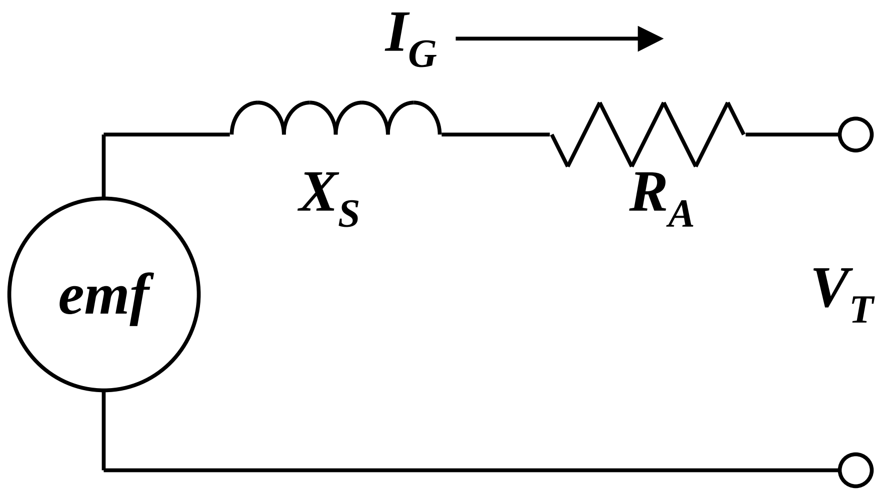
Chapter 8: Transient Analysis of Synchronous Machines

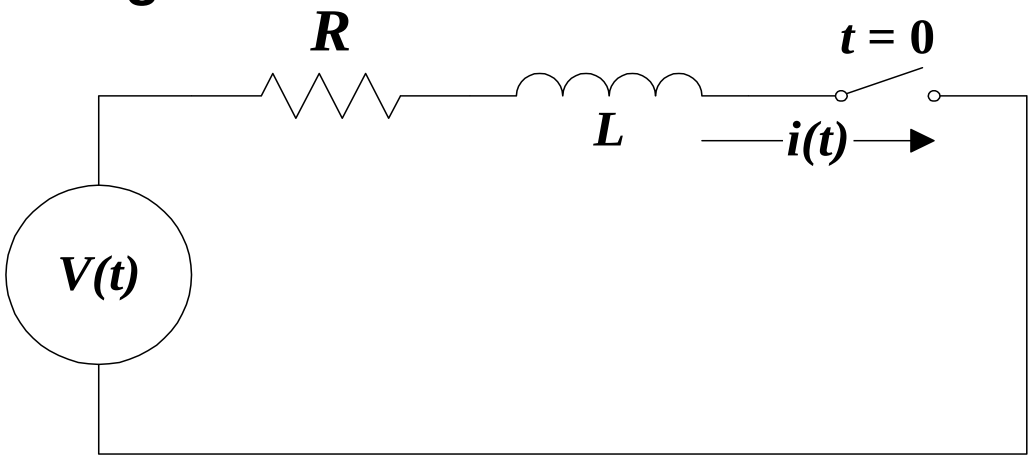
Synchronous Machines

- **Steady state modeling**
  - rotor *mmf* and stator *mmf* are stationary with respect to each other
  - flux linkage with the rotor are invariant with time
  - no voltages are induced on the rotor circuits
- **Transient modeling**
  - flux linkage changes with time
  - differential equations have time-varying coefficients
  - Parks transformation
  - dynamic behavior
    - sub-transient period, transient period, and steady-state period



Transient Analysis

- **Transient analysis will be applied in the dynamic study of generators**
- **Generators experience dynamic behavior during**
  - switching load
  - faults
- **Consider the transient behavior of an *RL* circuit with a switched voltage source**



Transient Analysis

- **The voltage source is sinusoidal:** $v(t) = V_m \sin(\omega t + \alpha)$
- **The KVL equation are:**

