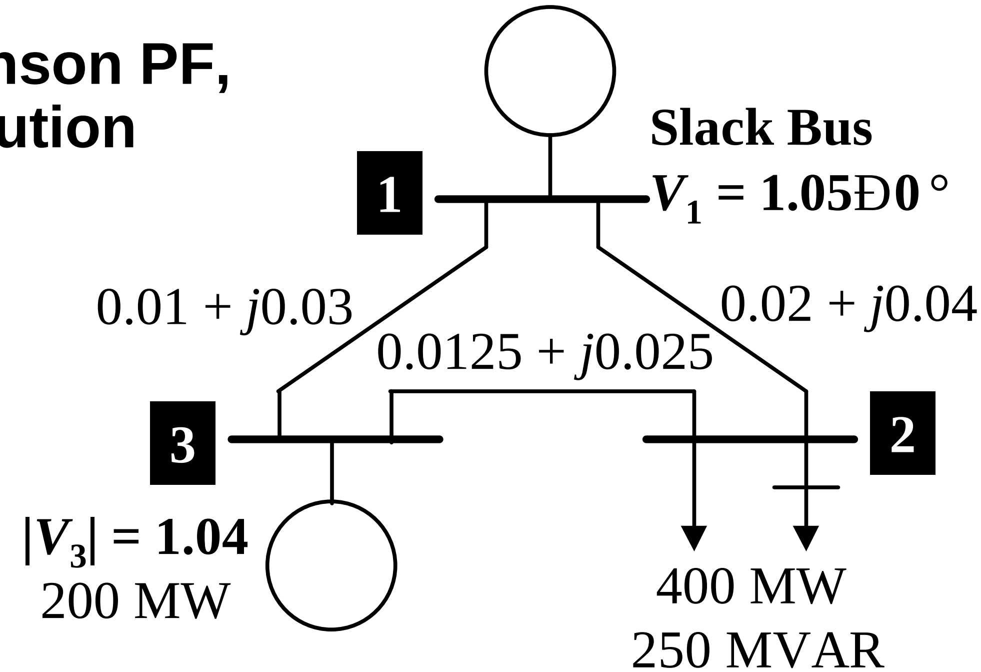
Example

- **Using the Newton-Raphson PF, find the power flow solution**



$$y_{12} = 10 - j20\ pu$$

$$y_{13} = 10 - j30\ pu$$

$$y_{23} = 16 - j32\ pu$$

$$S_2^{sch} = -\frac{400 + j250}{100} = -4.0 - j2.5\ pu$$

$$P_3^{sch} = \frac{200}{100} = 2.0\ pu$$

Example

$$Y_{bus} = \begin{bmatrix} 20 - j50 & -10 + j20 & -10 + j30 \\ -10 + j20 & 26 - j52 & -16 + j32 \\ -10 + j30 & -16 + j32 & 26 - j62 \end{bmatrix}$$

$$= \begin{bmatrix} 53.9\angle -1.90 & 22.4\angle 2.03 & 31.6\angle 1.89 \\ 22.4\angle 2.03 & 58.1\angle -1.11 & 35.8\angle 2.03 \\ 31.6\angle 1.89 & 35.8\angle 2.03 & 67.2\angle -1.17 \end{bmatrix}$$

angles are in radians

$$P_2 = |V_2||V_1||Y_{21}|\cos(q_{21} - d_2 + d_1) + |V_2|^2|Y_{22}|\cos(q_{22}) + |V_2||V_3||Y_{23}|\cos(q_{23} - d_2 + d_3)$$

$$P_3 = |V_3||V_1||Y_{31}|\cos(q_{31} - d_3 + d_1) + |V_3||V_2||Y_{32}|\cos(q_{32} - d_3 + d_2) + |V_3|^2|Y_{33}|\cos(q_{33})$$

$$Q_2 = -|V_2||V_1||Y_{21}|\sin(q_{21} - d_2 + d_1) - |V_2|^2|Y_{22}|\sin(q_{22}) - |V_2||V_3||Y_{23}|\sin(q_{23} - d_2 + d_3)$$

