Balanced 3-Phase Short Circuit

- **Consider a synchronous generator operating at 60 Hz with constant excitation**
- **Examine the impact on the stator currents when a three-phase short circuit is applied to the generator terminals**
  - The initial currents

$$i_a(0^+) = i_b(0^+) = i_c(0^+) = 0 \xrightarrow{Park} i_0(0^+) = i_d(0^+) = i_q(0^+) = 0$$

$$i_F(0^+) = \frac{V_F}{r_F}$$

  - The voltage after applying the fault

$$v_a = v_b = v_c = 0 \xrightarrow{Park} v_0 = v_d = v_q = 0$$

Balanced 3-Phase Short Circuit

- **Rearranging the equation and neglecting the zero sequence term**

$$\begin{bmatrix} v_d \\ -v_F \\ 0 \\ v_q \\ 0 \end{bmatrix} = -\begin{bmatrix} r & 0 & 0 & \omega L_q & \omega\sqrt{\frac{3}{2}}M_Q \\ 0 & r_F & 0 & 0 & 0 \\ 0 & 0 & r_D & 0 & 0 \\ -\omega L_d & -\omega\sqrt{\frac{3}{2}}M_F & -\omega\sqrt{\frac{3}{2}}M_D & r & 0 \\ 0 & 0 & 0 & 0 & r_Q \end{bmatrix}\begin{bmatrix} i_d \\ i_F \\ i_D \\ i_q \\ i_Q \end{bmatrix} -$$

$$-\begin{bmatrix} L_d & \sqrt{\frac{3}{2}}M_F & \sqrt{\frac{3}{2}}M_D & 0 & 0 \\ \sqrt{\frac{3}{2}}M_F & L_F & M_R & 0 & 0 \\ \sqrt{\frac{3}{2}}M_D & M_R & L_D & 0 & 0 \\ 0 & 0 & 0 & L_q & \sqrt{\frac{3}{2}}M_Q \\ 0 & 0 & 0 & \sqrt{\frac{3}{2}}M_Q & L_Q \end{bmatrix}\frac{d}{dt}\begin{bmatrix} i_d \\ i_F \\ i_D \\ i_q \\ i_Q \end{bmatrix}$$

Balanced 3-Phase Short Circuit

- **In matrix form (or state space form) the equation can be rewritten**

$$\mathbf{v} = -\mathbf{R}\mathbf{i} - \mathbf{L}\frac{d}{dt}\mathbf{i}$$

$$\frac{d}{dt}\mathbf{i} = -\mathbf{L}^{-1}\mathbf{R}\mathbf{i} - \mathbf{L}^{-1}\mathbf{v}$$

  - Using the Laplace transform or integrating the equation results in

$$\mathbf{i}_{abc} = \mathbf{P}^{-1}\,\mathbf{i}_{0dq}$$

$$i_a = i_d \cos\theta + i_q \sin\theta$$

$$i_b = i_d \cos\left(\theta - \tfrac{2\pi}{3}\right) + i_q \sin\left(\theta - \tfrac{2\pi}{3}\right)$$

$$i_c = i_d \cos\left(\theta + \tfrac{2\pi}{3}\right) + i_q \sin\left(\theta + \tfrac{2\pi}{3}\right)$$

Example

- **Consider a 500 MVA, 30 kV generator with no load and a constant excitation voltage of 400 V. A three-phase short circuit occurs at the terminals. Obtain the transient waveforms for the current in each phase and the field winding**