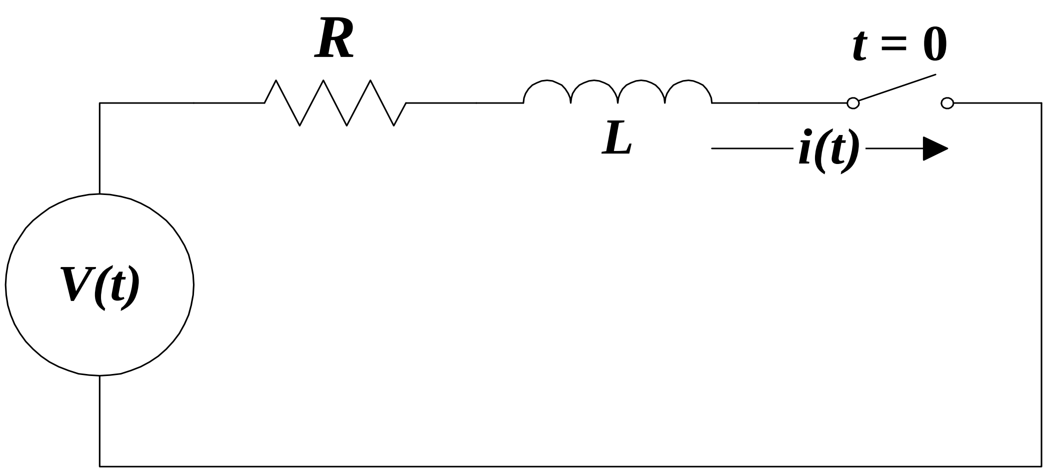
$$0 = V_m \sin(\omega t + \alpha) - R\, i(t) - L \frac{d\, i(t)}{dt}$$

$$i(t) = I_m \sin(\omega t + \alpha - \gamma) - I_m\, e^{-t/\tau} \sin(\alpha - \gamma)$$

$$\text{where:} \quad I_m = \frac{V_m}{Z}, \quad \tau = \frac{L}{R},$$

$$\gamma = \tan^{-1}\left(\frac{\omega L}{R}\right),$$

$$\text{and} \quad Z = \sqrt{R^2 + \omega^2 L^2}$$



Example

- **Solve for the time-domain solution of the current**
  - for a faulted generator having the following characteristics

    $R = 0.125\ \Omega \qquad L = 10\ \text{mH} \qquad v(t) = 151 \sin(377\, t + \alpha)$
  - which will give
    (a) zero dc offset current,
    (b) maximum dc offset current

Example

$$Z = 0.125 + j(377)(0.01) = 0.125 + j3.77 = 3.772\angle 88.1^\circ\ \Omega$$

$$I_m = \frac{151}{3.772} = 40\ \text{A} \qquad \tau = \frac{0.01}{0.125} = 0.08\ \text{s}$$

$$i(t) = 40\sin(377\,t + \alpha - 88.1^\circ) - 40\,e^{-t/0.08}\sin(\alpha - 88.1^\circ)$$

(a) Let $\alpha = 88.1^\circ \quad \Rightarrow \quad i(t) = 40\sin(377\,t)$

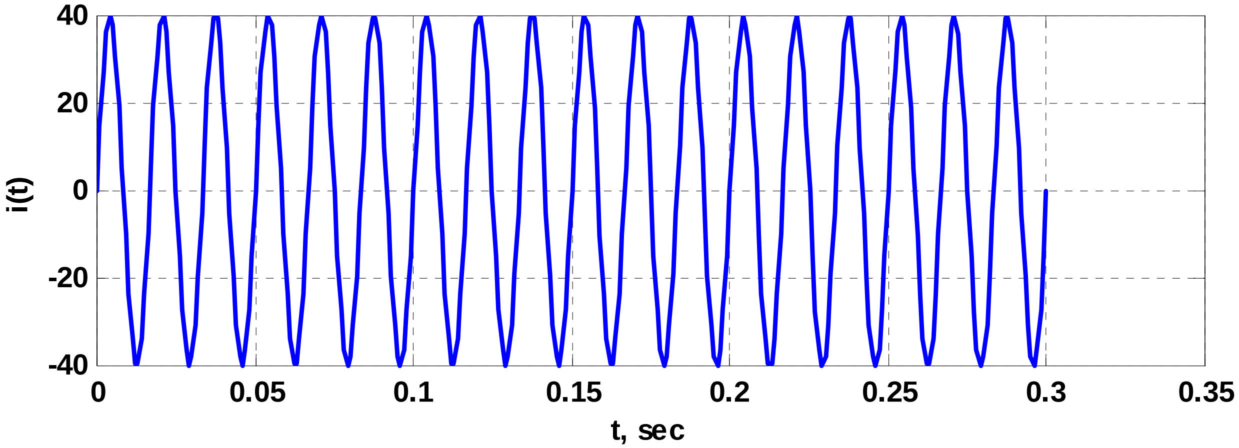
(b) Let $\alpha = (88.1^\circ - 90^\circ) = -1.9^\circ \quad \Rightarrow$