Example

$$\bar{x} = \begin{bmatrix} \bar{d}_2 \\ \bar{d}_3 \\ \bar{V}_2 \end{bmatrix} \qquad f(\bar{x}) = \begin{bmatrix} P_2(\bar{d}_2, \bar{d}_3, \bar{V}_2) \\ P_3(\bar{d}_2, \bar{d}_3, \bar{V}_2) \\ Q_2(\bar{d}_2, \bar{d}_3, \bar{V}_2) \end{bmatrix}$$

$$= \begin{bmatrix} |\bar{V}_2||1.05||22.3|\cos(2.03 - \bar{d}_{21}) + |\bar{V}_2|^2|58.1|\cos(-1.11) + |\bar{V}_2||1.04||35.8|\cos(2.03 - \bar{d}_2 + \bar{d}_3) \\ |V_3||1.05||31.6|\cos(1.89 - \bar{d}_3) + |1.04||\bar{V}_2||35.8|\cos(2.03 - \bar{d}_3 + \bar{d}_2) + |1.04|^2|67.2|\cos(-1.17) \\ -|\bar{V}_2||1.05||22.3|\sin(2.03 - \bar{d}_2) - |\bar{V}_2|^2|58.1|\sin(-1.11) - |\bar{V}_2||1.04||35.8|\sin(2.03 - \bar{d}_2 + \bar{d}_3) \end{bmatrix}$$

$$\Delta c = \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = c - f(\bar{x}) = \begin{bmatrix} P_2^{sch} \\ P_3^{sch} \\ Q_2^{sch} \end{bmatrix} - \begin{bmatrix} P_2(\bar{d}_2, \bar{d}_3, \bar{V}_2) \\ P_3(\bar{d}_2, \bar{d}_3, \bar{V}_2) \\ Q_2(\bar{d}_2, \bar{d}_3, \bar{V}_2) \end{bmatrix} = \begin{bmatrix} -4.0 \\ 2.0 \\ -2.5 \end{bmatrix} - \begin{bmatrix} P_2(\bar{d}_2, \bar{d}_3, \bar{V}_2) \\ P_3(\bar{d}_2, \bar{d}_3, \bar{V}_2) \\ Q_2(\bar{d}_2, \bar{d}_3, \bar{V}_2) \end{bmatrix}$$

Example

$$\frac{\partial P_2}{\partial \delta_2} = \sum_{j=1,\ j\neq 2}^{3} |V_2||V_j||Y_{2j}|\sin(\theta_{2j} - \delta_2 + \delta_j)$$

$$= |V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2) + |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

$$= |V_2||1.05||22.4|\sin(2.03 - \delta_2) + |V_2||1.04||35.8|\sin(2.03 - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3) = -|V_2||1.04||35.8|\sin(2.03 - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial |V_2|} = 2|V_2||Y_{22}|\cos(\theta_{22}) + \sum_{j=1,\ j\neq 2}^{3} |V_j||Y_{2j}|\cos(\theta_{2j} - \delta_2 + \delta_j)$$

$$= 2|V_2||Y_{22}|\cos(\theta_{22}) + |V_2||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$= 2|V_2||58.1|\cos(2.03) + |1.05||22.4|\cos(2.03 - \delta_2)$$

$$+ |1.04||35.8|\cos(2.03 - \delta_2 + \delta_3)$$

Example

$$\frac{\partial P_3}{\partial d_2} = -|V_3||V_2||Y_{32}|\sin(q_{32} - d_3 + d_2) = -|1.04||V_2||35.8|\sin(2.03 - d_2 + d_3)$$

$$\frac{\partial P_3}{\partial d_3} = \sum_{j=1,\, j\neq 3}^{3} |V_3||V_j||Y_{3j}|\sin(q_{3j} - d_3 + d_j)$$

$$= |V_3||V_1||Y_{31}|\sin(q_{31} - d_3 + d_1) + |V_3||V_2||Y_{32}|\sin(q_{32} - d_3 + d_2)$$

$$= |1.04||1.05||31.6|\sin(1.89 - d_3) + |1.04||V_2||35.8|\sin(2.03 - d_3 + d_2)$$

$$\frac{\partial P_3}{\partial |V_2|} = |V_3||Y_{32}|\cos(q_{32} - d_3 + d_2) = |1.04||35.8|\cos(2.03 - d_2 + d_3)$$

