Multi-machine Systems

- Each synchronous machine is represented by a constant voltage source behind the direct axis transient reactance
- The input powers are assumed to remain constant
- Using the pre-fault voltage, all loads are converted to equivalent admittances to ground and are assumed to remain constant
- Damping or asynchronous powers are ignored
- The mechanical rotor angle of each machine coincides with the electrical angle of the excitation voltage source

Modeling Steps

- **Solve the initial load flow and obtain the initial bus voltage magnitude and phase angle**
- **Calculate the machine currents prior to the disturbance**

$$I_{mach-i} = \frac{S^{*}_{mach-i}}{V^{*}_{mach-i}}$$

- **Obtain the voltages behind the transient reactances**

$$E'_{mach-i} = V_{mach-i} + j X'_{d} I_{mach-i}$$

- **Convert all loads to equivalent admittances**

$$y_{i0} = \frac{S^{*}_{i}}{|V_i|^2} = \frac{P_i - jQ_i}{|V_i|^2}$$

Modeling Steps

- **Combine the generator models with the network's bus admittance matrix with converted loads**

$$\begin{bmatrix} \mathbf{0} \\ \mathbf{I}_{mach} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{nn} & \mathbf{Y}_{n-mach} \\ \mathbf{Y}_{n-mach}^{T} & \mathbf{Y}_{mach-mach} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{n} \\ \mathbf{E}'_{mach} \end{bmatrix}$$

- **Use kron reduction (matrix form) to remove the network buses from the matrix**

$$\mathbf{Y}_{bus}^{reduced} = \mathbf{Y}_{mach-mach} - \mathbf{Y}_{n-mach}^{T} \left[\mathbf{Y}_{nn}\right]^{-1} \mathbf{Y}_{n-mach}$$

$$\mathbf{I}_{mach} = \mathbf{Y}_{bus}^{reduced} \mathbf{E}'_{mach}$$

Modeling Steps

- **Express in terms of the machines' excitation voltages, the power output**

$$S^*_{mach-i} = E'^*_{mach-i} I_{mach-i}$$

$$P_{mach-i} = \Re\left[E'^*_{mach-i} I_{mach-i}\right]$$

$$I_{mach-i} = \sum_{j=1}^{m} E'_{mach-j} Y_{ij}$$

$$P_{mach-i} = \sum_{j=1}^{m} \left|E'_{mach-i}\right| \left|E'_{mach-j}\right| \left|Y_{ij}\right| \cos\left(\theta_{ij} - \delta_i + \delta_j\right)$$

Multi-Machine Transient Stability

- **Set initial conditions**

$$P_{1-mach-i} = \sum_{j=1}^{m} \left|E'_{mach-i}\right|\left|E'_{mach-j}\right|\left|Y_{1-ij}\right|\cos\left(\theta_{ij} - \delta_i + \delta_j\right)$$

