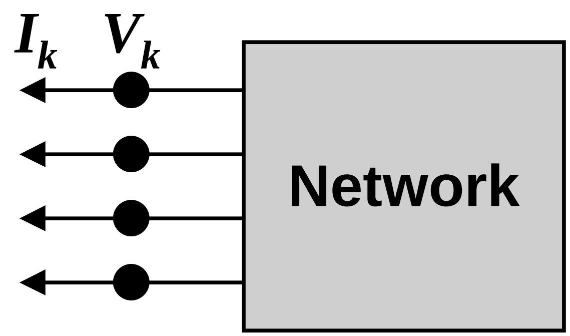
The Bus Admittance Matrix

- **The matrix equation for relating the nodal voltages to the currents that flow into and out of a network using the admittance values of circuit branches**

$$\mathbf{I}_{inj} = \mathbf{Y}_{bus} \cdot \mathbf{V}_{node}$$



- **Used to form the network model of an interconnected power system**
  - Nodes represent substation bus bars
  - Branches represent transmission lines and transformers
  - Injected currents are the flows from generator and loads

The Bus Admittance Matrix

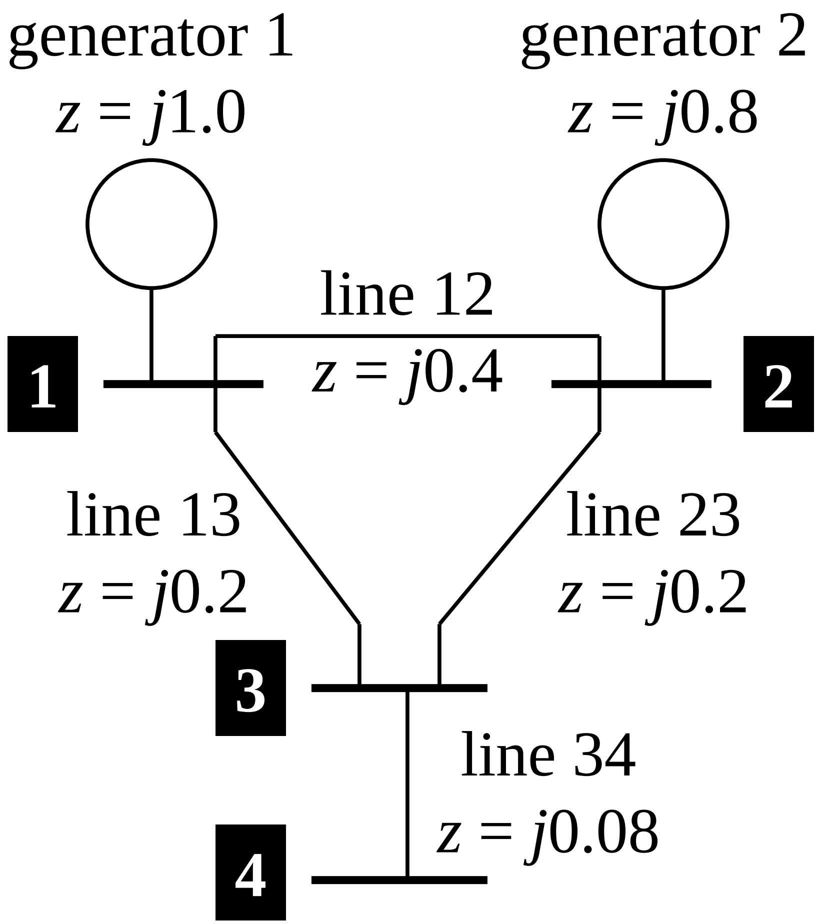
- **Constructing the Bus Admittance Matrix (or the Y bus matrix)**
  - form the nodal solution based upon Kirchhoff's current law

$$I_{k-inj} = y_{k0} V_k + y_{k1}(V_k - V_1) + y_{k2}(V_k - V_2) + \ldots + y_{kn}(V_k - V_n)$$

