

# Balanced 3-Phase Short Circuit

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- Consider a synchronous generator operating at 60 Hz with constant excitation
- Examine the impact on the stator currents when a three-phase short circuit is applied to the generator terminals
  - ♦ The initial currents

$$i_a(0^+) = i_b(0^+) = i_c(0^+) = 0 \xrightarrow{\text{Park}} i_0(0^+) = i_d(0^+) = i_q(0^+) = 0$$

$$i_F(0^+) = \frac{V_F}{r_F}$$

- ♦ The voltage after applying the fault

$$v_a = v_b = v_c = 0 \xrightarrow{\text{Park}} v_0 = v_d = v_q = 0$$

# Balanced 3-Phase Short Circuit

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- Rearranging the equation and neglecting the zero sequence term

$$\begin{bmatrix} v_d \\ -v_F \\ 0 \\ v_q \\ 0 \end{bmatrix} = - \begin{bmatrix} r & 0 & 0 & \omega L_q & \omega \sqrt{\frac{3}{2}} M_Q \\ 0 & r_F & 0 & 0 & 0 \\ 0 & 0 & r_D & 0 & 0 \\ -\omega L_d & -\omega \sqrt{\frac{3}{2}} M_F & -\omega \sqrt{\frac{3}{2}} M_D & r & 0 \\ 0 & 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_d \\ i_F \\ i_D \\ i_q \\ i_Q \end{bmatrix} -$$

$$- \begin{bmatrix} L_d & \sqrt{\frac{3}{2}} M_F & \sqrt{\frac{3}{2}} M_D & 0 & 0 \\ \sqrt{\frac{3}{2}} M_F & L_F & M_R & 0 & 0 \\ \sqrt{\frac{3}{2}} M_D & M_R & L_D & 0 & 0 \\ 0 & 0 & 0 & L_q & \sqrt{\frac{3}{2}} M_Q \\ 0 & 0 & 0 & \sqrt{\frac{3}{2}} M_Q & L_Q \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_d \\ i_F \\ i_D \\ i_q \\ i_Q \end{bmatrix}$$

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- In matrix form (or state space form) the equation can be rewritten

$$\mathbf{v} = -\mathbf{R}\mathbf{i} - \mathbf{L} \frac{d}{dt} \mathbf{i}$$

$$\frac{d}{dt} \mathbf{i} = -\mathbf{L}^{-1} \mathbf{R}\mathbf{i} - \mathbf{L}^{-1} \mathbf{v}$$

- ◆ Using the Laplace transform or integrating the equation results in

$$\mathbf{i}_{abc} = \mathbf{P}^{-1} \mathbf{i}_{0dq}$$

$$i_a = i_d \cos \theta + i_q \sin \theta$$

$$i_b = i_d \cos\left(\theta - \frac{2\pi}{3}\right) + i_q \sin\left(\theta - \frac{2\pi}{3}\right)$$

$$i_c = i_d \cos\left(\theta + \frac{2\pi}{3}\right) + i_q \sin\left(\theta + \frac{2\pi}{3}\right)$$

# Example

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- Consider a 500 MVA, 30 kV generator with no load and a constant excitation voltage of 400 V. A three-phase short circuit occurs at the terminals. Obtain the transient waveforms for the current in each phase and the field winding

# Example

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## Generator Parameters

$$\begin{array}{lll} L_d = 0.0072 \text{ H} & L_q = 0.0070 \text{ H} & L_F = 2.500 \text{ H} \\ L_D = 0.0068 \text{ H} & L_Q = 0.0016 \text{ H} & M_F = 0.100 \text{ H} \\ M_D = 0.0054 \text{ H} & M_Q = 0.0026 \text{ H} & M_R = 0.125 \text{ H} \\ r = 0.0020 \text{ } \Omega & r_Q = 0.0150 \text{ } \Omega & r_F = 0.4000 \text{ } \Omega \\ r_D = 0.0150 \text{ } \Omega & & L_0 = 0.0010 \text{ H} \end{array}$$

