Improving Power-Angle Characteristics

$$P_S + jQ_S = V_S I_S^* = V\angle\delta\left[\frac{V\angle-\delta - V\angle-(\delta/2)}{-jX/2}\right] = \frac{V^2 - V\angle(\delta/2)}{-jX/2}$$
$$= \frac{2V^2\sin(\delta/2)}{X} + j\frac{2V^2\{1-\cos(\delta/2)\}}{X} \tag{10.7}$$

The apparent power supplied by the source is given by

$$P_R + jQ_R = V_R I_R^* = V\left[\frac{V\angle-(\delta/2) - V}{-jX/2}\right]$$
$$= \frac{2V^2\sin(\delta/2)}{X} + j\frac{2V^2\{\cos(\delta/2)-1\}}{X} \tag{10.8}$$

Similarly the apparent power delivered at the receiving end is

$$P_e = P_S = P_R = \frac{2V^2}{X}\sin(\delta/2) \tag{10.9}$$

Hence the real power transmitted over the line is given by

$$Q_e = Q_S + Q_Q - Q_R = \frac{8V^2}{X}\{1-\cos(\delta/2)\} \tag{10.10}$$

Combining (10.6)-(10.8), we find the reactive power consumed by the line as

The power-angle characteristics of the shunt compensated line are shown in Fig. 10.3. In this figure $P_{max} = V^2/X$ is chosen as the power base.

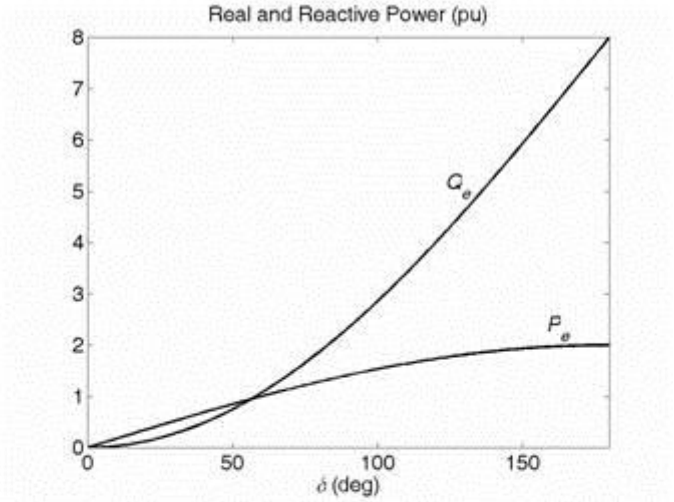


Fig. 10.3 Power-angle characteristics of ideal shunt compensated line.

Fig. 10.3 depicts P_e - δ and Q_Q - δ characteristics. It can be seen from fig 10.4 that for a real power transfer of 1 per unit, a reactive power injection of roughly 0.5359 per unit will be required from the shunt compensator if the midpoint voltage is regulated as per (10.1). Similarly for increasing the real power transmitted to 2 per unit, the shunt compensator has to inject 4 per unit of reactive power. This will obviously increase the device rating and may not be practical. Therefore power transfer enhancement using midpoint shunt compensation may not be feasible from the device rating point of view.

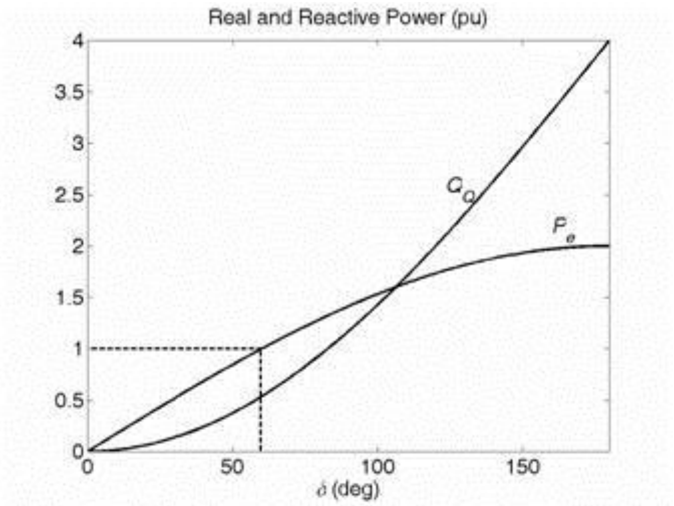


Fig. 10.4 Variations in transmitted real power and reactive power injection by the shunt compensator with load angle for perfect midpoint voltage regulation.