$$i(t) = 40\sin(377\,t - 90^\circ) - 40\,e^{-t/0.08}\sin(-90^\circ)$$

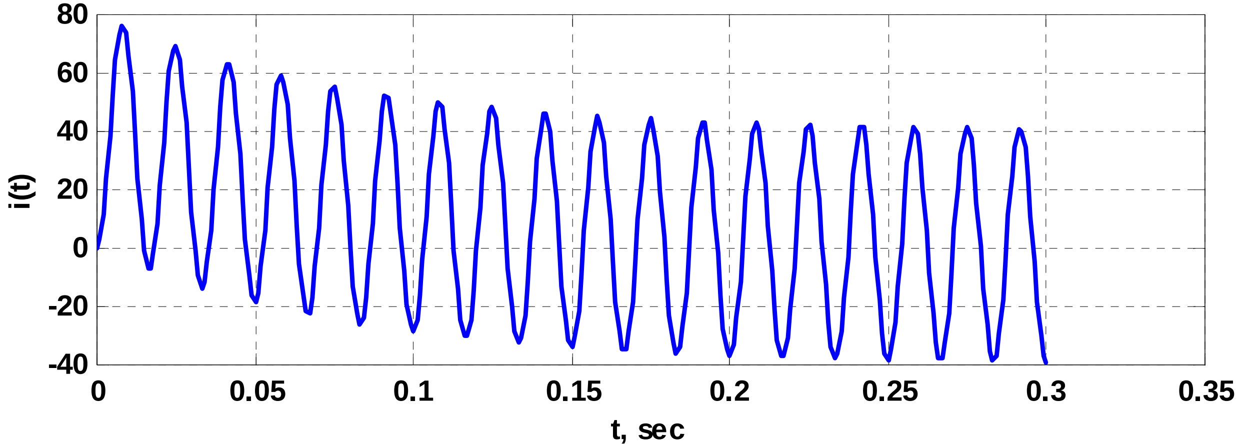
$$= -40\cos(377\,t) + 40\,e^{-t/0.08}$$

Example

(a) zero dc
offset current



(b) maximum dc
offset current



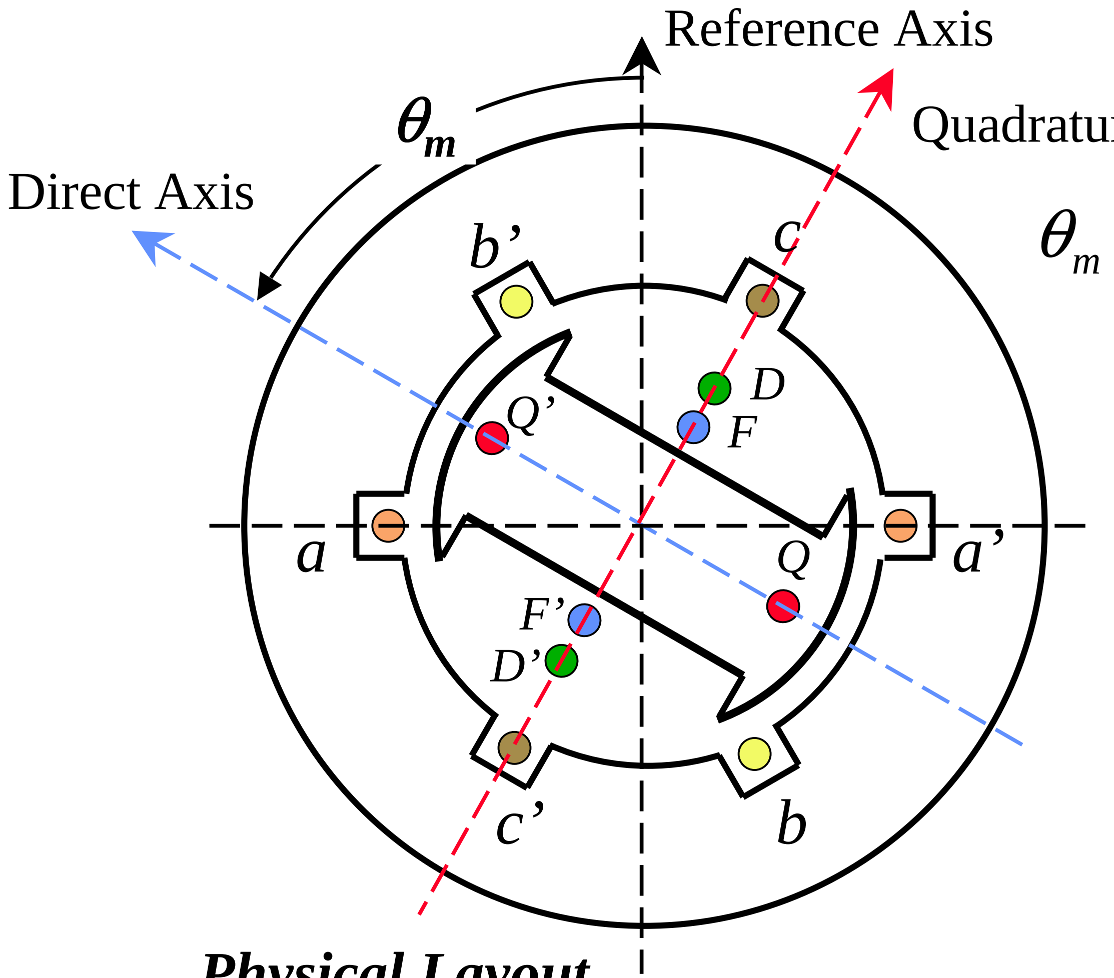
Transient Analysis

- **Synchronous Machines**
  - Models and analysis were previously developed for steady state behaviors
    - rotor and stator magnetic fields are stationary with respect to each other
    - the flux linkage in the rotor circuit are constant in time
    - the per phase equivalent circuit becomes a constant generated emf in series with a simple impedance
  - Under transient conditions (time varying) the above assumptions are no longer valid
    - changing stator current are reflected in a dynamic flux linkage
    - changing flux linkage induces transient currents in the rotor
    - transient rotor currents in turn react with the stator and the induced voltages

Synchronous Machine Model

- **The synchronous machine consist of:**
  - three ac stator windings mounted on the stator
  - one field winding mounted on the rotor
  - two fictitious windings which model short-circuited paths of the damper windings
- **When modeling, the following are assumed:**
  - a synchronously rotating reference frame with a speed of ω
  - the reference frame is along the axis of phase a at time zero
- **For transient analysis of an ideal synchronous machine**
  - The machine is represented as a group of magnetically coupled rotating circuits with inductances which depend on the angular position of the rotor