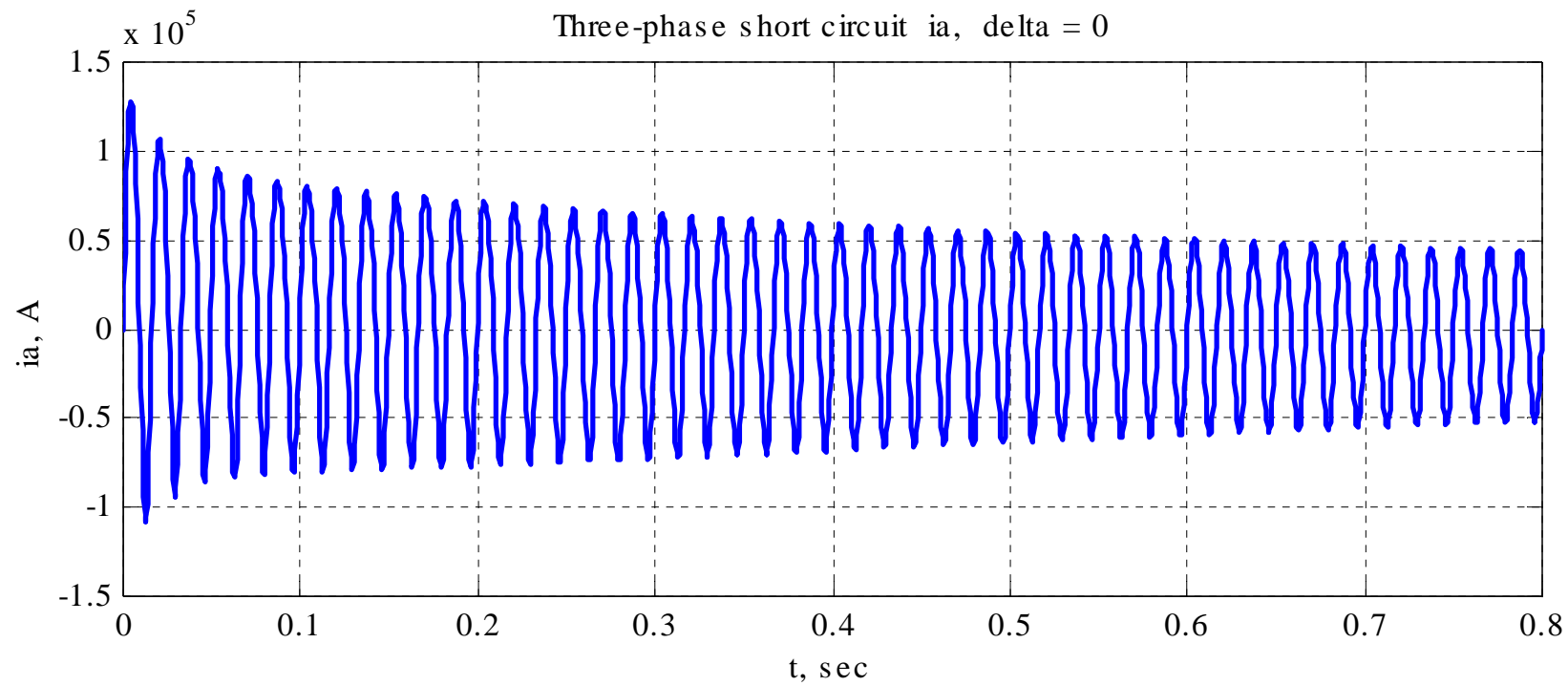
$$V_F = 400 \text{ V}$$

$$i_F(0^+) = V_F / r_F = 400 / 0.4 = 1000 \text{ A}$$

$$i_0(0^+) = i_d(0^+) = i_q(0^+) = 0$$

