The Power Flow Solution

- **Most common and important tool in power system analysis**
  - also known as the “Load Flow” solution
  - used for planning and controlling a system
  - assumptions: balanced condition and single phase analysis
- **Problem:**
  - determine the voltage magnitude and phase angle at each bus
  - determine the active and reactive power flow in each line
  - each bus has four state variables:
    - voltage magnitude
    - voltage phase angle
    - real power injection
    - reactive power injection

The Power Flow Solution

- Each bus has two of the four state variables defined or given

- **Types of buses:**
  - Slack bus (swing bus)
    - voltage magnitude and angle are specified, reference bus
    - solution: active and reactive power injections
  - Regulated bus (generator bus, P-V bus)
    - models generation-station buses
    - real power and voltage magnitude are specified
    - solution: reactive power injection and voltage angle
  - Load bus (P-Q bus)
    - models load-center buses
    - active and reactive powers are specified (negative values for loads)
    - solution: voltage magnitude and angle

Newton-Raphson PF Solution

- **Quadratic convergence**
  - mathematically superior to Guass-Seidel method
- **More efficient for large networks**
  - number of iterations required for solution is independent of system size
- **The Newton-Raphson equations are cast in natural power system form**
  - solving for voltage magnitude and angle, given real and reactive power injections

Newton-Raphson Method

- **A method of successive approximation using Taylor's expansion**
  - Consider the function: $f(x) = c$, where x is unknown
  - Let $x^{[0]}$ be an initial estimate, then $\Delta x^{[0]}$ is a small deviation from the correct solution

$$f\left(x^{[0]} + \Delta x^{[0]}\right) = c$$

  - Expand the left-hand side into a Taylor's series about $x^{[0]}$ yeilds

$$f\left(x^{[0]}\right) + \left[\frac{df}{dx}\right]\Delta x^{[0]} + \tfrac{1}{2}\left[\frac{d^2 f}{dx^2}\right]\left(\Delta x^{[0]}\right)^2 + \cdots = c$$

Newton-Raphson Method

- Assuming the error, $\Delta x^{[0]}$, is small, the higher-order terms are neglected, resulting in

$$f\left(x^{[0]}\right)+\left[\frac{df}{dx}\right]\Delta x^{[0]} \approx c \quad \Rightarrow \quad \Delta c^{[0]} \approx \left[\frac{df}{dx}\right]\Delta x^{[0]}$$