Let us now relax the condition that the midpoint voltage is regulated to 1.0 per unit. We then obtain some very interesting plots as shown in Fig. 10.5. In this figure, the x-axis shows the reactive power available from the shunt device, while the y-axis shows the maximum power that can be transferred over the line without violating the voltage constraint. There are three different P-Q relationships given for three midpoint voltage constraints. For a reactive power injection of 0.5 per unit, the power transfer can be increased from about 0.97 per unit to 1.17 per unit by lowering the midpoint voltage to 0.9 per unit. For a reactive power injection greater than 2.0 per unit, the best power transfer capability is obtained for V_M = 1.0 per unit. Thus there will be no benefit in reducing the voltage constraint when the shunt device is capable of injecting a large amount of reactive power. In practice, the level to which the midpoint voltage can be regulated depends on the rating of the installed shunt device as well the power being transferred.

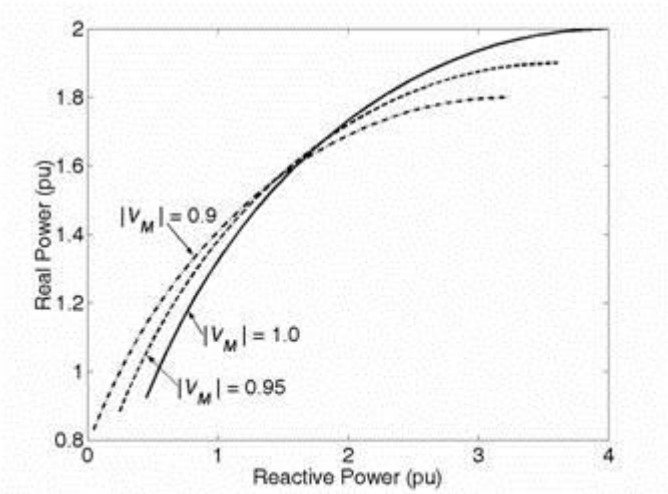
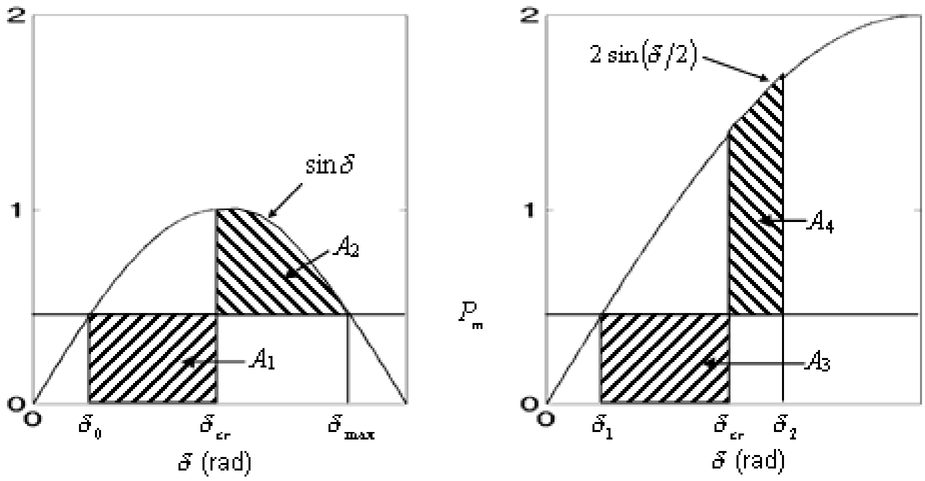


Fig. 10.5 Power transfer versus shunt reactive injection under midpoint voltage constraint.