Example

$$\frac{\partial Q_2}{\partial d_2} = \sum_{j=1,\, j\neq 2}^{3} |V_2||V_j||Y_{2j}| \cos(q_{2j} - d_2 + d_j)$$

$$= |V_2||V_1||Y_{21}| \cos(q_{21} - d_2 + d_1) + |V_2||V_3||Y_{23}| \cos(q_{23} - d_2 + d_3)$$

$$= |V_2||1.05||22.4| \cos(2.03 - d_2) + |V_2||1.04||35.8| \cos(2.03 - d_2 + d_3)$$

$$\frac{\partial Q_2}{\partial d_3} = -|V_2||V_3||Y_{23}| \cos(q_{23} - d_2 + d_3) = -|V_2||1.04||35.8| \cos(2.03 - d_2 + d_3)$$

$$\frac{\partial Q_2}{\partial |V_2|} = -2|V_2||Y_{22}| \sin(q_{22}) - \sum_{j=1,\, j\neq 2}^{3} |V_j||Y_{2j}| \sin(q_{2j} - d_2 + d_j)$$

$$= -2|V_2||Y_{22}| \sin(q_{22}) - |V_1||Y_{21}| \sin(q_{21} - d_2 + d_1) - |V_3||Y_{23}| \sin(q_{23} - d_2 + d_3)$$

$$= -2|V_2||58.1| \sin(-1.11) - |1.05||22.4| \sin(2.03 - d_2)$$

$$- |1.04||35.8| \sin(2.03 - d_2 + d_3)$$

Example

$$\bar{x}^{[k+1]} = \bar{x}^{[k]} + J^{-1} \cdot \Delta c^{[k]}$$

$$= \begin{bmatrix} \bar{d}_2 \\ \bar{d}_3 \\ \bar{V}_2 \end{bmatrix}^{[k+1]} = \begin{bmatrix} \bar{d}_2 \\ \bar{d}_3 \\ \bar{V}_2 \end{bmatrix}^{[k]} + \begin{bmatrix} \partial P_2 / \partial d_2 & \partial P_2 / \partial d_3 & \partial P_2 / \partial V_2 \\ \partial P_3 / \partial d_2 & \partial P_3 / \partial d_3 & \partial P_3 / \partial V_2 \\ \partial Q_2 / \partial d_2 & \partial Q_2 / \partial d_3 & \partial Q_2 / \partial V_2 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix}^{[k]}$$

Newton-Raphson PF Example

$$\bar{x}^{[0]} = \begin{bmatrix} 0.0 \\ 0.0 \\ 1.0 \end{bmatrix} \quad \Delta c^{[0]} = \begin{bmatrix} P_2^{sch} \\ P_3^{sch} \\ Q_2^{sch} \end{bmatrix} - \begin{bmatrix} P_2^{[0]} \\ P_3^{[0]} \\ Q_2^{[0]} \end{bmatrix} = \begin{bmatrix} -4.0 \\ 2.0 \\ -2.5 \end{bmatrix} - \begin{bmatrix} -1.14 \\ 0.562 \\ -2.28 \end{bmatrix} = \begin{bmatrix} -2.86 \\ 1.438 \\ -0.22 \end{bmatrix}$$

$$\Delta x^{[0]} = J^{-1} \Delta c^{[0]}$$

$$\Delta x^{[0]} = \begin{bmatrix} \Delta\, d_2^{[0]} \\ \Delta\, d_3^{[0]} \\ \Delta \left| V_2^{[0]} \right| \end{bmatrix} = \begin{bmatrix} 54.28 & -33.28 & 24.86 \\ -33.28 & 66.04 & -16.64 \\ -27.14 & 16.64 & 49.72 \end{bmatrix}^{-1} \begin{bmatrix} -2.86 \\ 1.438 \\ -0.22 \end{bmatrix} = \begin{bmatrix} -0.04526 \\ -0.00772 \\ -0.02655 \end{bmatrix}$$

$$\bar{x}^{[1]} = \begin{bmatrix} d_2^{[1]} \\ d_3^{[1]} \\ \left| V_2^{[1]} \right| \end{bmatrix} = \begin{bmatrix} 0.0 + (-0.04526) \\ 0.0 + (-0.00772) \\ 1.0 + (-0.02655) \end{bmatrix} = \begin{bmatrix} -0.04526 \\ -0.00772 \\ 0.9734 \end{bmatrix}$$