- where

$$\Delta c^{[0]} = c - f\left(x^{[0]}\right)$$

- rearranging the equations

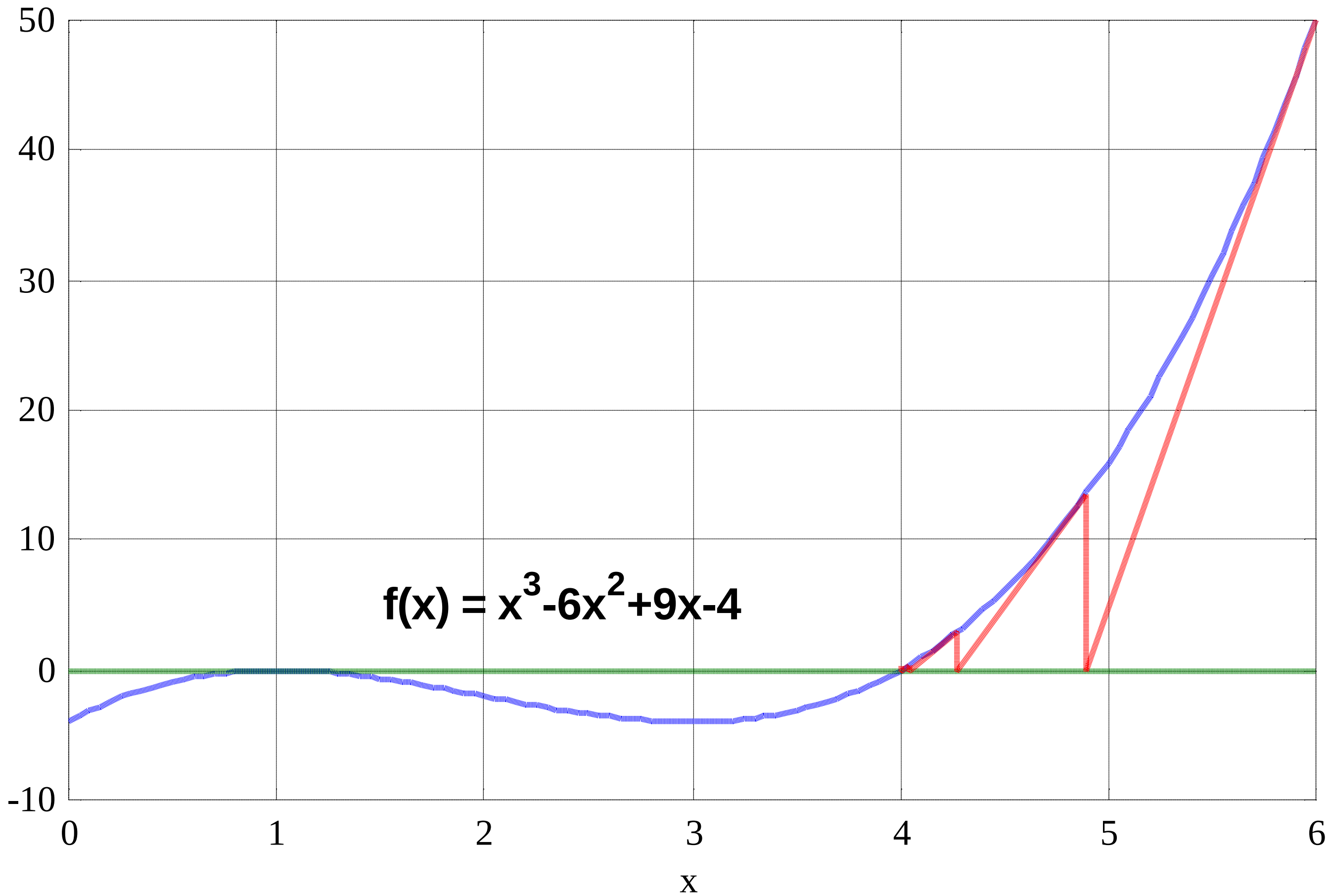
$$\Delta x^{[0]} = \frac{\Delta c^{[0]}}{\left[\frac{df}{dx}\right]}$$

$$x^{[1]} = x^{[0]} + \Delta x^{[0]}$$

Example

- **Find the root of the equation:** $f(x) = x^3 - 6x^2 + 9x - 4 = 0$

Newton-Raphson Method



Power Flow Equations

- **KCL for current injection**

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j = \sum_{j=1}^{n} |Y_{ij}||V_j| \angle q_{ij} + d_j$$

- **Real and reactive power injection**

$$P_i - j Q_i = V_i^* I_i$$

- **Substituting for I_i yields:**

$$P_i - j Q_i = (|V_i| \angle - d) \sum_{j=1}^{n} |Y_{ij}||V_j| \angle q_{ij} + d_j$$

Power Flow Equations

- **Divide into real and reactive parts**

$$P_i = \sum_{j=1}^{n} |V_i|\,|V_j|\,|Y_{ij}| \cos\left(q_{ij} - d_i + d_j\right)$$

$$Q_i = -\sum_{j=1}^{n} |V_i|\,|V_j|\,|Y_{ij}| \sin\left(q_{ij} - d_i + d_j\right)$$

Newton-Raphson Formation

- **Cast power equations into iterative form**

$$P_i^{[k]} = \sum_{j=1}^{n} \left|V_i^{[k]}\right| \left|V_j^{[k]}\right| \left|Y_{ij}\right| \cos\left(\theta_{ij} - \delta_i^{[k]} + \delta_j^{[k]}\right)$$

$$Q_i^{[k]} = -\sum_{j=1}^{n} \left|V_i^{[k]}\right| \left|V_j^{[k]}\right| \left|Y_{ij}\right| \sin\left(\theta_{ij} - \delta_i^{[k]} + \delta_j^{[k]}\right)$$