Improving Stability Margin

This is a consequence of the improvement in the power angle characteristics and is one of the major benefits of using midpoint shunt compensation. As mentioned before, the stability margin of the system pertains to the regions of acceleration and deceleration in the power-angle curve. We shall use this concept to delineate the advantage of mid point shunt compensation.

Consider the power angle curves shown in Fig. 10.6.



The curve of Fig. 10.6 (a) is for an uncompensated system, while that of Fig. 10.6 (b) for the compensated system. Both these curves are drawn assuming that the base power is V^2/X . Let us assume that the uncompensated system is operating on steady state delivering an electrical power equal to P_m with a load angle of δ_0 when a three-phase fault occurs that forces the real power to zero. To obtain the critical clearing angle for the uncompensated system is δ_{cr} , we equate the accelerating area A_1 with the decelerating area A_2 , where

$$A_1 = \int_{\delta_0}^{\delta_{cr}} P_m dt = P_m\left(\delta_{cr} - \delta_0\right)$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} \left(\sin\,\delta - P_m\right)dt = \left(\cos\delta_{cr} - \cos\delta_{max}\right) - P_m\left(\delta_{max} - \delta_{cr}\right)$$

with $\delta_{max} = \pi - \delta_0$. Equating the areas we obtain the value of δ_{cr} as

$$\delta_{cr} = \cos^{-1}\left[P_m\left(\delta_{max} - \delta_0\right) + \cos\delta_{max}\right] \tag{10.11}$$

Let us now consider that the midpoint shunt compensated system is working with the same mechanical power input $P_{m.}$ The operating angle in this case is δ_1 and the maximum power that can be transferred in this case is 2 per unit. Let the fault be cleared at the same clearing angle δ_{cr} as before. Then equating areas A_3 and A_4 in Fig. 10.6 (b) we get δ_2, where

$$A_3 = \int_{\delta_1}^{\delta_{cr}} P_m \, dt = P_m \left(\delta_{cr} - \delta_1\right)$$

$$A_4 = \int_{\delta_{cr}}^{\delta_2} \left[2\sin\left(\delta/2\right) - P_m\right] dt = 4\left[\cos\left(\delta_{cr}/2\right) - \cos\left(\delta_2/2\right)\right] - P_m\left(\delta_2 - \delta_{cr}\right)$$

Example 10.1

Improving Damping to Power Oscillations

The swing equation of a synchronous machine is given by (9.14). For any variation in the electrical quantities, the mechanical power input remains constant. Assuming that the magnitude of the midpoint voltage of the system is controllable by the shunt compensating device, the accelerating power in (9.14) becomes a function of two independent variables, | V_M | and δ . Again since the mechanical power is constant, its perturbation with the independent variables is zero. We then get the following small perturbation expression of the swing equation

$$\frac{2H}{\omega}\frac{d^2\Delta\delta}{dt^2}+\frac{\partial P_e}{\partial |V_M|}\Delta|V_M|+\frac{\partial P_e}{\partial \delta}\Delta\delta=0 \tag{10.12}$$

where Δ indicates a perturbation around the nominal values.

If the mid point voltage is regulated at a constant magnitude, Δ| V_M | will be equal to zero. Hence the above equation will reduce to

$$\frac{2H}{\omega}\frac{d^2\Delta\delta}{dt^2}+\frac{\partial P_e}{\partial \delta}\Delta\delta=0 \tag{10.13}$$

The 2nd order differential equation given in (10.13) can be written in the Laplace domain by neglecting the initial conditions as

$$\left(\frac{2H}{\omega}s^2+\frac{\partial P_e}{\partial \delta}\right)\Delta\delta(s)=0 \qquad (10.14$$