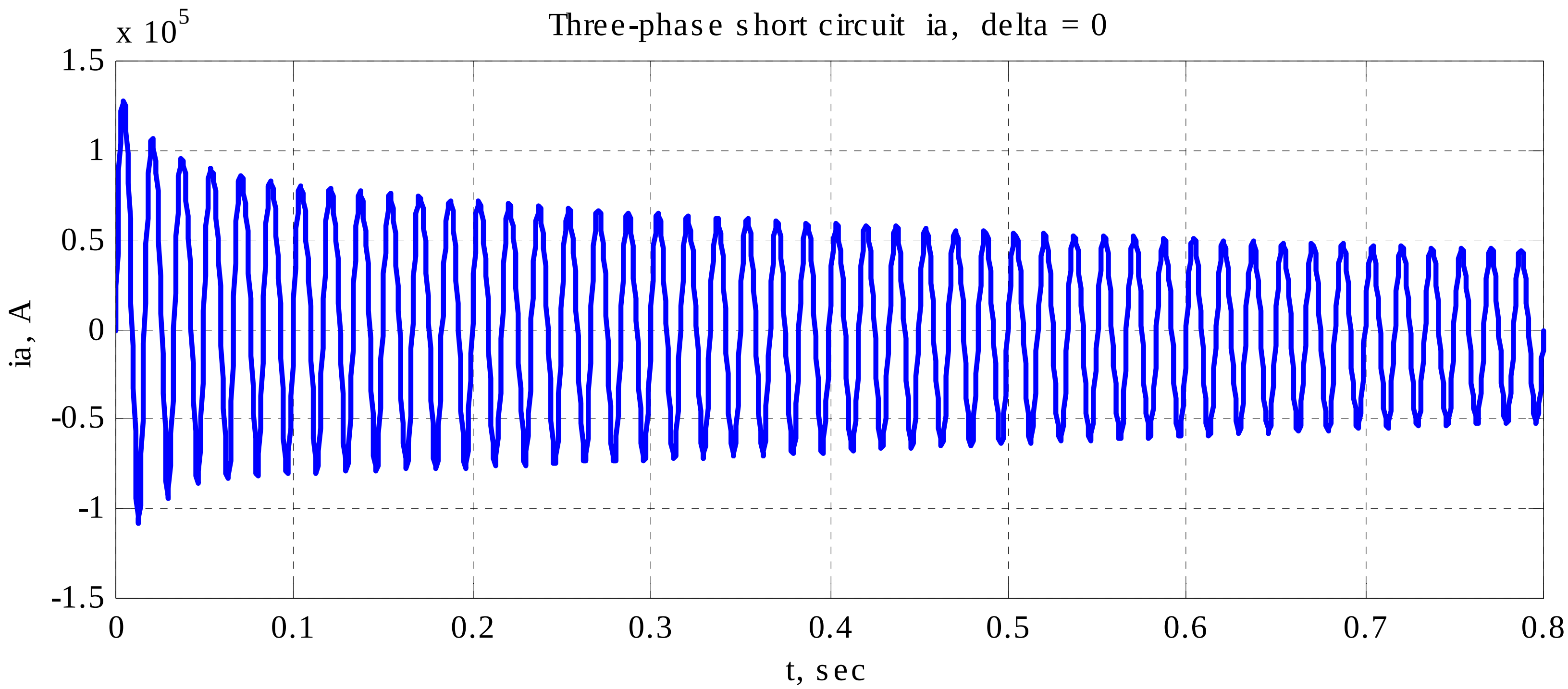
Example

<u>Generator Parameters</u>

$$L_d = 0.0072\ H \qquad L_q = 0.0070\ H \qquad L_F = 2.500\ H$$
$$L_D = 0.0068\ H \qquad L_Q = 0.0016\ H \qquad M_F = 0.100\ H$$
$$M_D = 0.0054\ H \qquad M_Q = 0.0026\ H \qquad M_R = 0.125\ H$$
$$r = 0.0020\ \Omega \qquad r_Q = 0.0150\ \Omega \qquad r_F = 0.4000\ \Omega$$
$$r_D = 0.0150\ \Omega \qquad L_0 = 0.0010\ H$$