Newton-Raphson PF Example

$$\bar{x}^{[1]} = \begin{bmatrix} -0.04526 \\ -0.00772 \\ 0.9734 \end{bmatrix} \quad \Delta c^{[1]} = \begin{bmatrix} P_2^{sch} \\ P_3^{sch} \\ Q_2^{sch} \end{bmatrix} - \begin{bmatrix} P_2^{[1]} \\ P_3^{[1]} \\ Q_2^{[1]} \end{bmatrix} = \begin{bmatrix} -4.0 \\ 2.0 \\ -2.5 \end{bmatrix} - \begin{bmatrix} -3.901 \\ 1.978 \\ -2.449 \end{bmatrix} = \begin{bmatrix} -0.099 \\ 0.0217 \\ -0.051 \end{bmatrix}$$

$$\Delta x^{[1]} = \begin{bmatrix} 51.72 & -31.77 & 21.30 \\ -32.98 & 65.66 & -15.38 \\ -28.54 & 17.40 & 48.10 \end{bmatrix}^{-1} \begin{bmatrix} -0.099 \\ 0.0217 \\ -0.051 \end{bmatrix} = \begin{bmatrix} -0.001795 \\ -0.000985 \\ -0.001767 \end{bmatrix}$$

$$\bar{x}^{[2]} = \begin{bmatrix} d_2^{[2]} \\ d_3^{[2]} \\ |V_2^{[2]}| \end{bmatrix} = \begin{bmatrix} -0.04526 + (-0.001795) \\ -0.00772 + (-0.000985) \\ 0.9734 + (-0.001767) \end{bmatrix} = \begin{bmatrix} -0.04706 \\ -0.00870 \\ 0.9717 \end{bmatrix}$$

Newton-Raphson PF Example

$$\bar{x}^{[2]} = \begin{bmatrix} -0.04706 \\ -0.00870 \\ 0.9717 \end{bmatrix} \quad \Delta c^{[2]} = \begin{bmatrix} P_2^{sch} \\ P_3^{sch} \\ Q_2^{sch} \end{bmatrix} - \begin{bmatrix} P_2^{[1]} \\ P_3^{[1]} \\ Q_2^{[1]} \end{bmatrix} = \begin{bmatrix} -4.0 \\ 2.0 \\ -2.5 \end{bmatrix} - \begin{bmatrix} -3.999 \\ 1.999 \\ -2.499 \end{bmatrix} = \begin{bmatrix} -0.0002 \\ 0.00004 \\ -0.0001 \end{bmatrix}$$

$$\Delta x^{[2]} = \begin{bmatrix} 51.60 & -31.69 & 21.14 \\ -32.93 & 65.60 & -15.35 \\ -28.55 & 17.40 & 47.95 \end{bmatrix}^{-1} \begin{bmatrix} -0.000216 \\ 0.000038 \\ -0.000143 \end{bmatrix} = \begin{bmatrix} -0.000038 \\ -0.000002 \\ -0.000004 \end{bmatrix}$$

$$\bar{x}^{[3]} = \begin{bmatrix} \delta_2^{[3]} \\ \delta_3^{[3]} \\ |V_2^{[3]}| \end{bmatrix} = \begin{bmatrix} -0.04706 + (-0.000038) \\ -0.00870 + (-0.000002) \\ 0.9717 + (-0.000004) \end{bmatrix} = \begin{bmatrix} -0.04706 \\ -0.008705 \\ 0.97168 \end{bmatrix}$$