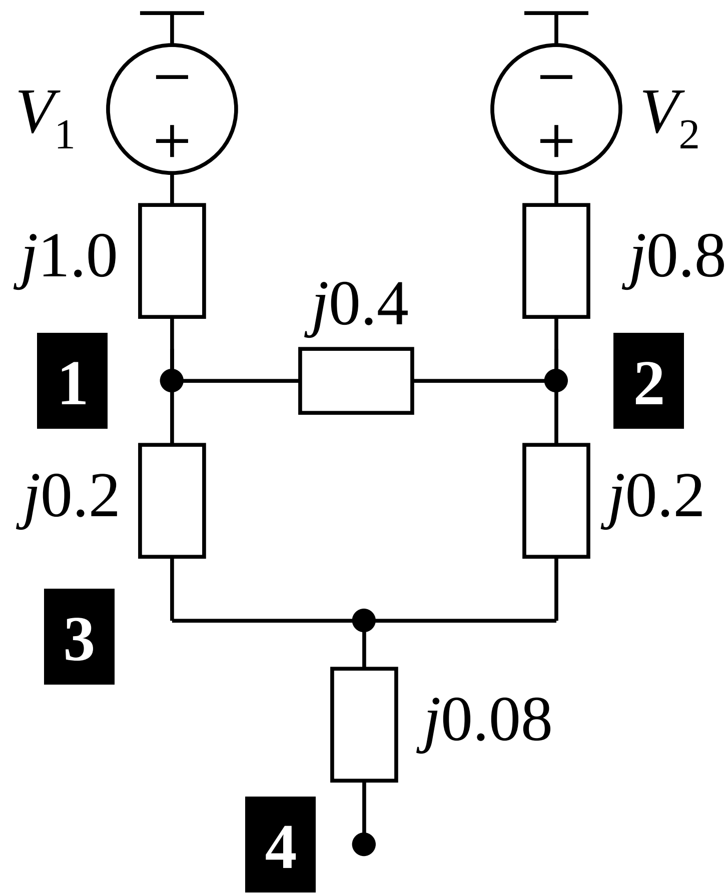
  - impedances are converted to admittances