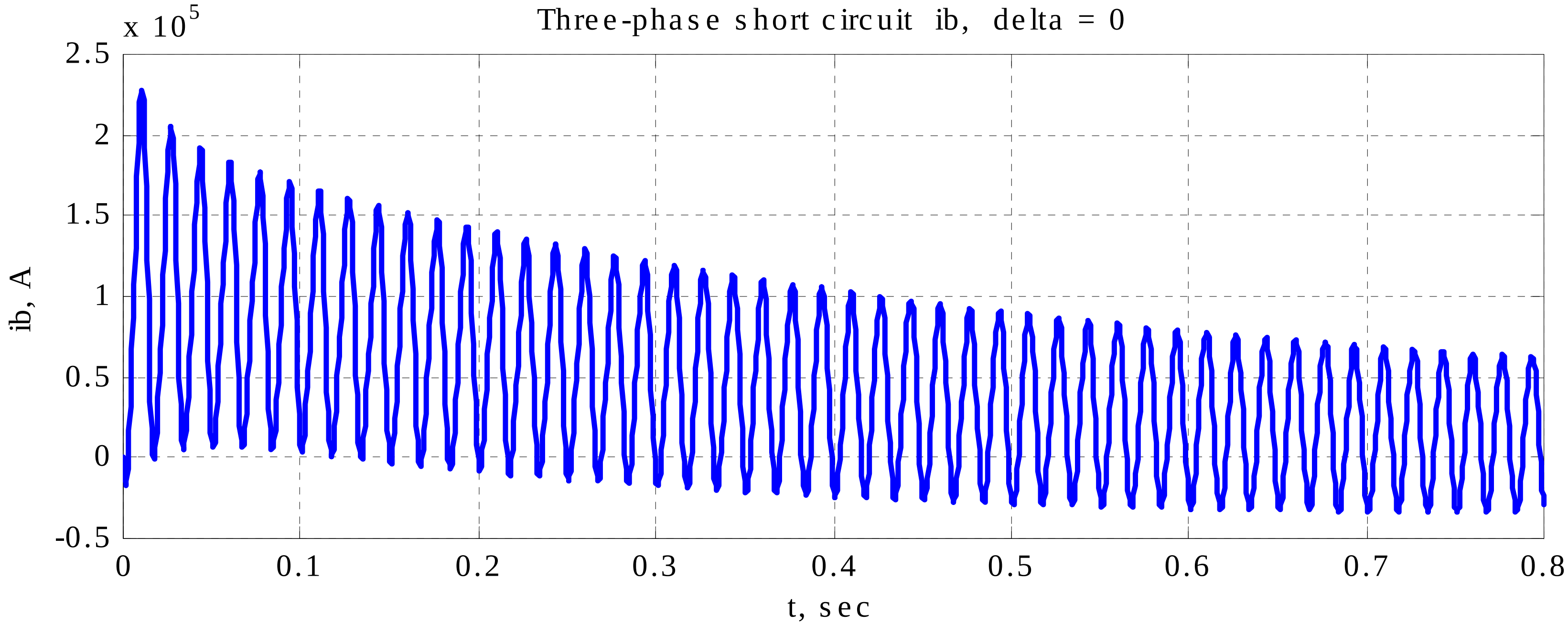
$$V_F = 400\ V$$
$$i_F\left(0^+\right) = V_F / r_F = 400/0.4 = 1000\ A$$
$$i_0\left(0^+\right) = i_d\left(0^+\right) = i_q\left(0^+\right) = 0$$