Newton-Raphson PF Example

$$\bar{x}^{[3]} = \begin{bmatrix} -0.04706 \\ -0.008705 \\ 0.97168 \end{bmatrix} \quad \Delta c^{[2]} = \begin{bmatrix} P_2^{sch} \\ P_3^{sch} \\ Q_2^{sch} \end{bmatrix} - \begin{bmatrix} P_2^{[1]} \\ P_3^{[1]} \\ Q_2^{[1]} \end{bmatrix} = \begin{bmatrix} -4.0 \\ 2.0 \\ -2.5 \end{bmatrix} - \begin{bmatrix} -4.0 \\ 2.0 \\ -2.5 \end{bmatrix} = \begin{bmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \end{bmatrix}$$

$$e_{\max} = 2.5 \times 10^{-4}$$

$$P_1 = |V_1|^2 |Y_{11}| \cos(q_{11}) + |V_1||V_2||Y_{12}| \cos(q_{12} - d_1 + d_2) + |V_1||V_3||Y_{13}| \cos(q_{13} - d_1 + d_3)$$

$$Q_1 = -|V_1|^2 |Y_{11}| \sin(q_{11}) - |V_1||V_2||Y_{12}| \sin(q_{12} - d_1 + d_2) - |V_1||V_3||Y_{13}| \sin(q_{13} - d_1 + d_3)$$

$$Q_3 = -|V_3||V_1||Y_{31}| \sin(q_{31} - d_3 + d_1) - |V_3||V_2||Y_{32}| \sin(q_{32} - d_3 + d_2) - |V_3|^2 |Y_{33}| \sin(q_{33})$$

$$P_1 = 2.1842\, pu$$

$$Q_1 = 1.4085\, pu$$

$$Q_3 = 1.4617\, pu$$

Fast Decoupled Power Flow

- **Transmission lines and transformers have high *X*/*R* ratios**
  - Real power change, ΔP
    - is less sensitive to changes in the voltage magnitude, $\Delta|V|$
    - is more sensitive to changes in the phase angle, $\Delta\delta$
  - Reactive power changes, ΔQ
    - is less sensitive to changes in the phase angle , $\Delta\delta$
    - is more sensitive to changes in the voltage magnitude, $\Delta|V|$
  - Jacobian submatrices J_{Qd} and J_{PV} tend to be much smaller in magnitude compared to J_{Pd} and J_{QV}
- **Jacobian submatrices J_{Qd} and J_{PV} can be set to zero**

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{P\,d} & 0 \\ 0 & J_{QV} \end{bmatrix} \begin{bmatrix} \Delta\, d \\ \Delta|V| \end{bmatrix} \qquad \begin{aligned} \Delta P &= J_{P\,d} \cdot \Delta\, d = {}^{\partial P}\!/\!_{\partial\, d}\,\Delta\, d \\ \Delta Q &= J_{QV} \cdot \Delta|V| = {}^{\partial Q}\!/\!_{\partial |V|}\,\Delta|V| \end{aligned}$$

Fast Decoupled Power Flow

- **J_{PV} elements**

$$\frac{\partial P_i}{\partial |V_j|} = |V_i||Y_{ij}|\cos(q_{ij} - d_i + d_j)$$

$$q_{ij} \approx 90^\circ \qquad d_i \approx d_j$$

$$\frac{\partial P_i}{\partial |V_j|} \approx |V_i||Y_{ij}|\cos(90^\circ) = 0.0$$

- **J_{Qd} elements**

$$\frac{\partial Q_i}{\partial | d_j|} = -|V_i||V_j||Y_{ij}|\cos(q_{ij} - d_i + d_j)$$

$$q_{ij} \approx 90^\circ \qquad d_i \approx d_j$$

$$\frac{\partial Q_i}{\partial | d_j|} \approx -|V_i||V_j||Y_{ij}|\cos(90^\circ) = 0.0$$

Fast Decoupled Power Flow

- **The matrix equation is separated into two decoupled equations**
  - requires considerably less time to solve compared to the full Newton-Raphson method
  - J_{Pd} and J_{QV} submatrices can be further simplified to eliminate the need for recomputing of the submatrices during each iteration
    - some terms in each element are relatively small and can be eliminated
    - the remaining equations consist of constant terms and one variable term
    - the one variable term can be moved and coupled with the change in power variable
    - the result is a Jacobian matrix with constant term elements