The roots of the above equation are located on the imaginary axis of the s-plane at locations ± $j\ \omega_m$ where

$$\omega_m=\sqrt{(\omega/2H)(\partial P_e/\partial\delta)}$$

This implies that the load angle will oscillate with a constant frequency of ω_m . Obviously, this solution is not acceptable. Thus in order to provide damping, the mid point voltage must be varied according to in sympathy with the rate of change in $\Delta\delta$. We can then write

$$\Delta|V_M|=K_M\frac{d\Delta\delta}{dt} \tag{10.15}$$

where K_M is a proportional gain. Substituting (10.15) in (10.12) we get

$$\frac{2H}{\omega}\frac{d^2\Delta\delta}{dt^2}+\frac{\partial P_e}{\partial|V_M|}K_M\frac{d\Delta\delta}{dt}+\frac{\partial P_e}{\partial\delta}\Delta\delta=0 \tag{10.16}$$

Provided that K_M is positive definite, the introduction of the control action (10.15) ensures that the roots of the second order equation will have negative real parts. Therefore through the feedback, damping to power swings can be provided by placing the poles of the above equation to provide the necessary damping ratio and undamped natural frequency of oscillations.

Example 10.2

Impact of Series Compensator on Voltage Profile

In the equivalent schematic diagram of a series compensated power system is shown in Fig. 10.10, the receiving end current is equal to the sending end current, i.e., $I_S = I_R$. The series voltage V_Q is injected in such a way that the magnitude of the injected voltage is made proportional to that of the line current. Furthermore, the phase of the voltage is forced to be in quadrature with the line current. We then have

$$V_Q=\lambda I_S e^{\mp j90^{\circ}} \tag{10.20}$$

The ratio λ/X is called the **compensation level** and is often expressed in percentage. This compensation level is usually measured with respect to the transmission line reactance. For example, we shall refer the compensation level as 50% when $\lambda = X/2$. In the analysis presented below, we assume that the injected voltage lags the line current. The implication of the voltage leading the current will be discussed later.

Applying KVL we get

$$V_S-V_Q-V_R=jXI_S \quad\Rightarrow\quad V_S-V_R=\mp j\lambda I_S+jXI_S$$

Assuming $V_S = V < \delta$ and $V_R = V <0°$, we get the following expression for the line current

$$I_S=\frac{V\angle\delta-V}{j(X\mp\lambda)} \tag{10.21}$$

When we choose $V_Q = \lambda\, I_S\, e^{-j90°}$, the line current equation becomes

$$I_S=\frac{V\angle\delta-V}{j(X-\lambda)}$$

Thus we see that λ is subtracted from X . This choice of the sign corresponds to the voltage source acting as a pure capacitor. Hence we call this as the **capacitive mode of operation** . In contrast, if we choose $V_Q = \lambda I_S\, e^{+j90^\circ}$, λ is added to X , and this mode is referred to as the **inductive mode of operation** . Since this voltage injection using (10.20) add λ to or subtract λ from the line reactance, we shall refer it as voltage injection in **constant reactance mode***.* We shall consider the implication of series voltage injection on the transmission line voltage through the following example.

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Improving Voltage Profile

Let the sending and receiving voltages be given by $V\angle\delta$ and $V\angle 0^\circ$ respectively. The ideal shunt compensator is expected to regulate the midpoint voltage to

$$V_M = V\angle(\delta/2) \tag{10.1}$$

against any variation in the compensator current. The voltage current characteristic of the compensator is shown in Fig. 10.2. This ideal behavior however is not feasible in practical systems where we get a slight droop in the voltage characteristic. This will be discussed later.

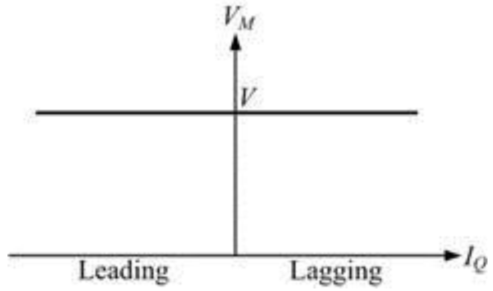


Fig. 10.2 Voltage-current characteristic of an ideal shunt compensator.