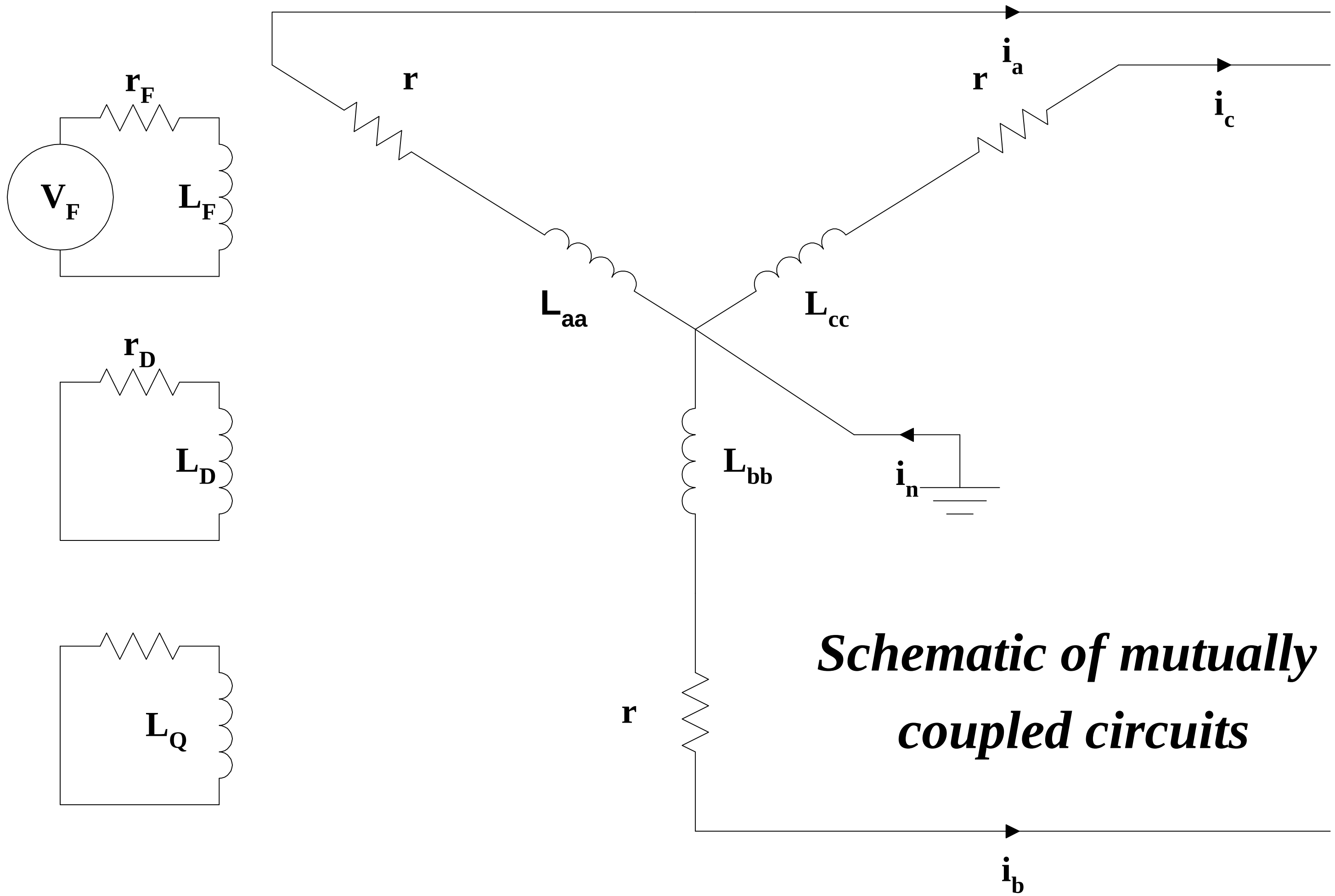
Synchronous Machine Model



$$\theta_m = \omega t + \delta + \frac{\pi}{2}$$

Physical Layout

Synchronous Machine Model



Synchronous Machine Model

- **The KVL equations for the model**

$$\begin{bmatrix} v_a \\ v_b \\ v_c \\ -v_F \\ 0 \\ 0 \end{bmatrix} = -\begin{bmatrix} r & 0 & 0 & 0 & 0 & 0 \\ 0 & r & 0 & 0 & 0 & 0 \\ 0 & 0 & r & 0 & 0 & 0 \\ 0 & 0 & 0 & r_F & 0 & 0 \\ 0 & 0 & 0 & 0 & r_D & 0 \\ 0 & 0 & 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_Q \end{bmatrix} - \frac{d}{dt} \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_F \\ \lambda_D \\ \lambda_Q \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{v}_{abc} \\ \mathbf{v}_{FDQ} \end{bmatrix} = -\begin{bmatrix} \mathbf{R}_{abc} & 0 \\ 0 & \mathbf{R}_{FDQ} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abc} \\ \mathbf{i}_{FDQ} \end{bmatrix} - \frac{d}{dt} \begin{bmatrix} \boldsymbol{\lambda}_{abc} \\ \boldsymbol{\lambda}_{FDQ} \end{bmatrix}$$