Jacobian J_{Pd} Diagonal Terms

$$\frac{\partial P_i}{\partial d_i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} |V_i||V_j||Y_{ij}| \sin\left(q_{ij} - d_i + d_j\right)$$

$$= -|V_i|^2 |Y_{ii}| \sin\left(q_{ii}\right) + \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin\left(q_{ij} - d_i + d_j\right)$$

$$\frac{\partial P_i}{\partial d_i} = -|V_i|^2 |Y_{ii}| \sin\left(q_{ii}\right) - Q_i \qquad Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin\left(q_{ij} - d_i + d_j\right)$$

$$|Y_{ii}| \sin\left(q_{ii}\right) = B_{ii} \qquad B_{ii} \gg Q_i \qquad \frac{\partial P_i}{\partial d_i} = -|V_i|^2 B_{ii}$$

$$|V_i|^2 \approx |V_i| \qquad \Rightarrow \qquad \frac{\partial P_i}{\partial d_i} = -|V_i| B_{ii}$$

Jacobian J_{Pd} Off-diagonal Terms

$$\frac{\partial P_i}{\partial d_j} = -|V_i||V_j||Y_{ij}|\sin\left(q_{ij} - d_i + d_j\right)$$

$$d_j - d_i \approx 0$$

$$\frac{\partial P_i}{\partial d_i} = -|V_i||V_j||Y_{ij}|\sin\left(q_{ij}\right)$$

$$|Y_{ij}|\sin\left(q_{ij}\right) = B_{ij} \qquad |V_j| \approx 1$$

$$\frac{\partial P_i}{\partial d_i} = -|V_i|B_{ij}$$

Jacobian J_{QV} Diagonal Terms

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Y_{ii}|\sin(\theta_{ii}) - \sum_{\substack{j=1 \\ j\neq i}}^{n} |V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i||Y_{ii}|\sin(\theta_{ii}) - |V_i|^{-1}\sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

$$\frac{\partial Q_i}{\partial |V_i|} = -|V_i||Y_{ii}|\sin(\theta_{ii}) + |V_i|^{-1} Q_i \qquad Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

$$|Y_{ii}|\sin(\theta_{ii}) = B_{ii} \qquad B_{ii} >> Q_i \qquad \Rightarrow \qquad \frac{\partial Q_i}{\partial |V_i|} = -|V_i| B_{ii}$$

Jacobian J_{QV} Off-diagonal Terms

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}|\sin(\theta_{ij} - \delta_i + \delta_j)$$

$$\delta_j - \delta_i \approx 0$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i||Y_{ij}|\sin(\theta_{ij})$$

$$|Y_{ij}|\sin(\theta_{ij}) = B_{ij}$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i|B_{ij}$$

Fast Decoupled Power Flow

- **Individual power change equations in $J_{P\delta}$ and J_{QV}**

$$\Delta P_i = \sum_{j=1}^{n} -|V_i| B_{ij} \Delta \delta_j \qquad \Rightarrow \qquad \frac{\Delta P_i}{|V_i|} = \sum_{j=1}^{n} - B_{ij} \Delta \delta_j$$

$$\Delta Q_i = \sum_{j=1}^{n} -|V_i| B_{ij} \Delta |V_j| \qquad \Rightarrow \qquad \frac{\Delta Q_i}{|V_i|} = \sum_{j=1}^{n} - B_{ij} \Delta |V_j|$$