Under the assumption that the shunt compensator regulates the midpoint voltage tightly as given by (10.1), we can write the following expressions for the sending and receiving end currents

$$I_S = \frac{V\angle\delta - V\angle(\delta/2)}{jX/2} \tag{10.2}$$

$$I_R = \frac{V\angle(\delta/2) - V}{jX/2} \tag{10.3}$$

Again from Fig. 10.1 we write

$$I_S + I_Q = I_R \tag{10.4}$$

Combing (10.2)-(10.4) and solving we get

$$I_Q = -j\frac{4V}{X}\{1-\cos(\delta/2)\}\angle(\delta/2) \tag{10.5}$$

We thus have to generate a current that is in phase with the midpoint voltage and has a magnitude of $(4V / X_L)\{1 - \cos(\delta/2)\}$. The apparent power injected by the shunt compensator to the ac bus is then

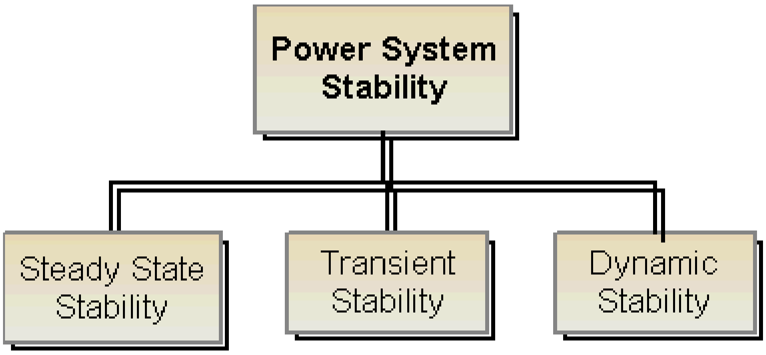
$$P_Q + jQ_Q = V_M I_Q^* = -j\frac{4V^2}{X}\left\{1-\cos\left(\delta/2\right)\right\} \tag{10.6}$$

Since the real part of the injected power is zero, we conclude that the ideal shunt compensator injects only reactive power to the ac system and no real power.

Introduction

The first electric power system was a dc system built by Edison in 1882. The subsequent power systems that were constructed in the late 19th century were all dc systems. However despite the initial popularity of dc systems by the turn of the 20th century ac systems started to outnumber them. The ac systems were though to be superior as ac machines were cheaper than their dc counterparts and more importantly ac voltages are easily transformable from one level to other using transformers. The early stability problems of ac systems were experienced in 1920 when insufficient damping caused spontaneous oscillations or hunting. These problems were solved using generator damper winding and the use of turbine-type prime movers.

The stability of a system refers to the ability of a system to return back to its steady state when subjected to a disturbance. As mentioned before, power is generated by synchronous generators that operate in synchronism with the rest of the system. A generator is synchronized with a bus when both of them have same frequency, voltage and phase sequence. We can thus define the power system stability as the ability of the power system to return to steady state without losing synchronism. Usually power system stability is categorized into **Steady State**, **Transient** and **Dynamic Stability.**



Steady State Stability studies are restricted to small and gradual changes in the system operating conditions. In this we basically concentrate on restricting the bus voltages close to their nominal values. We also ensure that phase angles between two buses are not too large and check for the overloading of the power equipment and transmission lines. These checks are usually done using power flow studies.

Transient Stability involves the study of the power system following a major disturbance. Following a large disturbance the synchronous alternator the machine power (load) angle changes due to sudden acceleration of the rotor shaft. The objective of the transient stability study is to ascertain whether the load angle returns to a steady value following the clearance of the disturbance.

The ability of a power system to maintain stability under continuous small disturbances is investigated under the name of **Dynamic Stability** (also known as small-signal stability). These small disturbances occur due random fluctuations in loads and generation levels. In an interconnected power system, these random variations can lead catastrophic failure as this may force the rotor angle to increase steadily.

In this chapter we shall discuss the transient stability aspect of a power system.