Example



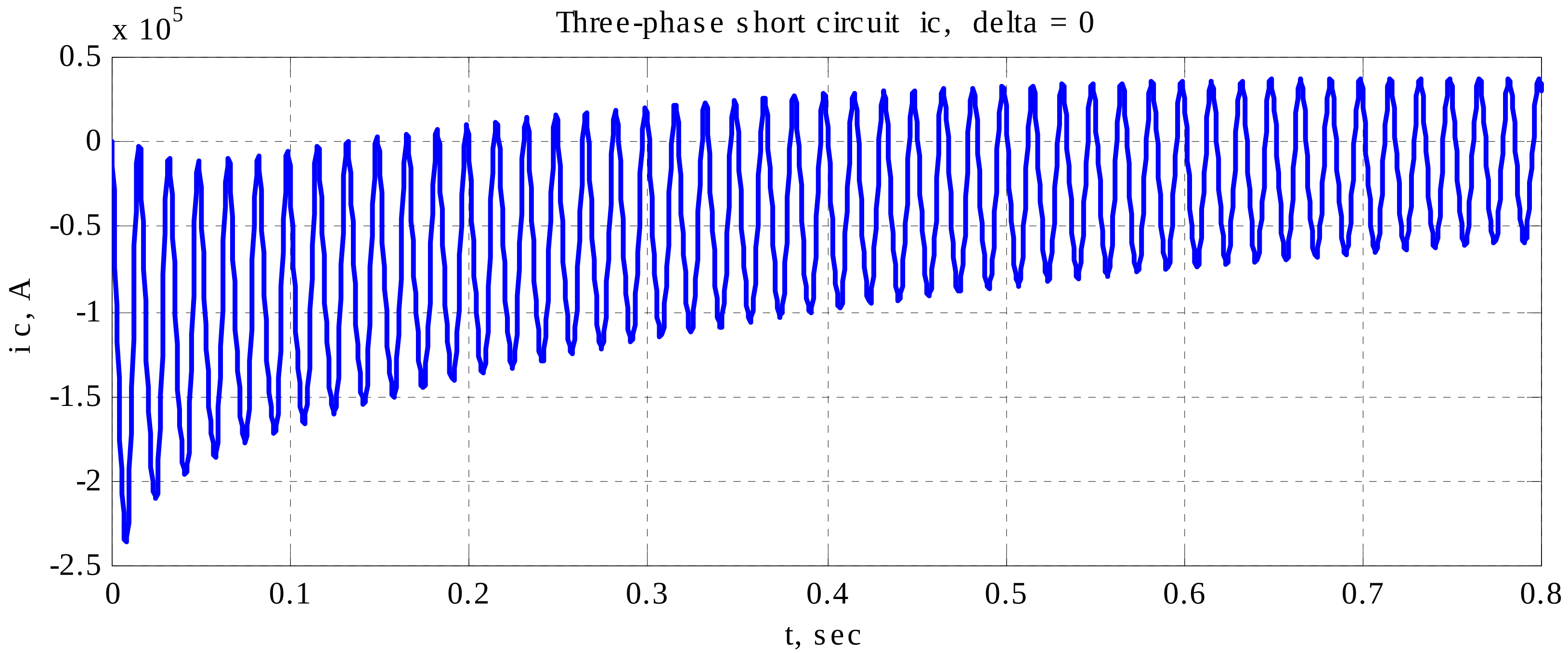
Three-phase short circuit ia, delta = 0
x 10^5
1.5
1
0.5
0
-0.5
-1
-1.5
ia, A
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
t, sec

Example



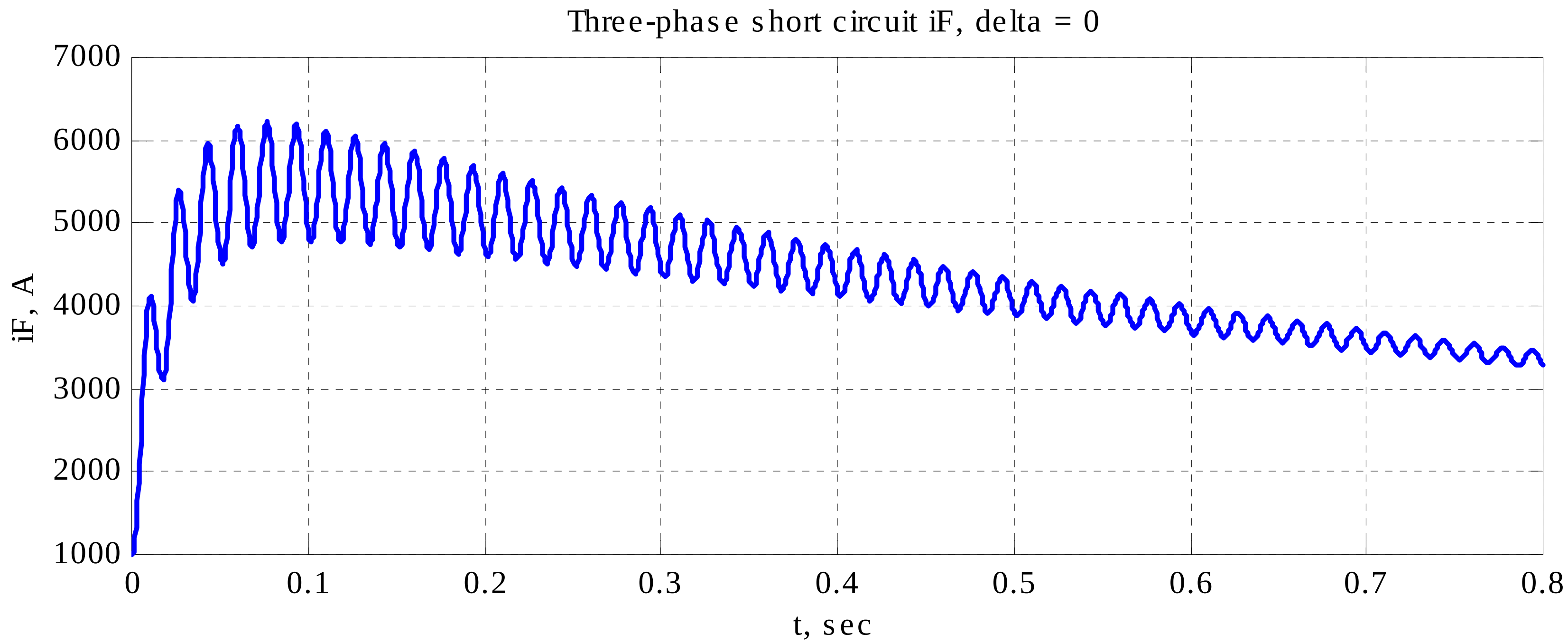
Three-phase short circuit ib, delta = 0
x 10^5
2.5
2
1.5
1
0.5
0
-0.5
ib, A
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
t, sec

