Power System Analysis

- **Fundamentals of Power Systems (EEL 3216)**
  - basic models of power apparatus,
    - transformers, synchronous machines, transmission lines
  - simple systems
    - one feeder radial to single load
- **What more is there?**
  - large interconnected systems
    - multiple loads; multiple generators
  - why have large interconnected systems?
    - reliability; economics
  - analysis of the large system
    - flow of power and currents; control and stability of the system
    - proper handling of fault conditions; economic operation

Modern Power Systems

- **Power Producer**
  - generation station
    - prime mover & generator
    - step-up transformer
- **Transmission Company**
  - HV transmission lines
  - switching stations
    - circuit breakers
    - transformers
- **Distribution Utility**
  - distribution substations
    - step-down transformers
  - MV distribution feeders
    - distribution transformers

Network Layout

- **HV Networks**
  - Large quantities of power shipped over great distances
  - Sharing of resources
    - Improved reliability
    - Economics of large scale
- **MV Networks**
  - Local distribution of power
  - Numerous systems
    - Economics of simplicity
    - Autonomous operation
- **Loads**
  - Industrial & Commercial
  - Residential

System Control

- **Network Protection**
  - Switchgear
    - instrumentation transformers
    - circuit breakers
    - disconnect switches
    - fuses
    - lightning arrestors
    - protective relays
- **Energy Management Systems**
  - Energy Control Center
    - computer control
    - SCADA - Supervisory Control And Data Acquisition

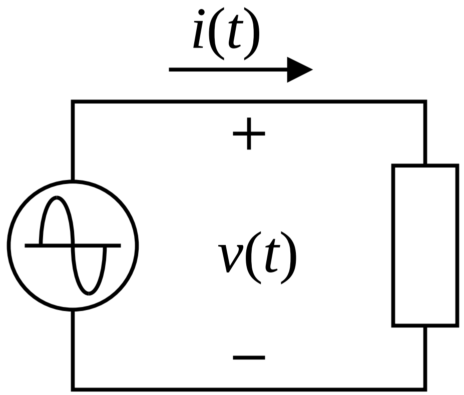
Computer Analysis

- **Practical power systems**
  - must be safe
  - reliable
  - economical
- **System Analysis**
  - for system planning
  - for system operations
  - requires component modeling
  - types of analysis
    - transmission line performance
    - power flow analysis
    - economic generation scheduling
    - fault and stability studies

Chapter 2

AC Power

Single-Phase Power Consumption



$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$p(t) = v(t)\, i(t) = V_m\; I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

$$\cos A \cos B = \tfrac{1}{2} \cos(A - B) + \tfrac{1}{2} \cos(A + B)$$

$$p(t) = \tfrac{1}{2} V_m\; I_m \; \{\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)\}$$

$$\theta = \theta_v - \theta_i \qquad V_m = \sqrt{2}\, |V| \qquad I_m = \sqrt{2}\, |I|$$

$$p(t) = \underbrace{|V|\,|I| \cos\theta\, \{1 + \cos 2(\omega t + \theta_v)\}}_{\text{energy flow into the circuit}} + \underbrace{|V|\,|I| \sin\theta \sin 2(\omega t + \theta_v)}_{\text{energy borrowed and returned by the circuit}}$$

Average Active (Real) Power

$$p(t) = |V|\,|I|\,\{1 + \cos 2(\omega t + \theta_v)\}\cos\theta + |V|\,|I|\sin 2(\omega t + \theta_v)\sin\theta$$

$$\overline{P} = \frac{1}{2\pi}\int_0^{2\pi} p(t)\,dt$$

$$= |V|\,|I|\int_0^{2\pi} \{1 + \cos 2(\omega t + \theta_v)\}\cos\theta + \sin 2(\omega t + \theta_v)\sin\theta\,dt$$

$$\int_0^{2\pi}\cos(\omega t)\,dt = 0 \qquad \int_0^{2\pi}\sin(\omega t)\,dt = 0$$