Synchronous Machine Model

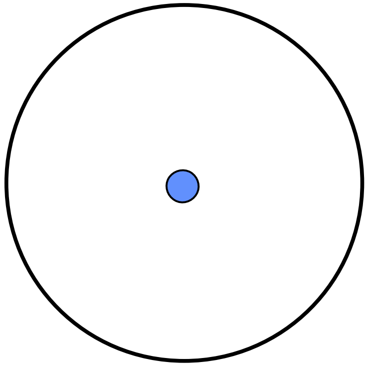
- **The magnetic inductance equations**

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \lambda_F \\ \lambda_D \\ \lambda_Q \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & L_{aF} & L_{aD} & L_{aQ} \\ L_{ba} & L_{bb} & L_{bc} & L_{bF} & L_{bD} & L_{bQ} \\ L_{ca} & L_{cb} & L_{cc} & L_{cF} & L_{cD} & L_{cQ} \\ L_{Fa} & L_{Fb} & L_{Fc} & L_{FF} & L_{FD} & L_{FQ} \\ L_{Da} & L_{Db} & L_{Dc} & L_{DF} & L_{DD} & L_{DQ} \\ L_{Qa} & L_{Qb} & L_{Qc} & L_{QF} & L_{QD} & L_{QQ} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_F \\ i_D \\ i_Q \end{bmatrix}$$

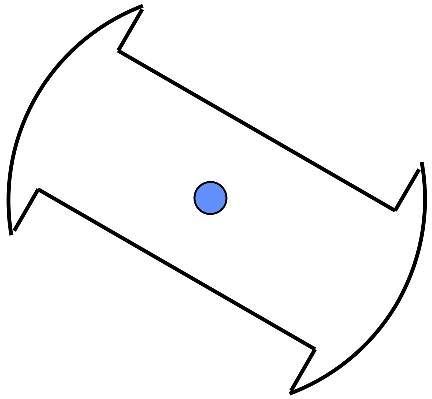
$$\begin{bmatrix} \boldsymbol{\lambda}_{abc} \\ \boldsymbol{\lambda}_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{SS} & \mathbf{L}_{SR} \\ \mathbf{L}_{RS} & \mathbf{L}_{RR} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{abc} \\ \mathbf{i}_{FDQ} \end{bmatrix}$$