Example



Three-phase short circuit ic, delta = 0
x 10^5
0.5
0
-0.5
-1
-1.5
-2
-2.5
i c, A
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
t, sec

Example



Three-phase short circuit iF, delta = 0
7000
6000
5000
4000
3000
2000
1000
iF, A
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
t, sec

Simplified Machine Model

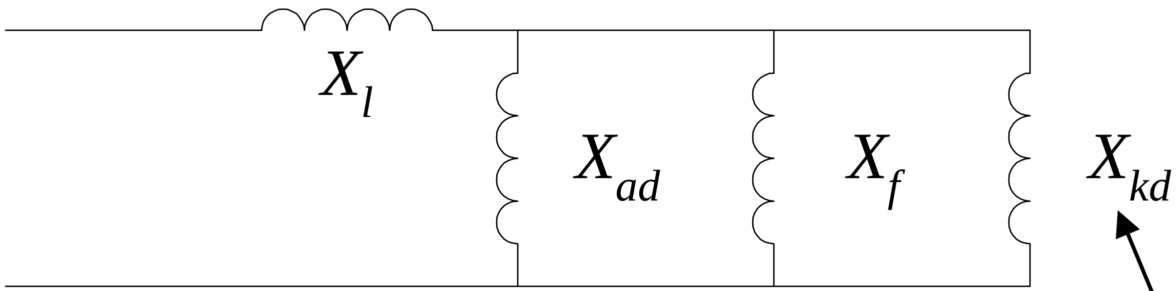
- **For steady-state operation, generators are represented with a constant emf behind a synchronous reactance, XS**
  - For salient-pole rotors, there is a direct axis and quadrature axis reactances
- **Under transient conditions, the machine reactance changes due to the effect of the armature (transformer) reaction and eddy currents in the damping circuits**
- **For analysis it is useful to imagine the synchronous reactance as three components**
  - direct axis sub-transient reactance
  - direct axis transient reactance
  - direct axis steady-state reactance
  - these transient reactances have an associated time-constant

Simplified Machine Model

- **Model visualization**
  - Consider the field and damper windings as the secondaries of a transformer (or the rotor of an induction motor)
  - The stator is the primary winding
  - For steady state conditions (synchronous speed) there is no transformer action, which can be modeled as an open circuit on the secondary side of the transformer
  - For dynamic conditions, the speed is not synchronous, and the field and damper windings look like short-circuited secondaries

Simplified Machine Model

- **The direct axis sub-transient reactance**
  - circuit model



  - equations

$$X''_d = X_l + \left(\frac{1}{X_{ad}} + \frac{1}{X_f} + \frac{1}{X_{kd}} \right)^{-1}$$