$$\overline{P} = |V|\,|I|\cos\theta$$

$$pf = \cos\theta = \frac{\overline{P}}{|V|\,|I|}$$

Apparent Power

$$\overline{P} = |V|\,|I|\cos\theta$$

$$S = |V|\,|I|$$

$$p(t) = |V|\,|I|\left\{1 + \cos 2(\omega t + \theta_v)\right\}\cos\theta + |V|\,|I|\sin 2(\omega t + \theta_v)\sin\theta$$

$$p_R(t) = |V|\,|I|\left\{1 + \cos 2(\omega t + \theta_v)\right\}\cos\theta = \overline{P}\left\{1 + \cos 2(\omega t + \theta_v)\right\}$$

$$p_X(t) = |V|\,|I|\sin 2(\omega t + \theta_v)\sin\theta = S\,\sin\theta\sin 2(\omega t + \theta_v)$$

Reactive Power

$$p_X(t) = |V||I|\sin\theta \sin 2(\omega t + \theta_v) = S\sin\theta \sin 2(\omega t + \theta_v)$$

$$Q \equiv S\sin\theta = |V||I|\sin\theta$$

$$p_X(t) = Q\sin 2(\omega t + \theta_v)$$

- **for a pure resistor**
  - the impedance angle is zero, power factor is unity
  - apparent power and real power are equal
- **for a purely inductive circuit**
  - the current lags the voltage by 90°, average power is zero
  - no transformation of energy
- **for a purely capacitive circuit**
  - the current leads the voltage by 90°, average power is zero

AC Power

- **Example**
  - the supply voltage is given by $v(t) = 480 \cos \omega t$
  - the load is inductive with impedance $Z = 1.20\angle 60^\circ\ \Omega$
  - determine the expression for the instantaneous current $i(t)$ and instantaneous power $p(t)$
  - plot $v(t)$, $i(t)$, $p(t)$, $p_R(t)$, $p_X(t)$ over an interval of 0 to 2π

Complex Power

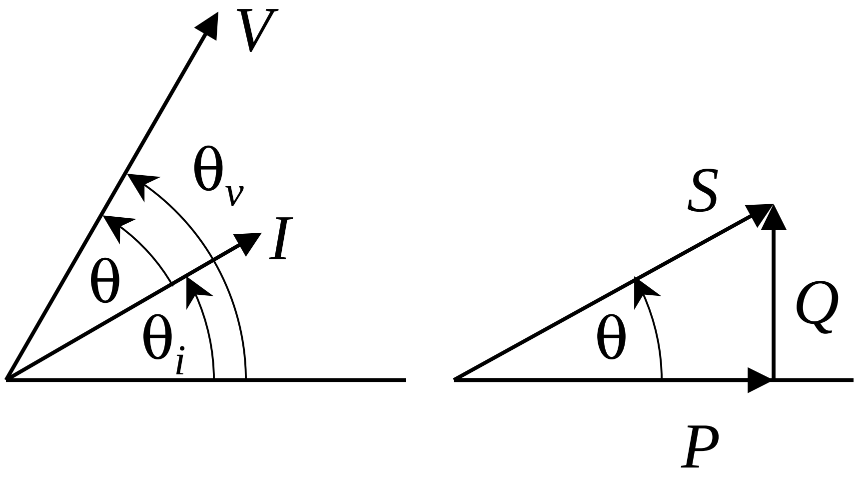
- **Real Power, P**
  - RMS based - thermally equivalent to DC power
- **Reactive Power, Q**
  - Oscillating power into and out of the load because of its reactive element (L or C).
  - Positive value for inductive load (lagging pf)
- **Complex Power, S**