- **Matrix function formation of the system of equations**

$$c = \begin{bmatrix} P_{inj}^{sch} \\ Q_{inj}^{sch} \end{bmatrix} \qquad x^{[k]} = \begin{bmatrix} \delta^{[k]} \\ V^{[k]} \end{bmatrix} \qquad f\left(x^{[k]}\right) = \begin{bmatrix} P_{inj}\left(x^{[k]}\right) \\ Q_{inj}\left(x^{[k]}\right) \end{bmatrix}$$

Newton-Raphson Formation

- **General formation of the equation to find a solution**

$$c = f\left(x_{solution}\right) \qquad x^{[0]} = \text{initial estimate of } x_{solution}$$

- **The iterative equation**

$$x^{[k+1]} = x^{[k]} + \frac{c - f\left(x^{[k]}\right)}{\left(df\left(x^{[k]}\right) \middle/ dx \right)}$$

- **The Jacobian - the first derivative of a set of functions**

$$df\left(x^{[k]}\right) \big/ dx$$ **a matrix of all combinatorial pairs**

The Jacobian Matrix

$$\frac{df(x)}{dx} \Rightarrow \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \partial P / \partial d & \partial P / \partial |V| \\ \partial Q / \partial d & \partial Q / \partial |V| \end{bmatrix} \begin{bmatrix} \Delta d \\ \Delta |V| \end{bmatrix}$$

$$\begin{bmatrix} \Delta P_1 \\ \vdots \\ \Delta P_{n-1} \\ \Delta Q_1 \\ \vdots \\ \Delta Q_{n-m} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_1}{\partial d_1} & \cdots & \frac{\partial P_1}{\partial d_{n-1}} & \frac{\partial P_1}{\partial |V_1|} & \cdots & \frac{\partial P_1}{\partial |V_{n-m}|} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_{n-1}}{\partial d_1} & \cdots & \frac{\partial P_{n-1}}{\partial d_{n-1}} & \frac{\partial P_{n-1}}{\partial |V_1|} & \cdots & \frac{\partial P_{n-1}}{\partial |V_{n-m}|} \\ \frac{\partial Q_1}{\partial d_1} & \cdots & \frac{\partial Q_1}{\partial d_{n-1}} & \frac{\partial Q_1}{\partial |V_1|} & \cdots & \frac{\partial Q_1}{\partial |V_{n-m}|} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_{n-m}}{\partial d_1} & \cdots & \frac{\partial Q_{n-m}}{\partial d_{n-1}} & \frac{\partial Q_{n-m}}{\partial |V_1|} & \cdots & \frac{\partial Q_{n-m}}{\partial |V_{n-m}|} \end{bmatrix} \begin{bmatrix} \Delta d_1 \\ \vdots \\ \Delta d_{n-1} \\ \Delta |V_1| \\ \vdots \\ \Delta |V_{n-m}| \end{bmatrix}$$

Jacobian Terms

- **Real power w.r.t. the voltage angle**

$$\frac{\partial P_i}{\partial d_i} = \sum_{j \neq i} |V_i| |V_j| |Y_{ij}| \sin(q_{ij} - d_i + d_j)$$

$$\frac{\partial P_i}{\partial d_j} = -|V_i| |V_j| |Y_{ij}| \sin(q_{ij} - d_i + d_j) \quad i \neq j$$

- **Real power w.r.t. the voltage magnitude**

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i| |Y_{ii}| \cos q_{ii} + \sum_{j \neq i} |V_j| |Y_{ij}| \cos(q_{ij} - d_i + d_j)$$

$$\frac{\partial P_i}{\partial |V_j|} = |V_i| |Y_{ij}| \cos(q_{ij} - d_i + d_j) \quad i \neq j$$