Salient Pole Machines

- **Rotors have two types of construction**
  - Cylindrical
  - Salient
- **The cylindrical rotor has an evenly spaced air gap and a constant self-inductance**
- **The Salient has a non-uniform air gap and a self-inductance that varies periodically**
  - maximum inductance when the direct axis coincides with the stator coil magnetic axis
  - minimum inductance when the quadrature axis coincides with the stator coil magnetic axis



Cylindrical



Salient

Salient Pole Machines

- **The salient pole machine can be represented by cosines of second harmonics**
  - Stator quantities

$$L_{aa} = L_S + L_m \cos 2\theta$$
$$L_{bb} = L_S + L_m \cos 2\left(\theta - \tfrac{2}{3}\pi\right)$$
$$L_{cc} = L_S + L_m \cos 2\left(\theta + \tfrac{2}{3}\pi\right)$$

$$L_{ab} = L_{ba} = -M_S - L_m \cos 2\left(\theta + \tfrac{1}{6}\pi\right)$$
$$L_{bc} = L_{cb} = -M_S - L_m \cos 2\left(\theta - \tfrac{1}{2}\pi\right)$$
$$L_{ca} = L_{ac} = -M_S - L_m \cos 2\left(\theta + \tfrac{5}{6}\pi\right)$$

Salient Pole Machines

- Rotor quantities - All the rotor self inductances are constant since the effects of the stator slots are negligible

$$L_{FF} = L_F$$

$$L_{DD} = L_D$$

$$L_{QQ} = L_Q$$

$$L_{FD} = L_{DF} = M_R$$

$$L_{FQ} = L_{QF} = 0$$

$$L_{DQ} = L_{QD} = 0$$

Salient Pole Machines

- Mutual inductance between the stator and rotor circuits

$$L_{aF} = M_F \cos 2\theta$$
$$L_{bF} = M_F \cos 2\left(\theta - \tfrac{2}{3}\pi\right)$$
$$L_{cF} = M_F \cos 2\left(\theta + \tfrac{2}{3}\pi\right)$$

$$L_{aD} = M_D \cos 2\theta \qquad L_{aQ} = M_Q \sin 2\theta$$
$$L_{bD} = M_D \cos 2\left(\theta - \tfrac{2}{3}\pi\right) \qquad L_{bQ} = M_Q \sin 2\left(\theta - \tfrac{2}{3}\pi\right)$$
$$L_{cD} = M_D \cos 2\left(\theta + \tfrac{2}{3}\pi\right) \qquad L_{cQ} = M_Q \sin 2\left(\theta + \tfrac{2}{3}\pi\right)$$

Park Transformation

- **Changes the *abc* frame of reference to the *dq0* frame of reference**
  - Voltages and currents on the stator are changed to equivalent values based on the rotor's frame of reference
- **The transformation is based on the two-axis theory**
  - the electrical quantities are projections onto three new axes:
    - direct axis - along the direct axis of the rotor field winding
    - quadrature axis - tangent to the direct axis of the rotor field winding
    - zero axis - a stationary axis

$$\mathbf{P} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ \cos\theta & \cos\left(\theta - \frac{2}{3}\pi\right) & \cos\left(\theta + \frac{2}{3}\pi\right) \\ \sin\theta & \sin\left(\theta - \frac{2}{3}\pi\right) & \sin\left(\theta + \frac{2}{3}\pi\right) \end{bmatrix}$$

Park Transformation

- **The Park transformation for current**

$$\begin{bmatrix} i_0 \\ i_d \\ i_q \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ \cos\theta & \cos\left(\theta - \frac{2}{3}\pi\right) & \cos\left(\theta + \frac{2}{3}\pi\right) \\ \sin\theta & \sin\left(\theta - \frac{2}{3}\pi\right) & \sin\left(\theta + \frac{2}{3}\pi\right) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

- **Similarly applied to all electrical quantities**

$$\mathbf{i}_{0dq} = \mathbf{P}\,\mathbf{i}_{abc}$$

$$\mathbf{v}_{0dq} = \mathbf{P}\,\mathbf{v}_{abc}$$ **in matrix notation**

$$\boldsymbol{\lambda}_{0dq} = \mathbf{P}\,\boldsymbol{\lambda}_{abc}$$

Park Transformation

- **The Park transformation matix is orthogonal:**

$$\mathbf{P}^{-1} = \mathbf{P}^{\mathrm{T}} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1/\sqrt{2} & \cos\theta & \sin\theta \\ 1/\sqrt{2} & \cos\left(\theta - \frac{2}{3}\pi\right) & \sin\left(\theta - \frac{2}{3}\pi\right) \\ 1/\sqrt{2} & \cos\left(\theta + \frac{2}{3}\pi\right) & \sin\left(\theta + \frac{2}{3}\pi\right) \end{bmatrix}$$