Section I: Power-Angle Relationship

The power-angle relationship has been discussed in Section 2.4.3. In this section we shall consider this relation for a lumped parameter lossless transmission line. Consider the **single-machine-infinite-bus** (SMIB) system shown in Fig. 9.1. In this the reactance *X* includes the reactance of the transmission line and the synchronous reactance or the transient reactance of the generator. The sending end voltage is then the internal emf of the generator. Let the sending and receiving end voltages be given by

$$V_S = V_1\angle\delta, \quad V_R = V_2\angle 0° \tag{9.1}$$

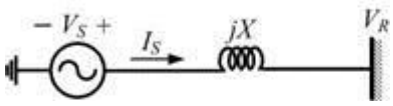


Fig. 9.1 An SMIB system.

We then have

$$I_S = \frac{V_1\angle\delta - V_2}{jX} = \frac{V_1\cos\delta - V_2 + jV_1\sin\delta}{jX} \tag{9.2}$$

The sending end real power and reactive power are then given by

$$P_S + jQ_S = V_S I_S^* = V_1(\cos\delta + j\sin\delta)\frac{V_1\cos\delta - V_2 - jV_1\sin\delta}{-jX}$$

This is simplified to

$$P_S + jQ_S = \frac{V_1V_2\sin\delta + j\left(V_1^2 - V_1V_2\cos\delta\right)}{X} \tag{9.3}$$

Since the line is loss less, the real power dispatched from the sending end is equal to the real power received at the receiving end. We can therefore write

$$P_e = P_S = P_R = \frac{V_1 V_2}{X} \sin \delta = P_{\max} \sin \delta \tag{9.4}$$

where $P_{max} = V_1\ V_2 / X$ is the maximum power that can be transmitted over the transmission line. The power-angle curve is shown in Fig. 9.2. From this figure we can see that for a given power P_0 . There are two possible values of the angle δ - δ_0 and δ_{max} . The angles are given by

$$\begin{aligned} \delta_0 &= \sin^{-1}\left(\frac{P_0}{P_{\max}}\right) \\ \delta_{\max} &= 180° - \delta_0 \end{aligned} \tag{9.5}$$

Chapter 9: Transient Stability

Section II: Swing Equation

Let us consider a three-phase synchronous alternator that is driven by a prime mover. The equation of motion of the machine rotor is given by

$$J\frac{d^2\theta}{dt^2} = T_m - T_e = T_a \tag{9.6}$$

where

J	is the total moment of inertia of the rotor mass in kgm^2
T_m	is the mechanical torque supplied by the prime mover in N-m
T_e	is the electrical torque output of the alternator in N-m
θ	is the angular position of the rotor in rad

Neglecting the losses, the difference between the mechanical and electrical torque gives the net accelerating torque T_a . In the steady state, the electrical torque is equal to the mechanical torque, and hence the accelerating power will be zero. During this period the rotor will move at **synchronous speed** ω_s in rad/s.

The angular position θ is measured with a stationary reference frame. To represent it with respect to the synchronously rotating frame, we define

$$\theta = \omega_s t + \delta \tag{9.7}$$

where δ is the angular position in rad with respect to the synchronously rotating reference frame. Taking the time derivative of the above equation we get

$$I_S = \frac{V_1\angle\delta - V_2}{jX} = \frac{V_1\cos\delta - V_2 + jV_1\sin\delta}{jX} \tag{9.8}$$

Defining the angular speed of the rotor as

we can write (9.8) as

$$\omega_r = \frac{d\theta}{dt}$$

$$\omega_r - \omega_s = \frac{d\delta}{dt} \tag{9.9}$$

We can therefore conclude that the rotor angular speed is equal to the synchronous speed only when $d\delta / dt$ is equal to zero. We can therefore term $d\delta / dt$ as the error in speed. Taking derivative of (9.8), we can then rewrite (9.6) as

$$J\frac{d^2\delta}{dt^2} = T_m - T_e = T_a \tag{9.10}$$

Multiplying both side of (9.11) by ω_m we get

$$J\omega_r\frac{d^2\delta}{dt^2} = P_m - P_e = P_a \tag{9.11}$$

where P_m , P_e and P_a respectively are the mechanical, electrical and accelerating power in MW.