$$P_{e-i}(\boldsymbol{\delta}) = P_{1-mach-i}$$

$$P_{m-i} = P_{e-i}(\boldsymbol{\delta})\Big|_{t=0}$$

- **Set machine inertia constants on a common system power base**

$$H_i = \frac{S_{G-i}}{S_B} H_{G-mach-i}$$

Multi-Machine Transient Stability

- **Solve the system of ODE's of the faulted network**

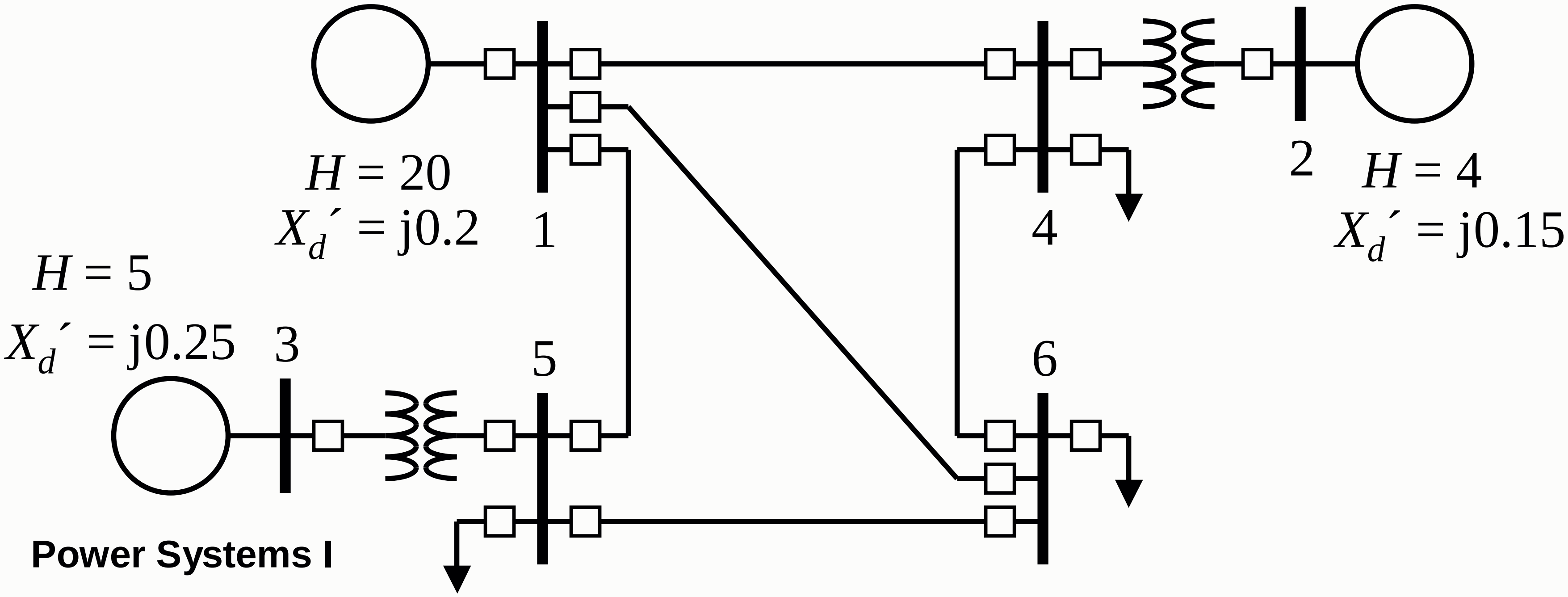
$$\frac{H_i}{\pi f_0}\frac{d^2\delta_i}{dt^2} = P_{mi} - \sum_{j=1}^{m} |E_i'||E_j'||Y_{2-ij}|\cos\left(\theta_{2-ij} - \delta_i + \delta_j\right)$$

- **Solve the system of ODE's of the post-fault network**

$$\frac{H_i}{\pi f_0}\frac{d^2\delta_i}{dt^2} = P_{mi} - \sum_{j=1}^{m} |E_i'||E_j'||Y_{3-ij}|\cos\left(\theta_{3-ij} - \delta_i + \delta_j\right)$$

Example

- **Consider the 3 machine system below**
  - select generator #1 as the swing machine with a constant angle of 0 degrees
  - determine the system stability when a fault on the line 5-6 near bus 6 is cleared in 0.4 and 0.5 seconds