$$y_{ij} = \frac{1}{z_{ij}} = \frac{1}{r_{ij} + j\, x_{ij}}$$

Matrix Formation Example

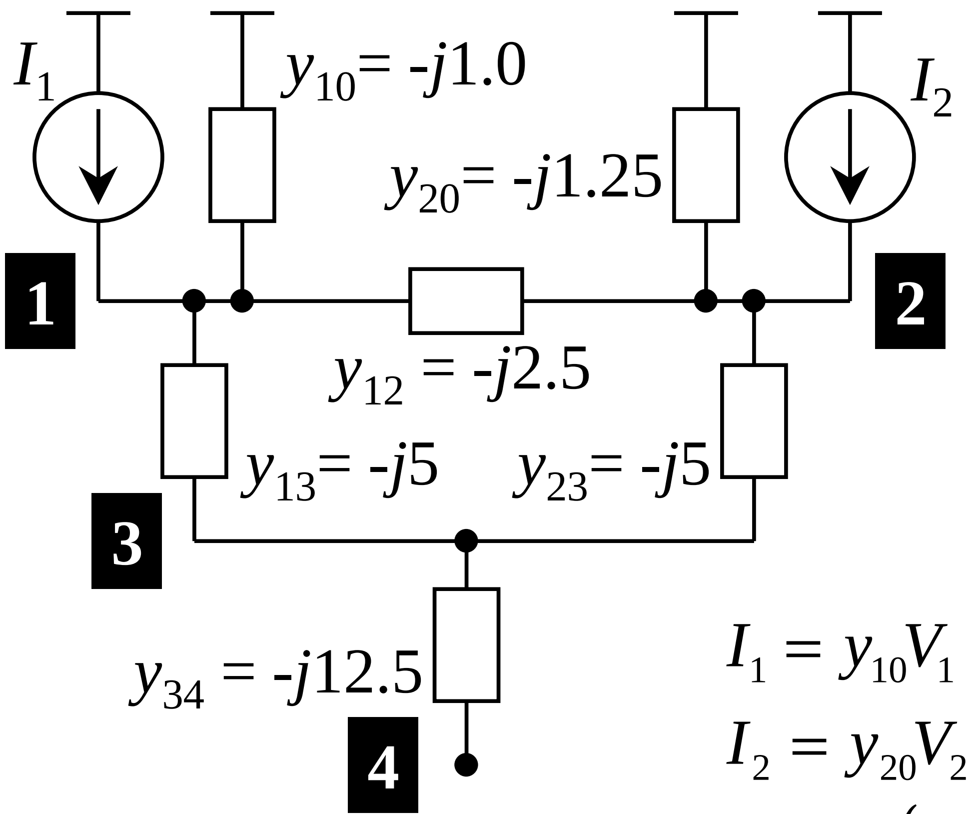


Network Diagram



Impedance Diagram

Matrix Formation Example



Admittance Diagram

KCL Equations

$$I_1 = y_{10}V_1 + y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3)$$
$$I_2 = y_{20}V_2 + y_{21}(V_2 - V_1) + y_{23}(V_2 - V_3)$$
$$0 = y_{31}(V_3 - V_1) + y_{32}(V_3 - V_2) + y_{34}(V_3 - V_4)$$
$$0 = y_{43}(V_4 - V_3)$$

Matrix Formation Example

Rearranging the KCL Equations

$$I_1 = \left(y_{10} + y_{12} + y_{13}\right)V_1 - y_{12}V_2 - y_{13}V_3$$

$$I_2 = -y_{21}V_1 + \left(y_{20} + y_{21} + y_{23}\right)V_2 - y_{23}V_3$$

$$0 = -y_{31}V_1 - y_{32}V_2 + \left(y_{31} + y_{32} + y_{34}\right)V_3 - y_{34}V_4$$

$$0 = -y_{43}V_3 + y_{43}V_4$$

Matrix Formation of the Equations

$$\begin{bmatrix} I_1 \\ I_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \left(y_{10} + y_{12} + y_{13}\right) & -y_{12} & -y_{13} & 0 \\ -y_{21} & \left(y_{20} + y_{21} + y_{23}\right) & -y_{23} & 0 \\ -y_{31} & -y_{32} & \left(y_{31} + y_{32} + y_{34}\right) & -y_{34} \\ 0 & 0 & -y_{43} & y_{43} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Matrix Formation Example

Completed Matrix Equation

$$Y_{11} = \left(y_{10} + y_{12} + y_{13}\right) = -j8.50 \qquad Y_{23} = Y_{32} = -y_{23} = j5.00$$

$$Y_{12} = Y_{21} = -y_{12} = j2.50 \qquad Y_{33} = \left(y_{31} + y_{32} + y_{34}\right) = -j22.50$$

$$Y_{13} = Y_{31} = -y_{13} = j5.00 \qquad Y_{34} = Y_{43} = -y_{34} = j12.50$$

$$Y_{22} = \left(y_{20} + y_{21} + y_{23}\right) = -j8.75 \qquad Y_{44} = y_{34} = -j12.50$$

$$\begin{bmatrix} I_1 \\ I_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -j8.50 & j2.50 & j5.00 & 0 \\ j2.50 & -j8.75 & j5.00 & 0 \\ j5.00 & j5.00 & -j22.50 & j12.50 \\ 0 & 0 & j12.50 & -j12.50 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Y-Bus Matrix Building Rules