- **Applying the Park transformation to the generator**

Park Transformation

- **Transforming the time-varying inductance to obtain a rotor frame of reference**

$$\begin{bmatrix} \boldsymbol{\lambda}_{0dq} \\ \boldsymbol{\lambda}_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{P} & 0 \\ 0 & \mathbf{I}_{3\times3} \end{bmatrix} \begin{bmatrix} \boldsymbol{\lambda}_{abc} \\ \boldsymbol{\lambda}_{FDQ} \end{bmatrix}$$

$$\text{or} \quad \begin{bmatrix} \boldsymbol{\lambda}_{abc} \\ \boldsymbol{\lambda}_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{P}^{-1} & 0 \\ 0 & \mathbf{I}_{3\times3} \end{bmatrix} \begin{bmatrix} \boldsymbol{\lambda}_{0dq} \\ \boldsymbol{\lambda}_{FDQ} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{P}^{-1} & 0 \\ 0 & \mathbf{I}_{3\times3} \end{bmatrix} \begin{bmatrix} \boldsymbol{\lambda}_{0dq} \\ \boldsymbol{\lambda}_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{SS} & \mathbf{L}_{SR} \\ \mathbf{L}_{RS} & \mathbf{L}_{RR} \end{bmatrix} \begin{bmatrix} \mathbf{P}^{-1} & 0 \\ 0 & \mathbf{I}_{3\times3} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{0dq} \\ \mathbf{i}_{FDQ} \end{bmatrix}$$

$$\begin{bmatrix} \boldsymbol{\lambda}_{0dq} \\ \boldsymbol{\lambda}_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{P} & 0 \\ 0 & \mathbf{I}_{3\times3} \end{bmatrix} \begin{bmatrix} \mathbf{L}_{SS} & \mathbf{L}_{SR} \\ \mathbf{L}_{RS} & \mathbf{L}_{RR} \end{bmatrix} \begin{bmatrix} \mathbf{P}^{-1} & 0 \\ 0 & \mathbf{I}_{3\times3} \end{bmatrix} \begin{bmatrix} \mathbf{i}_{0dq} \\ \mathbf{i}_{FDQ} \end{bmatrix}$$

Park Transformation

- **Resulting inductance matrix**

$$\begin{bmatrix} L_s - 2M_s & 0 & 0 & 0 & 0 & 0 \\ 0 & L_s + M_s + \frac{3}{2}L_m & 0 & \sqrt{\frac{3}{2}}M_F & \sqrt{\frac{3}{2}}M_D & 0 \\ 0 & 0 & L_s + M_s - \frac{3}{2}L_m & 0 & 0 & \sqrt{\frac{3}{2}}M_Q \\ 0 & \sqrt{\frac{3}{2}}M_F & 0 & L_F & M_R & 0 \\ 0 & \sqrt{\frac{3}{2}}M_D & 0 & M_R & L_D & 0 \\ 0 & 0 & \sqrt{\frac{3}{2}}M_Q & 0 & 0 & L_Q \end{bmatrix}$$