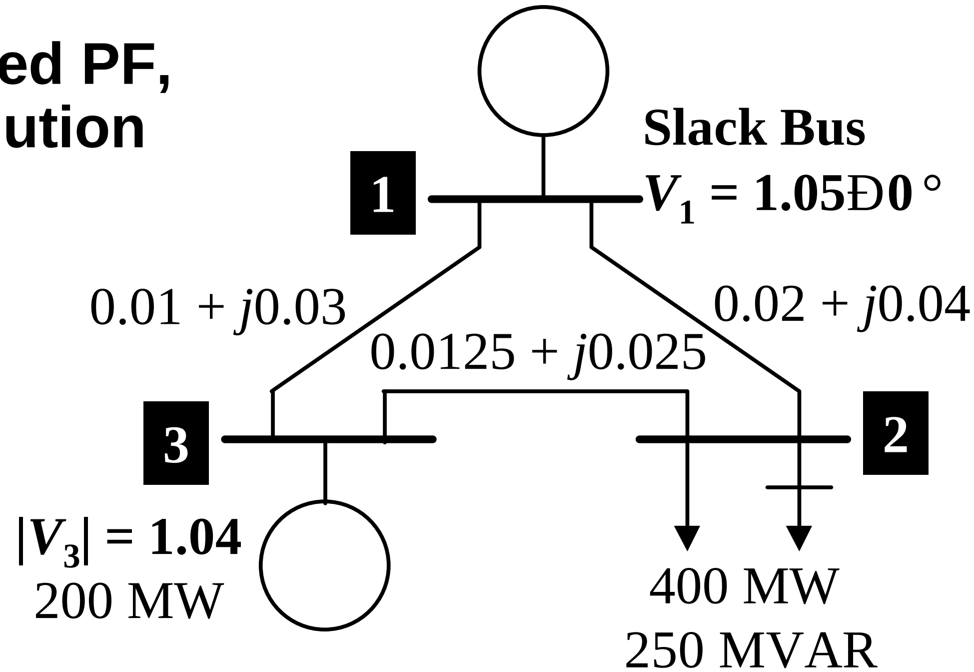
- **Matrix equation for $J_{P\delta}$ and J_{QV}**

$$\frac{\Delta P}{|V_i|} = -\mathbf{B}' \Delta \delta \qquad \Rightarrow \qquad \Delta \delta = -[\mathbf{B}']^{-1} \frac{\Delta P}{|V|}$$

$$\frac{\Delta Q}{|V_i|} = -\mathbf{B}'' \Delta |V| \qquad \Rightarrow \qquad \Delta |V| = -[\mathbf{B}'']^{-1} \frac{\Delta Q}{|V|}$$

Example

- **Using the fast decoupled PF, find the power flow solution**



$$y_{12} = 10 - j20 \ pu$$

$$y_{13} = 10 - j30 \ pu$$

$$y_{23} = 16 - j32 \ pu$$

$$S_2^{sch} = -\frac{400 + j250}{100} = -4.0 - j2.5 \ pu$$

$$P_3^{sch} = \frac{200}{100} = 2.0 \ pu$$

Example

$$B' = \begin{bmatrix} -52 & 32 \\ 32 & -62 \end{bmatrix}$$

$$[B']^{-1} = \begin{bmatrix} -0.028182 & -0.014545 \\ -0.014545 & -0.023636 \end{bmatrix}$$

$$B'' = [-52]$$

$$[B'']^{-1} = [-0.019231]$$

Example

Initial values:

$$V^{[0]} = \begin{bmatrix} 1.05\angle 0^\circ \\ 1.00\angle 0^\circ \\ 1.00\angle 0^\circ \end{bmatrix}$$

First iteration:

$$\bar{y} = \begin{bmatrix} P_2^{sch} \\ P_3^{sch} \\ Q_2^{sch} \end{bmatrix} = \begin{bmatrix} -4.0 \\ 2.0 \\ -2.5 \end{bmatrix} \qquad \bar{x}^{[k]} = \begin{bmatrix} d_2^{[k]} \\ d_3^{[k]} \\ V_2^{[k]} \end{bmatrix} \qquad \bar{x}^{[0]} = \begin{bmatrix} 0.0 \\ 0.0 \\ 1.0 \end{bmatrix}$$

$$f(\bar{x}) = \begin{bmatrix} P_{inj\,2}(\bar{x}) \\ P_{inj\,3}(\bar{x}) \\ Q_{inj\,2}(\bar{x}) \end{bmatrix} \qquad \begin{aligned} P_{inj\,i} &= \sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\cos(q_{ij} - d_i + d_j) \\ Q_{inj\,i} &= -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}|\sin(q_{ij} - d_i + d_j) \end{aligned}$$

