Stability

- **The ability of the power system to remain in synchronism and maintain the state of equilibrium following a disturbing force**
  - Steady-state stability: analysis of small and slow disturbances
    - gradual power changes
  - Transient stability: analysis of large and sudden disturbances
    - faults, outage of a line, sudden application or removal of load

Generator Dynamic Model

- **Under normal conditions, the relative position of the rotor axis and the stator magnetic field axis is fixed**
  - the angle between the two is the power angle or torque angle, δ
  - during a disturbance, the rotor will accelerate or decelerate w.r.t. the rotating stator field
  - acceleration or deceleration causes a change in the power angle

$$T_e = \frac{P_e}{\omega_e} = \frac{P_e}{2\pi(60Hz)} \qquad \frac{P_m}{\omega_{rotor}} = T_m$$

$$T_{accelation} = \Delta T = T_m - T_e$$

$$J\frac{d^2\theta_m}{dt^2} = \Delta T = T_m - T_e \quad \theta_m = \omega_{ms}t + \delta_m \quad \frac{\omega_{rotor}}{\omega_{ms}} = \frac{poles}{2}$$

Generator Dynamic Model

$$\omega_m = \frac{d\theta_m}{dt} = \omega_{ms} + \frac{d\delta_m}{dt} \qquad \alpha_m = \frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

$$J\frac{d^2\theta_m}{dt^2} = J\frac{d^2\delta_m}{dt^2} = T_m - T_e$$

$$J\omega_m\frac{d^2\delta_m}{dt^2} = \omega_m T_m - \omega_m T_e = P_m - P_e$$

$$W_{KE} = \tfrac{1}{2}J\omega_m^2 = \tfrac{1}{2}M\omega_m \qquad M = \frac{2\,W_{KE}}{\omega_m} = J\omega_m$$

$$\omega_m \approx \omega_{ms} \rightarrow M \approx \frac{2\,W_{KE}}{\omega_{ms}} = J\omega_{ms}$$

Generator Dynamic Model

$$M \frac{d^2 \delta_m}{dt^2} = P_m - P_e$$

$$\delta = \delta_e = \frac{poles}{2} \delta_m \quad \rightarrow \quad \frac{p}{2} M \frac{d^2 \delta}{dt^2} = P_m - P_e$$

$$\frac{p}{2} M \frac{d^2 \delta}{dt^2} \quad = \quad \frac{p}{2} \frac{2 W_{KE}}{\omega_{ms}} \frac{d^2 \delta}{dt^2} \quad = \quad \frac{2 W_{KE}}{\omega_s} \frac{d^2 \delta}{dt^2}$$

$$\frac{2 W_{KE}}{\omega_s} \frac{d^2 \delta}{dt^2} = P_m - P_e \quad \rightarrow \quad \frac{2 W_{KE}}{\omega_s \, S_B} \frac{d^2 \delta}{dt^2} = \frac{P_m}{S_B} - \frac{P_e}{S_B}$$

Generator Dynamic Model

$$\frac{2\,W_{KE}}{\omega_s\ S_B}\frac{d^2\delta}{dt^2} = P_{m(pu)} - P_{e(pu)}$$

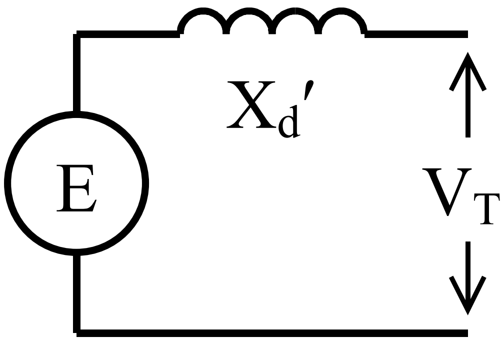
$$\frac{W_{KE}}{S_B} = \frac{\text{kinetic energy in MJ at rated speed}}{\text{machine power rating in MVA}} = H$$

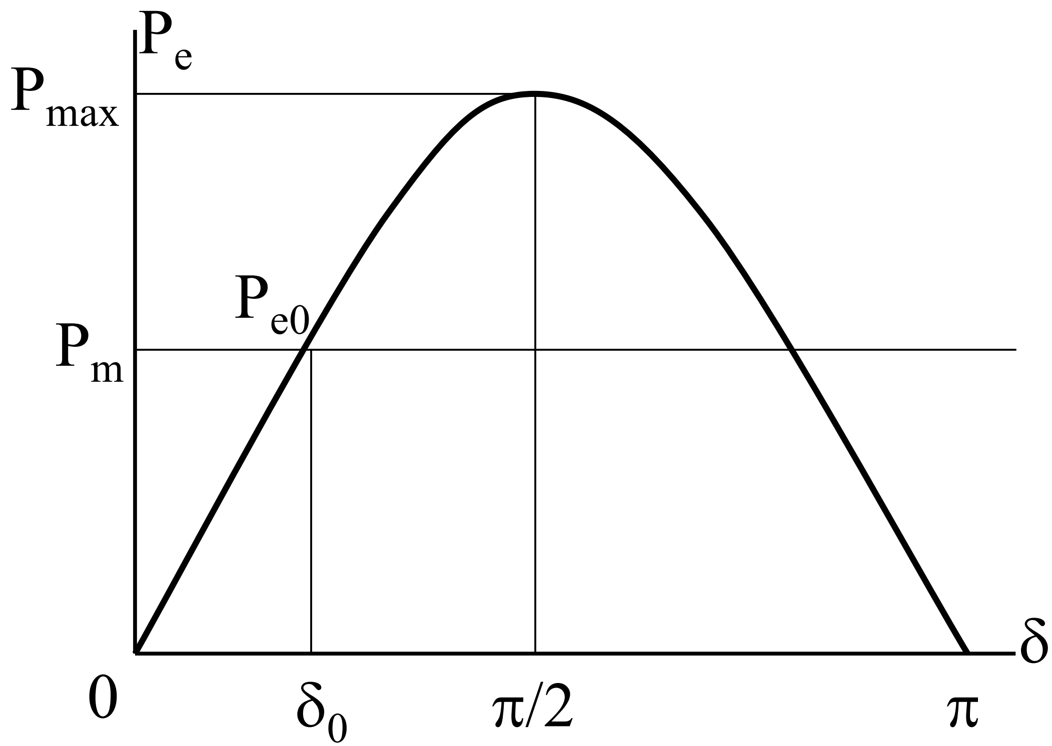
$$\frac{2\,H}{\omega_s}\frac{d^2\delta}{dt^2} = P_{m(pu)} - P_{e(pu)}$$

$$\rightarrow \quad \frac{H}{\pi\ f}\frac{d^2\delta}{dt^2} = P_{m(pu)} - P_{e(pu)} \qquad (radians)$$

$$\rightarrow \quad \frac{H}{180\ f}\frac{d^2\delta}{dt^2} = P_{m(pu)} - P_{e(pu)} \qquad (degrees)$$

Synchronous Machine Model





power angle curve

Round Rotor Machine Model

$$E' = |E'| \angle \delta$$

$$V_G = |V_G| \angle 0^\circ$$

$$B = \frac{1}{X'_d}$$

$$P_e = |E'||V_G||B| \cos(\delta - 90^\circ) = \frac{|E'||V_G|}{X'_d} \sin\delta = P_{\max} \sin\delta$$

The Swing Equation

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_e$$ Dynamic Generator Model

$$P_e = P_{\max}\sin\delta$$ Synchronous Machine Model

$$\frac{H}{\pi f_0}\frac{d^2\delta}{dt^2} = P_m - P_{\max}\sin\delta$$ Forming the Swing Equation