- **Square matrix with dimensions equal to the number of buses**
- **Convert all network impedances into admittances**
- **Diagonal elements:**

$$Y_{ii} = \sum_{j=0}^{n} y_{ij} \qquad j \neq i$$

- **Off-diagonal elements:**

$$Y_{ij} = Y_{ji} = -y_{ij}$$

- **Matrix is symmetrical along the leading diagonal**

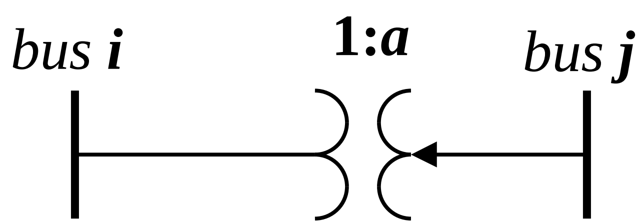
Example

System Data

Line	Start	End	X value
g1	1	0	1.00
g2	5	0	1.25
L1	1	2	0.40
L2	1	3	0.50
L3	2	3	0.25
L4	2	5	0.20
L5	3	4	0.125
L6	4	5	0.50

Tap-Changing Transformers

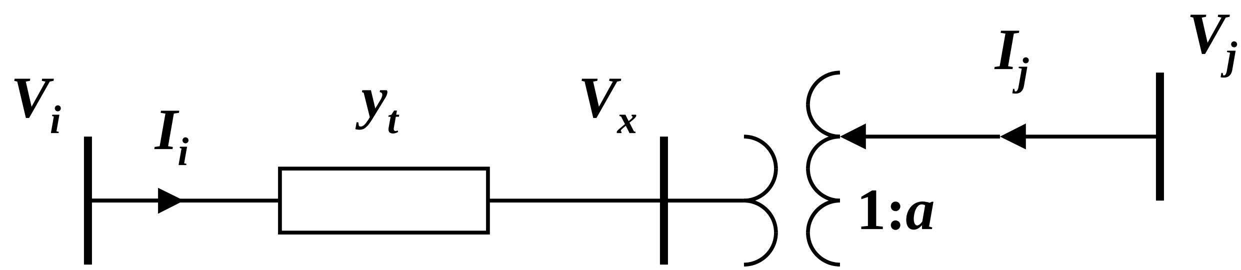
- **The tap-changing transform gives some control of the power network by changing the voltages and current magnitudes and angles by small amounts**
  - The flow of real power along a network branch is controlled by the angular difference of the terminal voltages
  - The flow of reactive power along a network branch is controlled by the magnitude difference of the terminal voltages
  - Real and reactive powers can be adjusted by voltage-regulating transformers and by phase-shifting transformers



a can be a complex number

Modeling of Tap-Changers

- the off-nominal tap ratio is given as 1:***a***
- the nominal turns-ratio (N_1/N_2) was addressed with the conversion of the network to per unit
- the transformer is modeled as two elements joined together at a fictitious bus ***x***



- basic circuit equations:

$$V_x = \tfrac{1}{a} V_j \qquad I_i = -a^* \cdot I_j \qquad I_i = y_t (V_i - V_x)$$

Modeling of Tap-Changers

- **Making substitutions**

$$V_x = \tfrac{1}{a} V_j \qquad\qquad I_i = y_t \left(V_i - V_x\right)$$

$$I_i = y_t \left(V_i - \tfrac{1}{a} V_j\right)$$

$$I_i = -a^* \cdot I_j$$

$$I_j = -\tfrac{1}{a^*} I_i$$

$$I_j = -\frac{y_t}{a^*}\left(V_i - \tfrac{1}{a} V_j\right) = -\frac{y_t}{a^*} V_i + \frac{y_t}{|a|^2} V_j$$

YBus Formation of Tap-Changers

- **Matrix formation**

$$I_i = \{y_t\} V_i + \left\{-\frac{y_t}{a}\right\} V_j$$

$$I_j = \left\{-\frac{y_t}{a^*}\right\} V_i + \left\{\frac{y_t}{|a|^2}\right\} V_j$$

$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{bmatrix} y_t & -y_t/a \\ -y_t/a^* & y_t/|a|^2 \end{bmatrix} \cdot \begin{bmatrix} V_i \\ V_j \end{bmatrix}$$

Pi-Circuit Model of Tap-Changers

- **Valid for real values of a**
- **Taking the y-bus formation, break the diagonal elements into two components**
  - the off-diagonal element represent the impedance across the two buses
  - the remainder form the shunt element