$$V\ I^{*} = |V|\,|I|\angle(\theta_v - \theta_i) = |V|\,|I|\angle\theta = S$$

$$S = |V|\,|I|\cos\theta + j\,|V|\,|I|\sin\theta = P + jQ$$

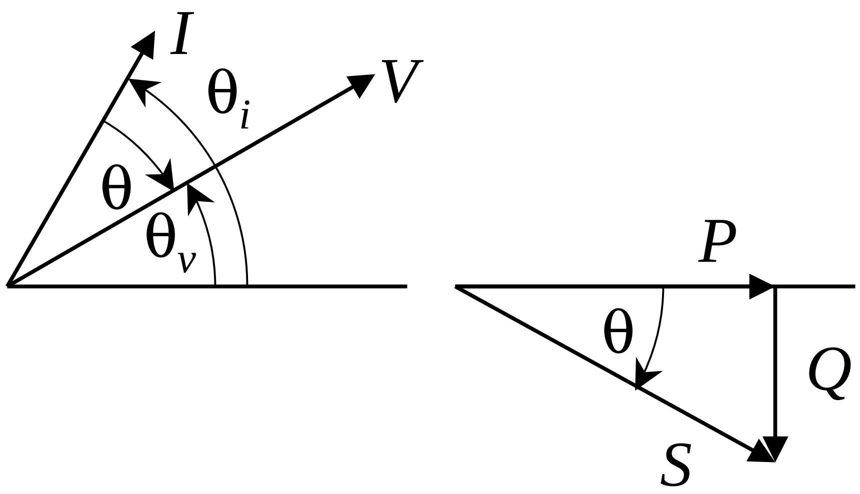
$$|S| = \sqrt{P^2 + Q^2}$$

Complex Power



Lagging Power Factor

Leading Power Factor



The Complex Power Balance

- **From the conservation of energy**
  - Real power supplied by the source is equal to the sum of the real powers absorbed by the load and the real losses in the system
  - Reactive power must also be balanced
    - The balance is between the sum of leading and the sum of lagging reactive power producing elements
  - The total complex power delivered to the loads in parallel is the sum of the complex powers delivered to each

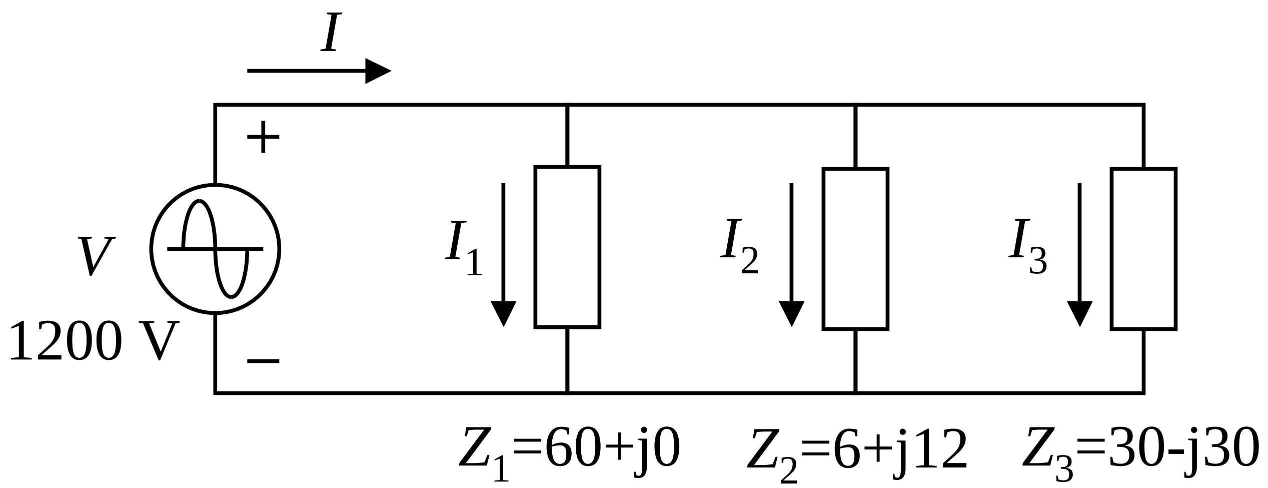
$$0 = \sum P_{gen} - \sum P_{loads} - \sum P_{losses}$$