We now define a normalized inertia constant as

$$H = \frac{\text{Stored kinetic energy at synchronous speed in mega - joules}}{\text{Generator MVA rating}} = \frac{J\omega_s^2}{2S_{rated}} \tag{9.12}$$

Substituting (9.12) in (9.10) we get

$$2H\frac{S_{rated}}{\omega_s^2}\omega_r\frac{d^2\delta}{dt^2} = P_m - P_e = P_a \tag{9.13}$$

In steady state, the machine angular speed is equal to the synchronous speed and hence we can replace ω_r in the above equation by ω_s. Note that in (9.13) P_m , P_e and P_a are given in MW. Therefore dividing them by the generator MVA rating S_{rated} we can get these quantities in per unit. Hence dividing both sides of (9.13) by S_{rated} we get

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e = P_a \quad \text{per unit} \tag{9.14}$$

Equation (7.14) describes the behaviour of the rotor dynamics and hence is known as the swing equation. The angle δ is the angle of the internal emf of the generator and it dictates the amount of power that can be transferred. This angle is therefore called the **load angle** .

Section III: Equal Area Criterion

The real power transmitted over a lossless line is given by (9.4). Now consider the situation in which the synchronous machine is operating in steady state delivering a power P_e equal to P_m when there is a fault occurs in the system. Opening up of the circuit breakers in the faulted section subsequently clears the fault. The circuit breakers take about 5/6 cycles to open and the subsequent post-fault transient last for another few cycles. The input power, on the other hand, is supplied by a prime mover that is usually driven by a steam turbine. The time constant of the turbine mass system is of the order of few seconds, while the electrical system time constant is in milliseconds. Therefore, for all practical purpose, the mechanical power is remains constant during this period when the electrical transients occur. The transient stability study therefore concentrates on the ability of the power system to recover from the fault and deliver the constant power P_m with a possible new load angle δ .

Consider the power angle curve shown in Fig. 9.3. Suppose the system of Fig. 9.1 is operating in the steady state delivering a power of P_m at an angle of δ_0 when due to malfunction of the line, circuit breakers open reducing the real power transferred to zero. Since P_m remains constant, the accelerating power P_a becomes equal to P_m . The difference in the power gives rise to the rate of change of stored kinetic energy in the rotor masses. Thus the rotor will accelerate under the constant influence of non-zero accelerating power and hence the load angle will increase. Now suppose the circuit breaker re-closes at an angle δ_c. The power will then revert back to the normal operating curve. At that point, the electrical power will be more than the mechanical power and the accelerating power will be negative. This will cause the machine decelerate. However, due to the inertia of the rotor masses, the load angle will still keep on increasing. The increase in this angle may eventually stop and the rotor may start decelerating, otherwise the system will lose synchronism.

Note that

$$\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = 2\left(\frac{d\delta}{dt}\right)\left(\frac{d^2\delta}{dt^2}\right)$$

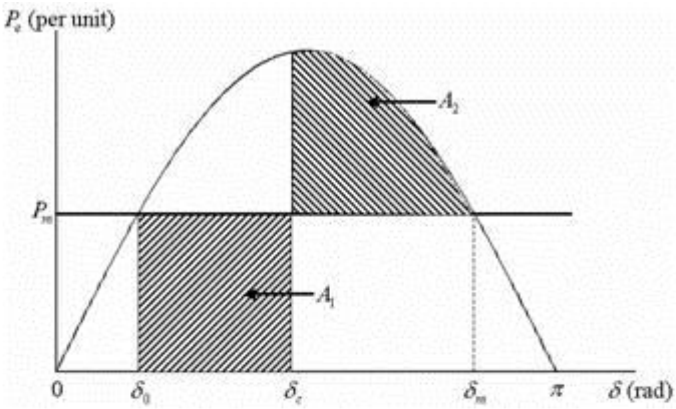


Fig. 9.3 Power-angle curve for equal area criterion.