Park Transformation

- **Applying the transformation to the machine model KVL**

$$\begin{bmatrix} \mathbf{v}_{abc} \\ \mathbf{v}_{FDQ} \end{bmatrix} = -\begin{bmatrix} \mathbf{R}_{abc} & 0 \\ 0 & \mathbf{R}_{FDQ} \end{bmatrix}\begin{bmatrix} \mathbf{i}_{abc} \\ \mathbf{i}_{FDQ} \end{bmatrix} - \frac{d}{dt}\begin{bmatrix} \boldsymbol{\lambda}_{abc} \\ \boldsymbol{\lambda}_{FDQ} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{P}^{-1} & 0 \\ 0 & \mathbf{I}_{3\times3} \end{bmatrix}\begin{bmatrix} \mathbf{v}_{0dq} \\ \mathbf{v}_{FDQ} \end{bmatrix} = -\begin{bmatrix} \mathbf{R}_{abc} & 0 \\ 0 & \mathbf{R}_{FDQ} \end{bmatrix}\begin{bmatrix} \mathbf{P}^{-1} & 0 \\ 0 & \mathbf{I}_{3\times3} \end{bmatrix}\begin{bmatrix} \mathbf{i}_{0dq} \\ \mathbf{i}_{FDQ} \end{bmatrix} - \frac{d}{dt}\begin{bmatrix} \mathbf{P}^{-1} & 0 \\ 0 & \mathbf{I}_{3\times3} \end{bmatrix}\begin{bmatrix} \boldsymbol{\lambda}_{0dq} \\ \boldsymbol{\lambda}_{FDQ} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{v}_{0dq} \\ \mathbf{v}_{FDQ} \end{bmatrix} = -\begin{bmatrix} \mathbf{P} & 0 \\ 0 & \mathbf{I}_{3\times3} \end{bmatrix}\begin{bmatrix} \mathbf{R}_{abc} & 0 \\ 0 & \mathbf{R}_{FDQ} \end{bmatrix}\begin{bmatrix} \mathbf{P}^{-1} & 0 \\ 0 & \mathbf{I}_{3\times3} \end{bmatrix}\begin{bmatrix} \mathbf{i}_{0dq} \\ \mathbf{i}_{FDQ} \end{bmatrix} - \begin{bmatrix} \mathbf{P} & 0 \\ 0 & \mathbf{I}_{3\times3} \end{bmatrix}\frac{d}{dt}\begin{bmatrix} \mathbf{P}^{-1} & 0 \\ 0 & \mathbf{I}_{3\times3} \end{bmatrix}\begin{bmatrix} \boldsymbol{\lambda}_{0dq} \\ \boldsymbol{\lambda}_{FDQ} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{v}_{0dq} \\ \mathbf{v}_{FDQ} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{abc} & 0 \\ 0 & \mathbf{R}_{FDQ} \end{bmatrix}\begin{bmatrix} \mathbf{i}_{0dq} \\ \mathbf{i}_{FDQ} \end{bmatrix} - \begin{bmatrix} \mathbf{P}\frac{d}{dt}\mathbf{P}^{-1} & 0 \\ 0 & \mathbf{I}_{3\times3} \end{bmatrix}\begin{bmatrix} \boldsymbol{\lambda}_{0dq} \\ \boldsymbol{\lambda}_{FDQ} \end{bmatrix} - \frac{d}{dt}\begin{bmatrix} \boldsymbol{\lambda}_{0dq} \\ \boldsymbol{\lambda}_{FDQ} \end{bmatrix}$$

Park Transformation

- **Evaluate the expression** $\mathbf{P}\frac{d}{dt}\mathbf{P}^{-1}$

$$\mathbf{P}\tfrac{d}{dt}\mathbf{P}^{-1}$$

$$= \tfrac{2}{3}\omega \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ \cos\theta & \cos\left(\theta - \tfrac{2\pi}{3}\right) & \cos\left(\theta + \tfrac{2\pi}{3}\right) \\ \sin\theta & \sin\left(\theta - \tfrac{2\pi}{3}\right) & \sin\left(\theta + \tfrac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} 0 & -\sin\theta & \cos\theta \\ 0 & -\sin\left(\theta - \tfrac{2\pi}{3}\right) & \cos\left(\theta - \tfrac{2\pi}{3}\right) \\ 0 & -\sin\left(\theta + \tfrac{2\pi}{3}\right) & \cos\left(\theta + \tfrac{2\pi}{3}\right) \end{bmatrix}$$

$$= \omega \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

Park Transformation

- **Substituting the original terms into the transformation**

$$\begin{bmatrix} v_0 \\ v_d \\ v_q \\ -v_F \\ 0 \\ 0 \end{bmatrix} = -\begin{bmatrix} r & 0 & 0 & 0 & 0 & 0 \\ 0 & r & \omega L_q & 0 & 0 & \omega\sqrt{\tfrac{3}{2}}M_Q \\ 0 & -\omega L_d & r & -\omega\sqrt{\tfrac{3}{2}}M_F & -\omega\sqrt{\tfrac{3}{2}}M_D & 0 \\ 0 & 0 & 0 & r_F & 0 & 0 \\ 0 & 0 & 0 & 0 & r_D & 0 \\ 0 & 0 & 0 & 0 & 0 & r_Q \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_D \\ i_Q \end{bmatrix}$$

$$-\begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & \sqrt{\tfrac{3}{2}}M_F & \sqrt{\tfrac{3}{2}}M_D & 0 \\ 0 & 0 & L_q & 0 & 0 & \sqrt{\tfrac{3}{2}}M_Q \\ 0 & \sqrt{\tfrac{3}{2}}M_F & 0 & L_F & M_R & 0 \\ 0 & \sqrt{\tfrac{3}{2}}M_D & 0 & M_R & L_D & 0 \\ 0 & 0 & \sqrt{\tfrac{3}{2}}M_Q & 0 & 0 & L_Q \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_D \\ i_Q \end{bmatrix}$$

Park Transformation

- **Observations**
  - The transformation has constant coefficients provided that the speed is assumed to be constant
  - The first equation (the zero sequence) is not coupled to the other equations, and it can be treated separately
  - While the transformation technique is a mathematical process, it gives insight into the internal phenomena of the rotor and the effects of transients