$$0 = \sum Q_{leading} + \sum Q_{caps} - \sum Q_{lagging} - \sum Q_{ind}$$

$$0 = \sum S_{gen} - \sum S_{loads} - \sum S_{losses}$$

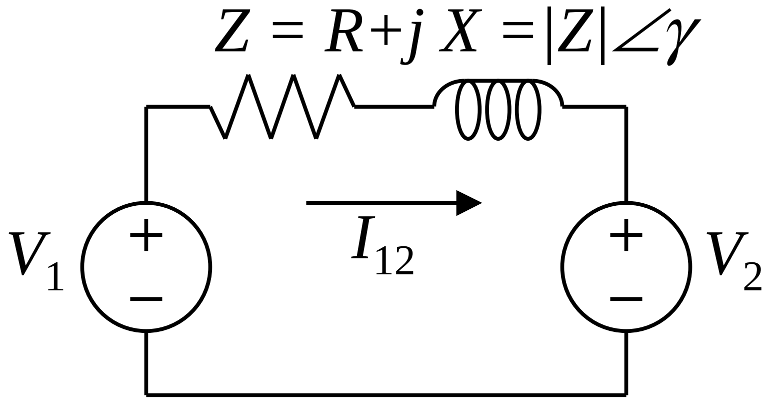
Complex Power

- **Example**
  - in the circuit below, find the power absorbed by each load and the total complex power
  - find the capacitance of the capacitor to be connected across the loads to improve the overall power factor to 0.9 lagging



Complex Power Flow

- **Consider the following circuit**



  - For the assumed direction of current

$$V_1 = |V_1|\angle\delta_1 \qquad V_2 = |V_2|\angle\delta_2$$

$$I_{12} = \frac{|V_1|\angle\delta_1 - |V_2|\angle\delta_2}{|Z|\angle\gamma} = \frac{|V_1|}{|Z|}\angle(\delta_1 - \gamma) - \frac{|V_2|}{|Z|}\angle(\delta_2 - \gamma)$$

  - The complex power

$$S_{12} = V_1\, I_{12}^* = |V_1|\angle\delta_1\left[\frac{|V_1|}{|Z|}\angle(\gamma - \delta_1) - \frac{|V_2|}{|Z|}\angle(\gamma - \delta_2)\right]$$

$$= \frac{|V_1|^2}{|Z|}\angle\gamma - \frac{|V_1||V_2|}{|Z|}\angle(\gamma + \delta_1 - \delta_2)$$

Complex Power Flow

- The real and reactive power at the sending end

$$P_{12} = \frac{|V_1|^2}{|Z|}\cos\gamma - \frac{|V_1||V_2|}{|Z|}\cos(\gamma + \delta_1 - \delta_2)$$

$$Q_{12} = \frac{|V_1|^2}{|Z|}\sin\gamma - \frac{|V_1||V_2|}{|Z|}\sin(\gamma + \delta_1 - \delta_2)$$

- Transmission lines have small resistance compared to the reactance. Often, it is assumed $R = 0$ ($Z = X\angle 90°$)

$$P_{12} = \frac{|V_1||V_2|}{X}\sin(\delta_1 - \delta_2) \qquad Q_{12} = \frac{|V_1|}{|X|}\left[\,|V_1| - |V_2|\cos(\delta_1 - \delta_2)\,\right]$$

Complex Power Flow

- **For a typical power system with small R / X ratio, the follow observations are made**
  - Small changes in δ_1 or δ_2 will have significant effect on the real power flow
  - Small changes in voltage magnitude will not have appreciable effect on the real power flow
  - Assuming no resistance, the theoretical maximum power (static transmission capacity) occurs when the angular difference, δ, is 90° and is given by:

$$P_{\max} = \frac{|V_1||V_2|}{X}$$

  - For maintaining stability, the power system operates with small load angle δ
  - The reactive power flow is determined by the magnitude difference of the terminal voltages