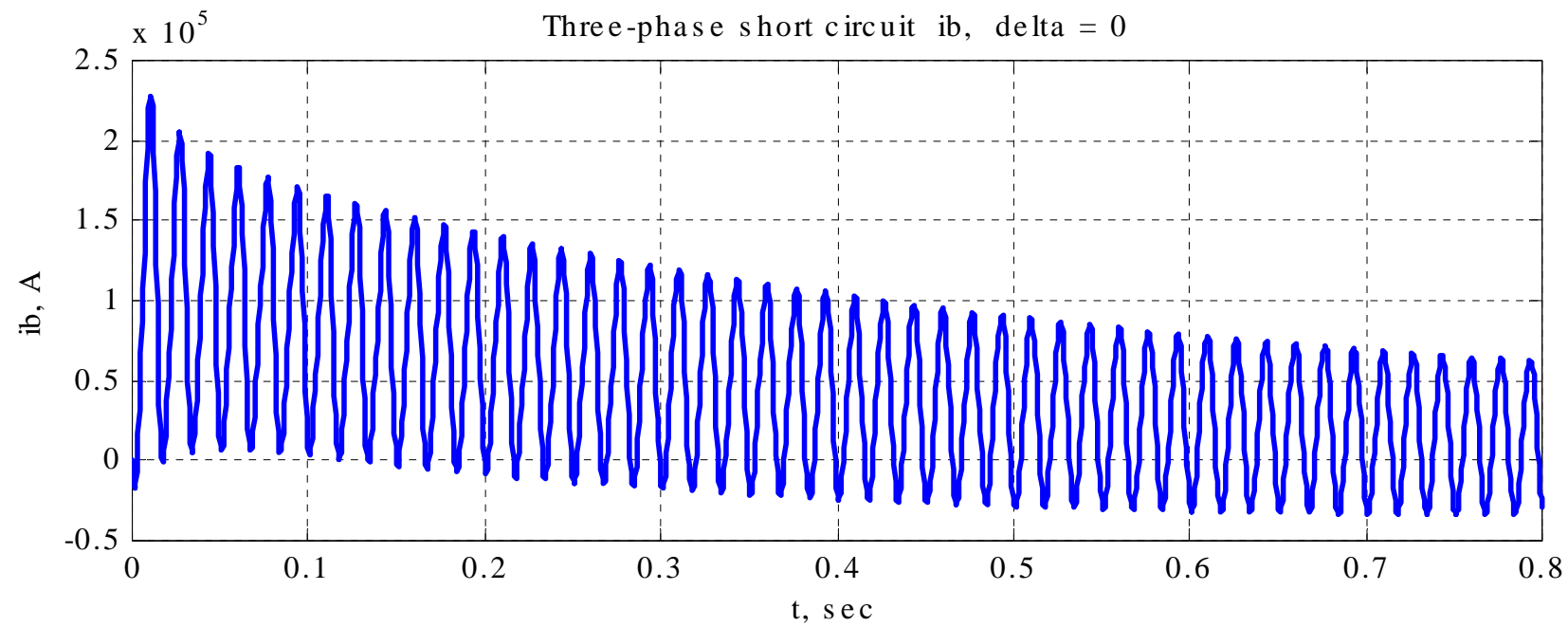
# Example

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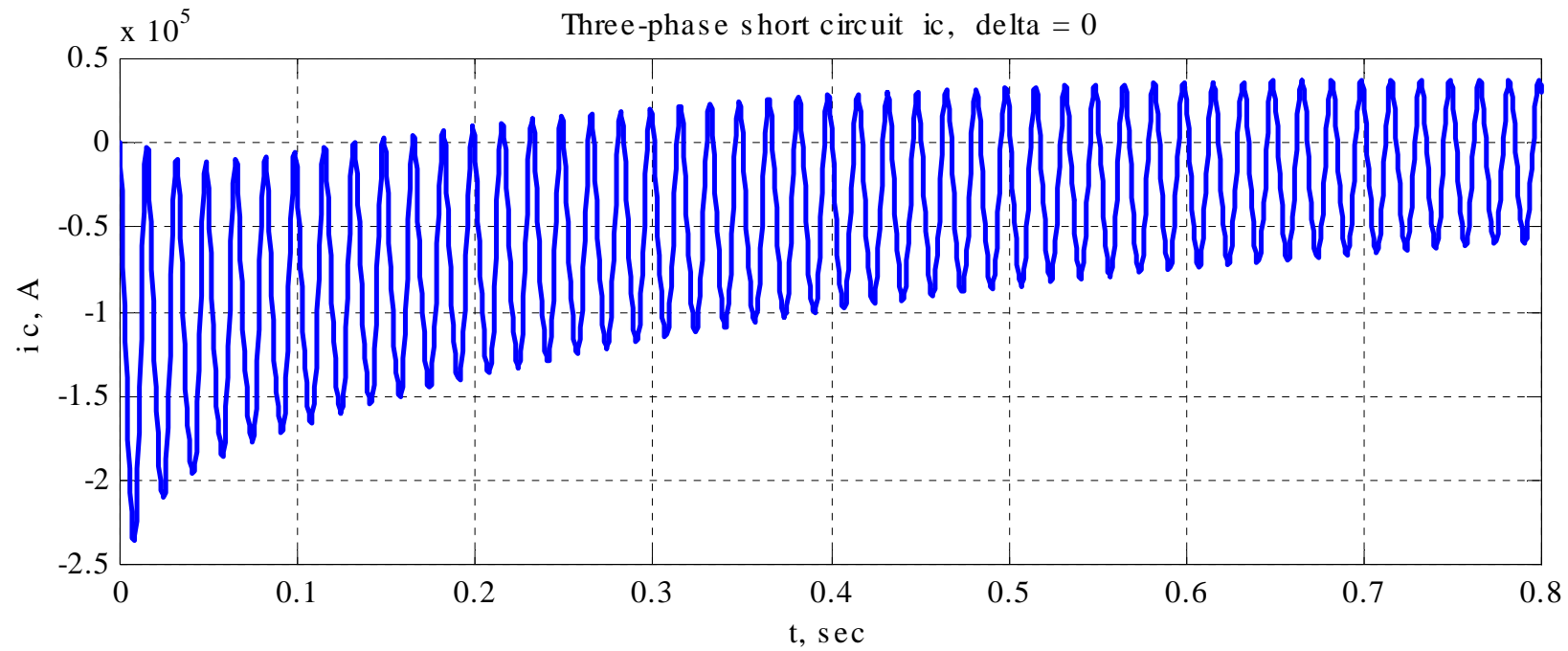