Example

- **Load Data**

	P	Q
1	0	0
2	0	0
3	0	0
4	1.0	0.7
5	0.9	0.3
6	1.6	1.1

- **Generator Data**

	V	P
1	1.06	---
2	1.04	1.5
3	1.03	1.0

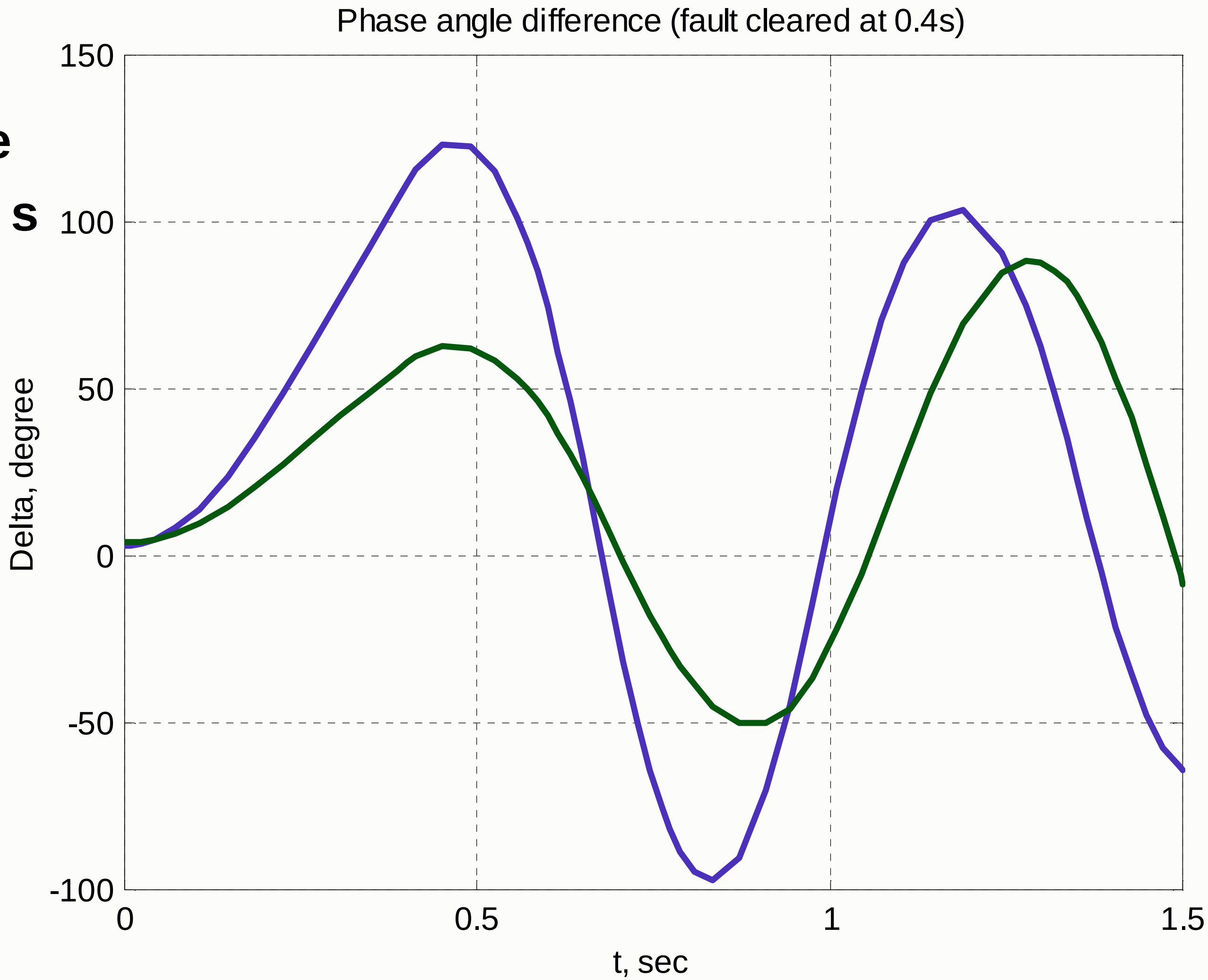
- **Line Data**

		R	X	½B
1	4	0.035	0.225	0.0065
1	5	0.025	0.105	0.0045
1	6	0.040	0.215	0.0055
2	4	0.000	0.035	0.0000
3	5	0.000	0.042	0.0000
4	6	0.028	0.125	0.0035
5	6	0.026	0.175	0.0300

Example

Power angle / time

fault clearing in 0.4 s



Example

Power angle / time
fault clearing in 0.5 s