Jacobian Terms

- **Reactive power w.r.t. the voltage angle**

$$\frac{\partial Q_i}{\partial d_i} = \sum_{j \neq i} |V_i|\,|V_j|\,|Y_{ij}| \cos\left(q_{ij} - d_i + d_j\right)$$

$$\frac{\partial Q_i}{\partial d_j} = -|V_i|\,|V_j|\,|Y_{ij}| \cos\left(q_{ij} - d_i + d_j\right) \quad i \neq j$$

- **Reactive power w.r.t. the voltage magnitude**

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i|\,|Y_{ii}| \sin q_{ii} + \sum_{j \neq i} |V_j|\,|Y_{ij}| \sin\left(q_{ij} - d_i + d_j\right)$$

$$\frac{\partial Q_i}{\partial |V_j|} = -|V_i|\,|Y_{ij}| \sin\left(q_{ij} - d_i + d_j\right) \quad i \neq j$$

Iteration process

- **Power mismatch or power residuals**
  - difference in schedule to calculated power

$$\Delta P_i^{[k]} = P_i^{sch} - P_i^{[k]}$$

$$\Delta Q_i^{[k]} = Q_i^{sch} - Q_i^{[k]}$$

- **New estimates for the voltages**

$$\delta_i^{[k+1]} = \delta_i^{[k]} + \Delta\,\delta_i^{[k]}$$

$$\left|V_i^{[k+1]}\right| = \left|V_i^{[k]}\right| + \Delta\left|V_i^{[k]}\right|$$

Bus Type and the Jacobian Formation

- **Slack Bus / Swing Bus**
  - one generator bus must be selected and defined as the voltage and angular reference
    - The voltage and angle are known for this bus
    - The angle is arbitrarily selected as zero degrees
    - bus is not included in the Jacobian matrix formation
- **Generator Bus**
    - have known terminal voltage and real (actual) power injection
    - the bus voltage angle and reactive power injection are computed
    - bus is included in the real power parts of the Jacobian matrix
- **Load Bus**
    - have known real and reactive power injections
    - bus is fully included in the Jacobian matrix

Newton-Raphson Steps

1. Set flat start

- For load buses, set voltages equal to the slack bus or $1.0\angle 0°$
- For generator buses, set the angles equal the slack bus or $0°$

2. Calculate power mismatch

- For load buses, calculate P and Q injections using the known and estimated system voltages
- For generator buses, calculate P injections
- Obtain the power mismatches, ΔP and ΔQ

3. Form the Jacobian matrix

- Use the various equations for the partial derivatives w.r.t. the voltage angles and magnitudes

Newton-Raphson Steps

4. Find the matrix solution (choose a or b)

- a. inverse the Jacobian matrix and multiply by the mismatch power
- b. perform gaussian elimination on the Jacobian matrix with the b vector equal to the mismatch power

compute $\Delta\delta$ **and** ΔV

5. Find new estimates for the voltage magnitude and angle

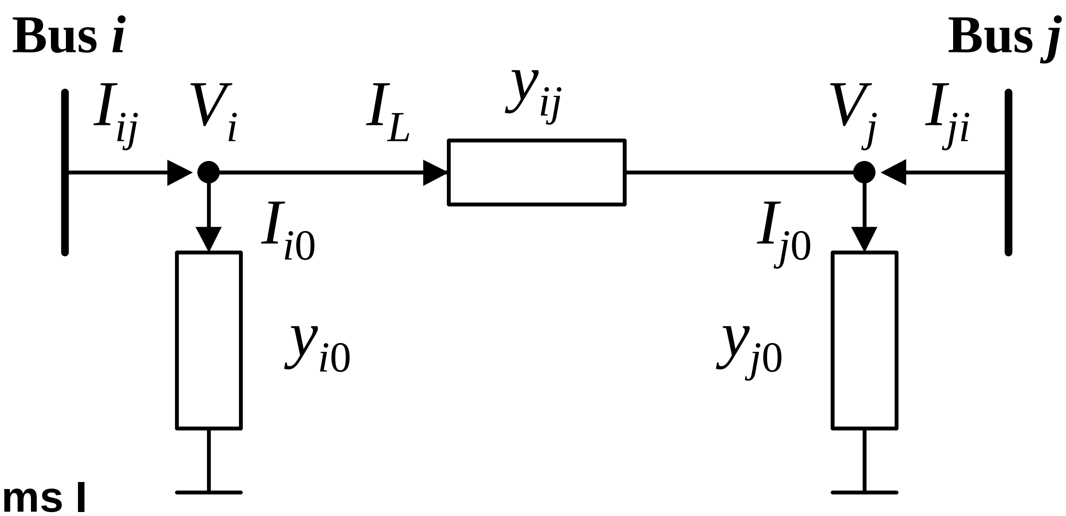
6. Repeat the process until the mismatch (residuals) are less than the specified accuracy