# Example

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# Example

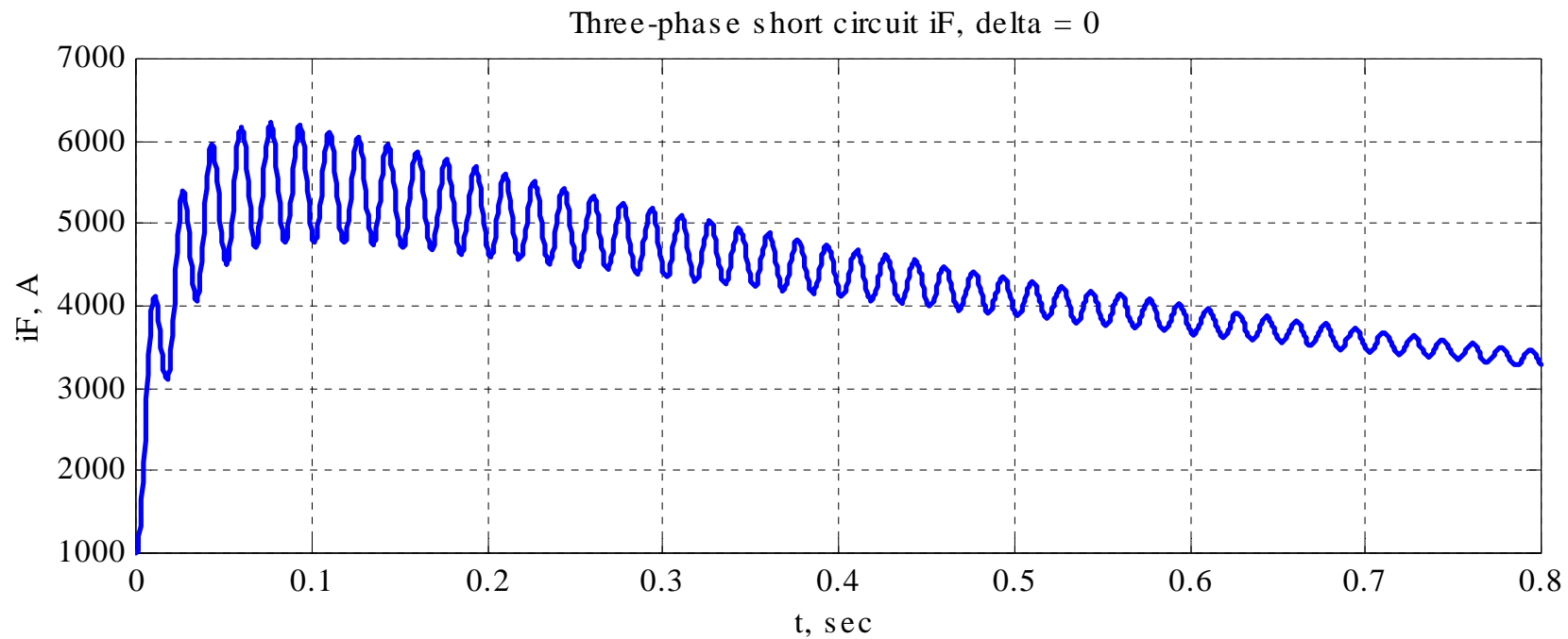
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# Example