Three-Phase Power

- **Balanced three-phase power**
  - Assumes balanced loads
  - Assumes voltage and currents with phases that have 120° separation

$$P_{3\phi} = 3\left|V_p\right|\left|I_p\right|\cos\theta = \sqrt{3}\left|V_{LL}\right|\left|I_L\right|\cos\theta$$

$$Q_{3\phi} = 3\left|V_p\right|\left|I_p\right|\sin\theta = \sqrt{3}\left|V_{LL}\right|\left|I_L\right|\sin\theta$$

$$S_{3\phi} = 3V_p\ I_p^* = \sqrt{3}\,V_{LL}\ I_L^*$$

Chapter 3

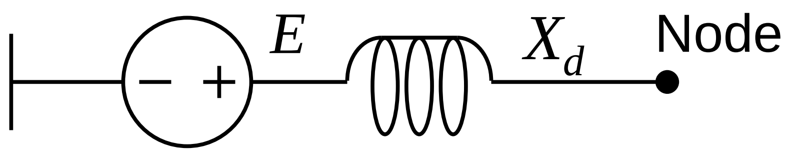
Power Apparatus Modeling

System Modeling

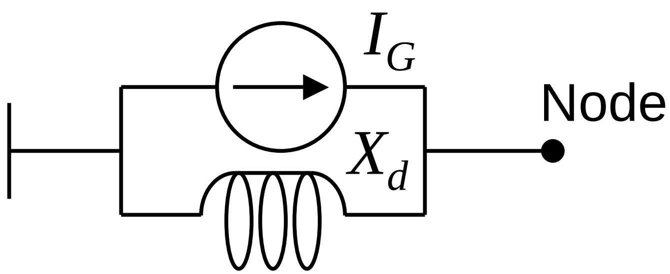
- **Systems are represented on a per-phase basis**
  - A single-phase representation is used for a balanced system
    - the system is modeled as one phase of a wye-connected network
  - Symmetrical components are used for unbalanced systems
    - unbalance systems may be caused by: generation, network components, loads, or unusual operating conditions such as faults
  - The per-unit system of measurements is used
- **Review of basic network component models**
  - generators
  - transformers
  - loads
  - transmission lines

Generator Models

- **Generator may be modeled in three different ways**
  - Power Injection Model - the real, P, and reactive, Q, power of the generator is specified at the node that the generator is connected
    - either the voltage or injected current is specified at the connected node, allowing the other quantity to be determined
  - Thevenin Model - induced AC voltage, E, behind the synchronous reactance, X_d