Example

$$f\left(\bar{x}\right)=\begin{bmatrix} \left|V_2\right|^2\left|Y_{22}\right|\cos\left(q_{22}\right)+\left|V_2\right|\left|V_1\right|\left|Y_{21}\right|\cos\left(q_{21}-d_2+d_1\right)+\left|V_2\right|\left|V_3\right|\left|Y_{23}\right|\cos\left(q_{23}-d_2+d_3\right) \\ \left|V_3\right|^2\left|Y_{33}\right|\cos\left(q_{33}\right)+\left|V_3\right|\left|V_1\right|\left|Y_{31}\right|\cos\left(q_{31}-d_3+d_1\right)+\left|V_3\right|\left|V_2\right|\left|Y_{32}\right|\cos\left(q_{32}-d_3+d_2\right) \\ -\left|V_2\right|^2\left|Y_{22}\right|\sin\left(q_{22}\right)-\left|V_2\right|\left|V_1\right|\left|Y_{21}\right|\sin\left(q_{21}-d_2+d_1\right)-\left|V_2\right|\left|V_3\right|\left|Y_{23}\right|\sin\left(q_{23}-d_2+d_3\right) \\ \end{bmatrix}$$

$$=\begin{bmatrix} \left|V_2\right|^2\left|58.1\right|\cos\left(-1.11\right)+\left|V_2\right|\left|1.05\right|\left|22.4\right|\cos\left(2.03-d_2\right)+\left|V_2\right|\left|1.04\right|\left|35.8\right|\cos\left(2.03-d_2+d_3\right) \\ \left|1.04\right|^2\left|67.2\right|\cos\left(-1.17\right)+\left|1.04\right|\left|1.05\right|\left|31.6\right|\cos\left(1.89-d_3\right)+\left|1.04\right|\left|V_2\right|\left|35.8\right|\cos\left(2.03-d_3+d_2\right) \\ -\left|V_2\right|^2\left|58.1\right|\sin\left(-1.11\right)-\left|V_2\right|\left|1.05\right|\left|22.4\right|\sin\left(2.03-d_2\right)-\left|V_2\right|\left|1.04\right|\left|35.8\right|\sin\left(2.03-d_2+d_3\right) \\ \end{bmatrix}$$

Example

$$\Delta y^{[0]} = \begin{bmatrix} P_2^{sch} \\ P_3^{sch} \\ Q_2^{sch} \end{bmatrix} - \begin{bmatrix} P_2^{[0]} \\ P_3^{[0]} \\ Q_2^{[0]} \end{bmatrix} = \begin{bmatrix} -4.0 \\ 2.0 \\ -2.5 \end{bmatrix} - \begin{bmatrix} -1.14 \\ 0.562 \\ -2.28 \end{bmatrix} = \begin{bmatrix} -2.86 \\ 1.438 \\ -0.22 \end{bmatrix}$$

$$\begin{bmatrix} \Delta d_2^{[0]} \\ \Delta d_3^{[0]} \end{bmatrix} = \begin{bmatrix} 0.028182 & 0.014545 \\ 0.014545 & 0.023636 \end{bmatrix} \begin{bmatrix} -2.86/1.0 \\ 1.438/1.04 \end{bmatrix} = \begin{bmatrix} -0.06048 \\ -0.00891 \end{bmatrix}$$

$$\left[\Delta \left|V_2^{[0]}\right|\right] = [0.019231][-0.22/1.0] = [-0.004231]$$

$$d_2^{[1]} = 0.0 + (-0.06048) = -0.06048$$

$$d_3^{[1]} = 0.0 + (-0.00891) = -0.00891$$

$$\left|V_2^{[1]}\right| = 1.0 + (-0.004231) = 0.995769$$

Example

Remaining iterations:

Iter	$\delta 2$	$\delta 3$	|V2|	ΔP2	ΔP3	ΔQ2
1	-0.060482	-0.008909	0.995769	-2.860000	1.438400	-0.220000
2	-0.056496	-0.007952	0.965274	0.175895	-0.070951	-1.579042
3	-0.044194	-0.008690	0.965711	0.640309	-0.457039	0.021948
4	-0.044802	-0.008986	0.972985	-0.021395	0.001195	0.365249
5	-0.047665	-0.008713	0.973116	-0.153368	0.112899	0.006657
6	-0.047614	-0.008645	0.971414	0.000520	0.002610	-0.086136
7	-0.046936	-0.008702	0.971333	0.035980	-0.026190	-0.004067
8	-0.046928	-0.008720	0.971732	0.000948	-0.001411	0.020119
9	-0.047087	-0.008707	0.971762	-0.008442	0.006133	0.001558
10	-0.047094	-0.008702	0.971669	-0.000470	0.000510	-0.004688