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# Simplified Machine Model

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- **For steady-state operation, generators are represented with a constant emf behind a synchronous reactance,  $X_S$** 
  - ◆ For salient-pole rotors, there is a direct axis and quadrature axis reactances
- **Under transient conditions, the machine reactance changes due to the effect of the armature (transformer) reaction and eddy currents in the damping circuits**
- **For analysis it is useful to imagine the synchronous reactance as three components**
  - ◆ direct axis sub-transient reactance
  - ◆ direct axis transient reactance
  - ◆ direct axis steady-state reactance
  - ◆ these transient reactances have an associated time-constant

# Simplified Machine Model

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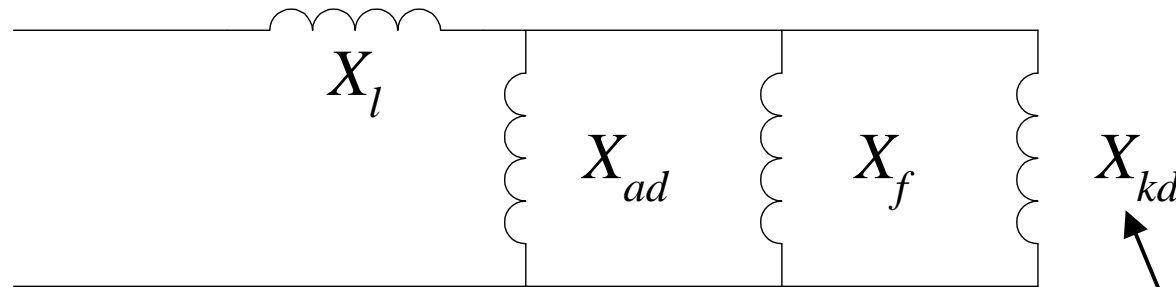
- **Model visualization**

- ◆ Consider the field and damper windings as the secondaries of a transformer (or the rotor of an induction motor)
- ◆ The stator is the primary winding
- ◆ For steady state conditions (synchronous speed) there is no transformer action, which can be modeled as an open circuit on the secondary side of the transformer
- ◆ For dynamic conditions, the speed is not synchronous, and the field and damper windings look like short-circuited secondaries

# Simplified Machine Model

- The direct axis sub-transient reactance

- ◆ circuit model



- ◆ equations

$$X_d'' = X_l + \left( \frac{1}{X_{ad}} + \frac{1}{X_f} + \frac{1}{X_{kd}} \right)^{-1}$$

$$\tau_d'' = \frac{X_{kd} + \left( \frac{1}{X_{ad}} + \frac{1}{X_f} + \frac{1}{X_l} \right)^{-1}}{R_k}$$