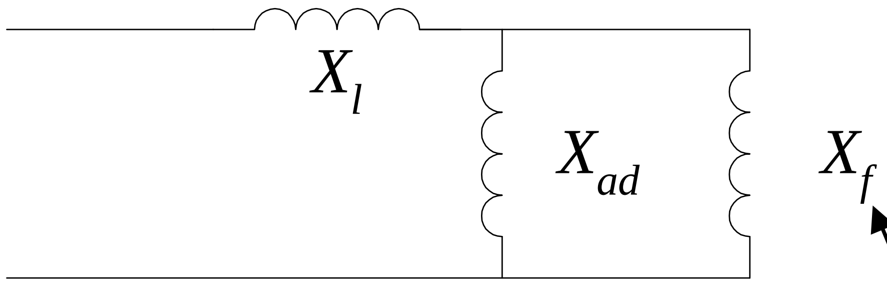
time constant
is very small,
around 0.035s

$$\tau''_d = \frac{X_{kd} + \left(\frac{1}{X_{ad}} + \frac{1}{X_f} + \frac{1}{X_l} \right)^{-1}}{R_k}$$

damper winding reactance and resistance

Simplified Machine Model

- **The direct axis transient reactance**
  - circuit model



  - equations

$$X'_d = X_l + \left(\frac{1}{X_{ad}} + \frac{1}{X_f} \right)^{-1}$$

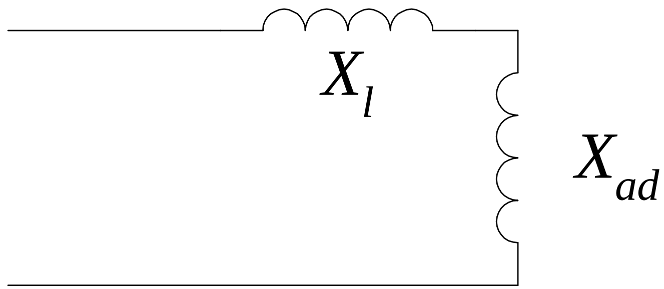
time constant
is in the order
of 1s to 2s

$$\tau''_d = \frac{X_f + \left(\frac{1}{X_{ad}} + \frac{1}{X_l} \right)^{-1}}{R_f}$$

field winding reactance and resistance

Simplified Machine Model

- **The direct axis steady-state reactance**
  - circuit model



  - equation

$$X_d = X_l + X_{ad}$$

  - equivalent circuit for the steady state

Simplified Machine Model

- **Similar models are used for the quadrature axis:**
  - quadrature axis sub-transient reactance, X_q''
  - quadrature axis transient reactance, X_q'
  - quadrature axis steady-state reactance, X_q
- **For an unloaded generator, the stator current following the occurrance of a short-circuit on the terminals:**

$$i_{ac}(t) = \sqrt{2}\, E_0 \left[\left(\frac{1}{X_d''} - \frac{1}{X_d'} \right) e^{-t/\tau_d''} + \left(\frac{1}{X_d'} - \frac{1}{X_d} \right) e^{-t/\tau_d'} + \frac{1}{X_d} \right] \sin(\omega t + \delta)$$

Simplified Machine Model

- **Example**
  - a three-phase, 60 Hz machine has the stator windings initially open-circuited, and the field current adjusted so that the terminal voltage is at rated value (i.e., 1.0 pu)
  - The machine has the following time constants:

    $X_d'' = 0.15$ pu $\qquad t_d'' = 0.035$ sec

    $X_d' = 0.40$ pu $\qquad t_d' = 1.0$ sec

    $X_d = 1.20$ pu
  - Determine the subtransient, transient, and steady state short-circuit currents

Simplified Machine Model

$$I_d'' = \frac{E_0}{X_d''} = \frac{1.0}{0.15} = 6.666\,\mathrm{pu}$$

$$I_d' = \frac{E_0}{X_d'} = \frac{1.0}{0.4} = 2.5\,\mathrm{pu}$$

$$I_d = \frac{E_0}{X_d} = \frac{1.0}{1.2} = 0.8333\,\mathrm{pu}$$