$$\left|\Delta P_i^{[k]}\right| \leq e$$

$$\left|\Delta Q_i^{[k]}\right| \leq e$$

Line Flows and Losses

- **After solving for bus voltages and angles, power flows and losses on the network branches are calculated**
  - Transmission lines and transformers are network branches
  - The direction of positive current flow are defined as follows for a branch element (demonstrated on a medium length line)
  - Power flow is defined for each end of the branch
    - Example: the power leaving bus i and flowing to bus j



Line Flows and Losses

- **current and power flows:**

$i \rightarrow j$

$$I_{ij} = I_L + I_{i0} = y_{ij}(V_i - V_j) + y_{i0}V_i$$

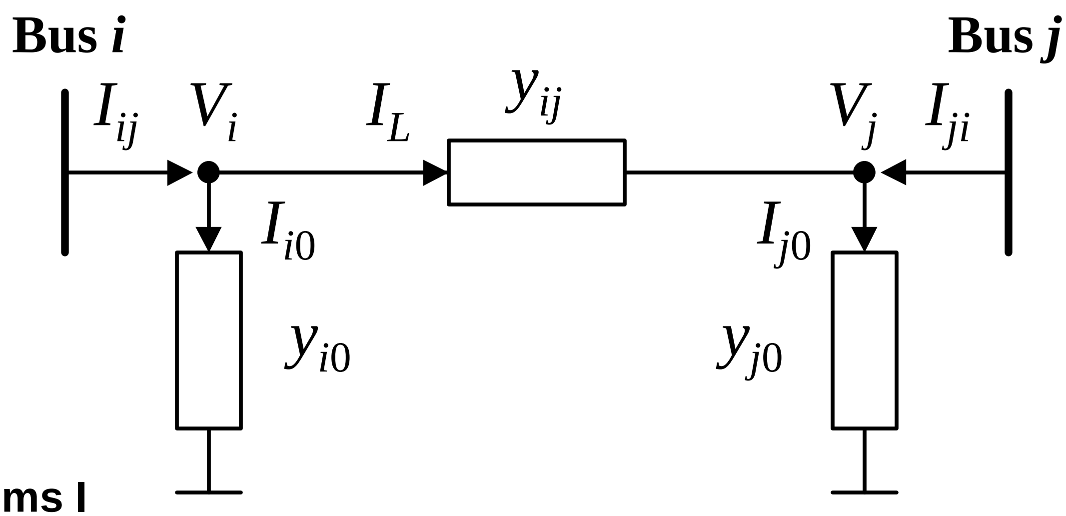
$$S_{ij} = V_i I_{ij}^* = V_i^2 (y_{ij} + y_{i0})^* - V_i y_{ij}^* V_j^*$$

$j \rightarrow i$

$$I_{ji} = -I_L + I_{j0} = y_{ij}(V_j - V_i) + y_{j0}V_j$$

$$S_{ji} = V_j I_{ji}^* = V_j^2 (y_{ij} + y_{j0})^* - V_j y_{ij}^* V_i^*$$

- **power loss:** $S_{Loss\,ij} = S_{ij} + S_{ji}$



Example

- **Using N-R method, find the phasor voltages at buses 2 and 3**
- **Find the slack bus real and reactive power**
- **Calculate line flows and line losses**
  - 100 MVA base