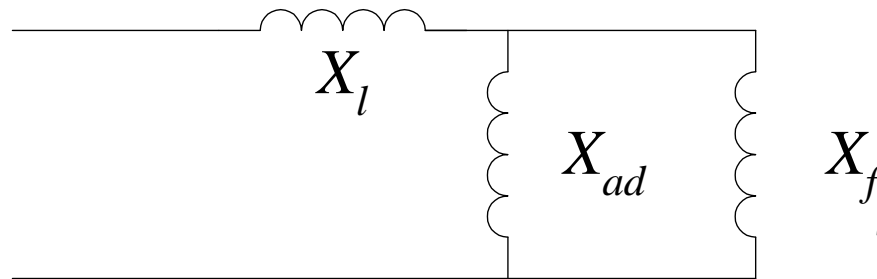
**damper  
winding  
reactance  
and  
resistance**

time constant  
is very small,  
around 0.035s

# Simplified Machine Model

- The direct axis transient reactance

- ◆ circuit model



- ◆ equations

$$X'_d = X_l + \left( \frac{1}{X_{ad}} + \frac{1}{X_f} \right)^{-1}$$

$$\tau''_d = \frac{X_f + \left( \frac{1}{X_{ad}} + \frac{1}{X_l} \right)^{-1}}{R_f}$$

time constant  
is in the order  
of 1s to 2s

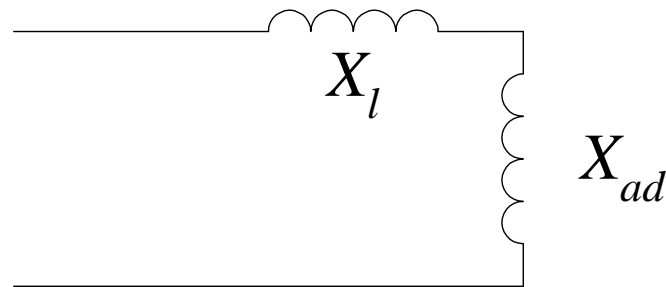
field  
winding  
reactance  
and  
resistance

# Simplified Machine Model

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- The direct axis steady-state reactance

- ◆ circuit model



- ◆ equation

$$X_d = X_l + X_{ad}$$

- ◆ equivalent circuit for the steady state

# Simplified Machine Model

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- **Similar models are used for the quadrature axis:**
  - ◆ quadrature axis sub-transient reactance,  $X_q''$
  - ◆ quadrature axis transient reactance,  $X_q'$
  - ◆ quadrature axis steady-state reactance,  $X_q$
- **For an unloaded generator, the stator current following the occurrence of a short-circuit on the terminals:**

$$i_{ac}(t) = \sqrt{2} E_0 \left[ \left( \frac{1}{X_d''} - \frac{1}{X_d'} \right) e^{-t/\tau_d''} + \left( \frac{1}{X_d'} - \frac{1}{X_d} \right) e^{-t/\tau_d'} + \frac{1}{X_d} \right] \sin(\omega t + \delta)$$

# Simplified Machine Model

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- **Example**

- ◆ a three-phase, 60 Hz machine has the stator windings initially open-circuited, and the field current adjusted so that the terminal voltage is at rated value (i.e., 1.0 pu)
- ◆ The machine has the following time constants:  
 $X_d'' = 0.15 \text{ pu}$                        $t_d'' = 0.035 \text{ sec}$   
 $X_d' = 0.40 \text{ pu}$                        $t_d' = 1.0 \text{ sec}$   
 $X_d = 1.20 \text{ pu}$
- ◆ Determine the subtransient, transient, and steady state short-circuit currents



# Simplified Machine Model

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$$I_d'' = \frac{E_0}{X_d''} = \frac{1.0}{0.15} = 6.666 \text{ pu}$$

$$I_d' = \frac{E_0}{X_d'} = \frac{1.0}{0.4} = 2.5 \text{ pu}$$

$$I_d = \frac{E_0}{X_d} = \frac{1.0}{1.2} = 0.8333 \text{ pu}$$