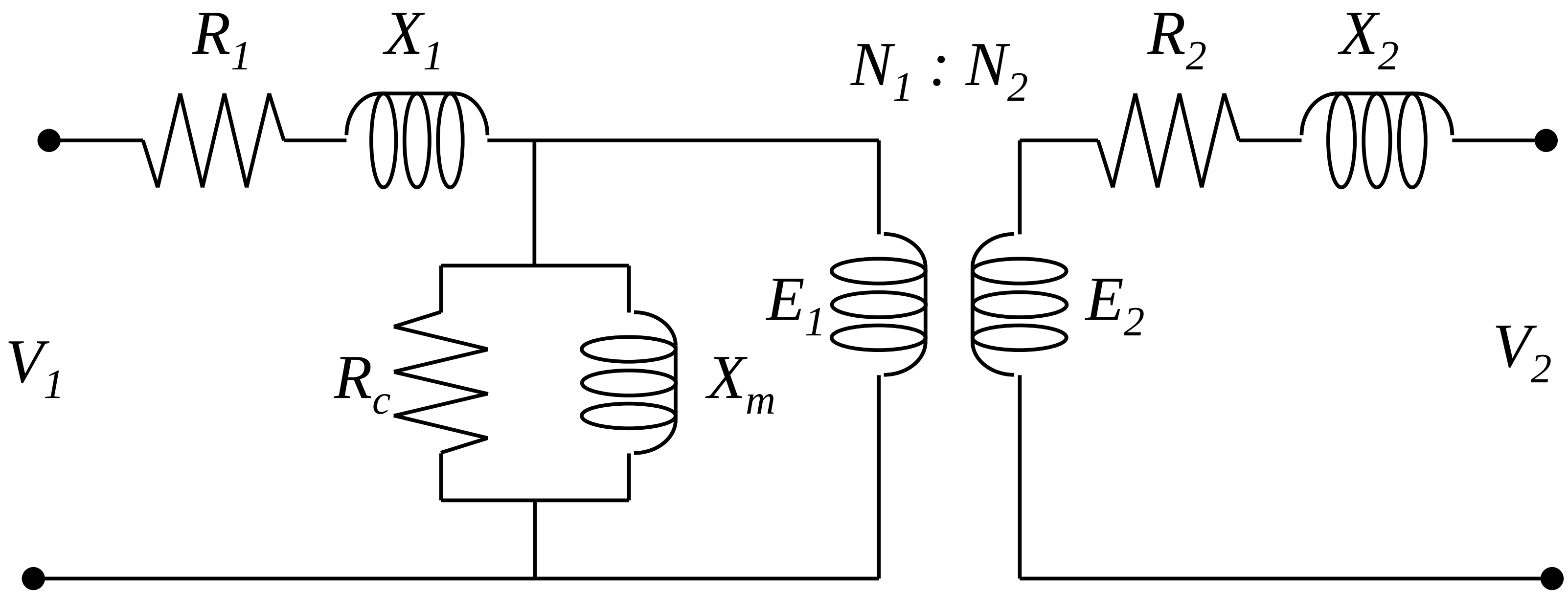


  - Norton Model - injected AC current, I_G, in parallel with the synchronous reactance



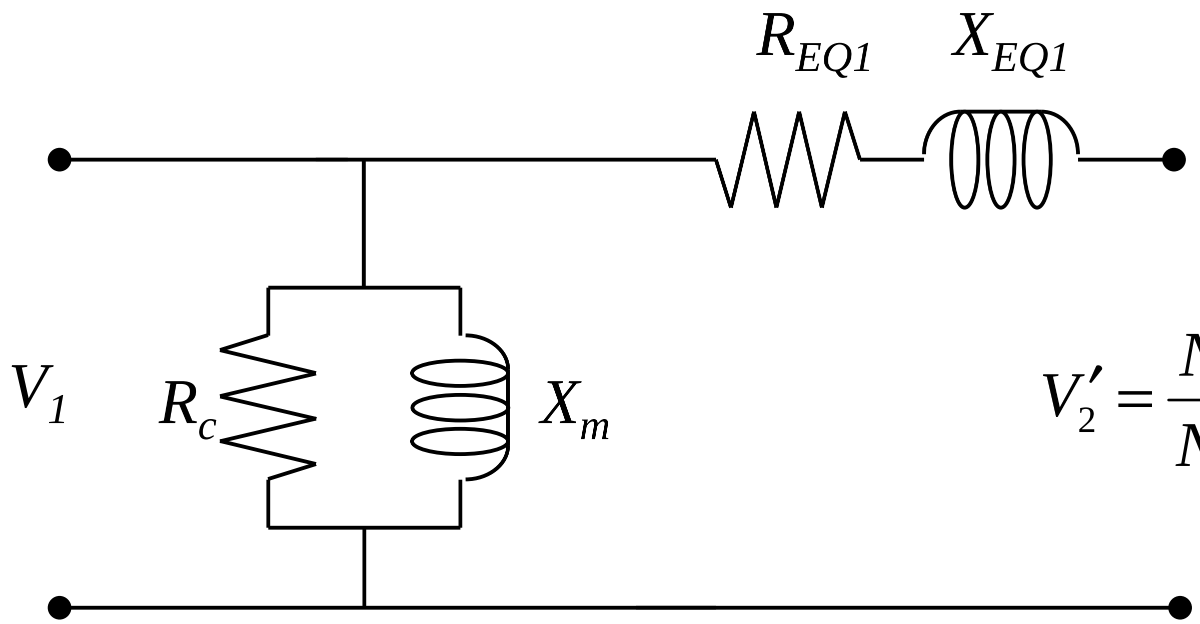
Transformer Model

- **Equivalent circuit of a two winding transformer**



Transformer Model

- **Approximate circuit referred to the primary**



$$V_2' = \frac{N_1}{N_2} V_2$$

Load Models

- **Models are selected based on both the type of analysis and the load characteristics**
- **Constant impedance,** Z_{load}
  - Load is made up of R, L, and C elements connected to a network node and the ground (or neutral point of the system)
- **Constant current,** I_{load}
  - The load has a constant current magnitude I, and a constant power factor, independent of the nodal voltage
  - Also considered as a current injection into the network
- **Constant power,** S_{load}
  - The load has a constant real, P, and reactive, Q, power component independent of nodal voltage or current injection
  - Also considered as a negative power injection into the network

