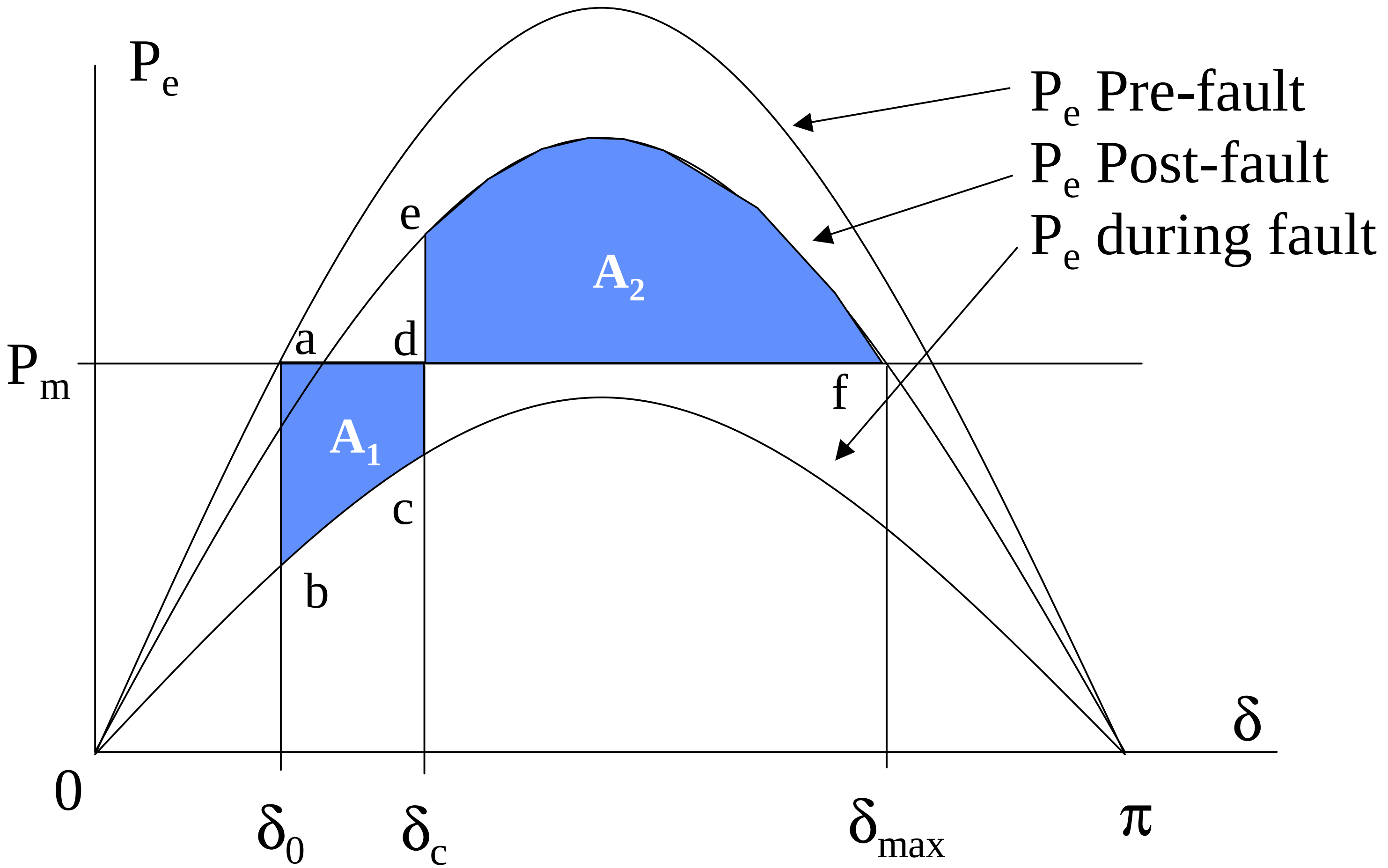
3-Phase Fault



Equal Area Criterion



Critical Clearing Time



Critical Clearing Time

$$P_m\left(\delta_c-\delta_0\right)-\int_{\delta_0}^{\delta_c} P_{2\max}\sin\delta\, d\delta=\int_{\delta_c}^{\delta_{\max}} P_{3\max}\sin\delta\, d\delta-P_m\left(\delta_{\max}-\delta_c\right)$$

$$\cos\delta_c=\frac{P_m\left(\delta_{\max}-\delta_c\right)+P_{3\max}\cos\delta_{\max}-P_{2\max}\cos\delta_0}{P_{3\max}-P_{2\max}}$$