Per Unit System

- **Almost all power system analyses are performed in per-units**

$$x_{per\ unit}(pu) = \frac{x_{engineering\,(actual)}(engr.\,unit)}{x_{base}(engr.\,unit)} = \frac{x\%}{100}$$

- **Per unit system for power systems**
  - Based on a per-phase, wye-connect, three-phase system
  - 3-phase power base, $S_{3\phi}$
    - common power base is 100 MVA
  - Line-to-line voltage base, V_{LL}
    - voltage base is usually selected from the equipment rated voltage
  - Phase current base, I_L
  - Phase impedance base, Z

$$I_{L-base} = \frac{S_{3\phi-base}}{\sqrt{3}\,V_{LL-base}}$$

$$Z_{base} = \frac{(V_{LL-base})^2}{S_{3\phi-base}} = \frac{(V_{LN-base})^2}{S_{1\phi-base}}$$

Per Unit System

- **Equipment impedances are frequently given in per units or percentages of the impedance base**
  - The impedance base for equipment is derived from the rated power and the rated voltage
  - When modeling equipment in a system, the per unit impedance must be converted so that the equipment and the system are on a common base

$$Z_{pu}^{old} = \frac{Z_\Omega}{Z_{base}^{old}} = Z_\Omega \frac{S_{base}^{old}}{\left(V_{base}^{old}\right)^2} \qquad Z_{pu}^{new} = \frac{Z_\Omega}{Z_{base}^{new}} = Z_\Omega \frac{S_{base}^{new}}{\left(V_{base}^{new}\right)^2}$$

$$Z_{pu}^{new} = \frac{S_{base}^{new}}{\left(V_{base}^{new}\right)^2} \cdot \frac{\left(V_{base}^{old}\right)^2}{S_{base}^{old}} \cdot Z_{pu}^{old} = Z_{pu}^{old} \frac{S_{base}^{new}}{S_{base}^{old}} \cdot \left(\frac{V_{base}^{old}}{V_{base}^{new}}\right)^2$$

  - It is normal for the voltage bases to be the same: $Z_{pu}^{new} = Z_{pu}^{old} \dfrac{S_{base}^{new}}{S_{base}^{old}}$

Per Unit System

- **The advantages of the per unit system for analysis**
  - Gives a clear idea of relative magnitudes of various quantities
  - The per-unit impedance of equipment of the same general type based upon their own ratings fall in a narrow range regardless of the rating of equipment.
    - Whereas their impedances in ohms vary greatly with the ratings.
  - The per-unit impedance, voltages, and currents of transformers are the same regardless of whether they are referred to the primary or the secondary side.
    - Different voltage levels disappear across the entire system.
    - The system reduces to a system of simple impedances
  - The circuit laws are valid in per-unit systems, and the power and voltages equations are simplified since the factors of $\sqrt{3}$ and 3 are eliminated in the per-unit system

Per Unit System

- **Example**
  - the one-line diagram of a three-phase power system is shown
  - use a common base of 100 MVA and 22 kV at the generator
    - draw an impedance diagram with all impedances marked in per-unit
    - the manufacturer's data for each apparatus is given as follows

G:	90 MVA	22 kV	18%
T1:	50 MVA	22/220 kV	10%
L1:	48.4 ohms		
T2:	40 MVA	220/11 kV	6%
T3:	40 MVA	22/110 kV	6.4%
L2:	65.43 ohms		
T4:	40 MVA	110/11 kV	8%
M:	66.5 MVA	10.45 kV	18.5%
Ld:	57 MVA	10.45 kV	0.6 pf lag