Hence multiplying both sides of (9.14) by $d\delta/dt$ and rearranging we get

$$\frac{H}{\omega_s}\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = (P_m - P_e)\frac{d\delta}{dt}$$

Multiplying both sides of the above equation by *dt* and then integrating between two arbitrary angles δ_0 and δ_c we get

$$\frac{H}{\omega_s}\left(\frac{d\delta}{dt}\right)^2\Bigg|_{\delta_0}^{\delta_c} = \int_{\delta_0}^{\delta_c}(P_m - P_e)d\delta \tag{9.15}$$

Now suppose the generator is at rest at δ_0. We then have $d\delta$ / dt = 0. Once a fault occurs, the machine starts accelerating. Once the fault is cleared, the machine keeps on accelerating before it reaches its peak at δ_c , at which point we again have $d\delta$ / dt = 0. Thus the area of accelerating is given from (9.15) as

$$A_1 = \int_{\delta_0}^{\delta_c} (P_m - P_e) d\delta = 0 \qquad (9.16)$$

In a similar way, we can define the area of deceleration. In Fig. 9.3, the area of acceleration is given by A_1 while the area of deceleration is given by A_2 . This is given by

$$A_2 = \int_{\delta_c}^{\delta_m} (P_e - P_m) d\delta = 0 \qquad (9.17)$$

Contd... Equal Area Criterion

Now consider the case when the line is reclosed at δ_c such that the area of acceleration is larger than the area of deceleration, i.e., $A_1 > A_2$. The generator load angle will then cross the point δ_m , beyond which the electrical power will be less than the mechanical power forcing the accelerating power to be positive. The generator will therefore start accelerating before is slows down completely and will eventually become unstable. If, on the other hand, $A_1 < A_2$, i.e., the decelerating area is larger than the accelerating area, the machine will decelerate completely before accelerating again. The rotor inertia will force the subsequent acceleration and deceleration areas to be smaller than the first ones and the machine will eventually attain the steady state. If the two areas are equal, i.e., $A_1 = A_2$, then the accelerating area is equal to decelerating area and this is defines the **boundary of the stability limit.** The clearing angle δ_c for this mode is called the **Critical Clearing Angle** and is denoted by δ_{cr}. We then get from Fig. 9.3 by substituting $\delta_c = \delta_{cr}$

$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_e) d\delta = \int_{\delta_{cr}}^{\delta_m} (P_e - P_m) d\delta \qquad (9.18)$$

We can calculate the critical clearing angle from the ab move equation. Since the critical clearing angle depends on the equality of the areas, this is called the **equal area criterion**.

Example 9.3:

Consider the system of Example 9.1. Let us assume that the system is operating with $P_m = P_e = 0.9$ per unit when a circuit breaker opens inadvertently isolating the generator from the infinite bus. During this period the real power transferred becomes zero. From Example 9.1 we have calculated δ_0 = 23.96 ° = 0.4182 rad and the maximum power transferred as

$$P_{max} = \frac{1.1082 \times 1}{0.5} = 2.2164$$ per unit

We have to find the critical clearing angle.

From (9.15) the accelerating area is computed as by note that $P_e = 0$ during this time. This is then given by

$$A_1 = \int_{0.4182}^{\delta_{cr}} 0.9\,d\delta = 0.9\delta_{cr} - 0.9\times 0.4182 = 0.9\delta_{cr} - 0.3764$$

To calculate the decelerating area we note that $\delta_m = \pi - 0.4182 = 2.7234$ rad. This area is computed by noting that $P_e = 2.2164\sin(\delta)$ during this time. Therefore

$$A_2 = \int_{\delta_{cr}}^{2.7234} (2.2164\sin\delta - 0.9)\,d\delta$$
$$= -2.2164\times\cos(2.7234) + 2.2164\cos(\delta_{cr}) - 0.9\times 2.7234 + 0.9\delta_{cr}$$
$$= 2.2164\cos(\delta_{cr}) + 0.9\delta_{cr} - 0.4257$$

Equating $A_1 = A_2$ and rearranging we get

$$\delta_{cr} = \cos^{-1}\left(\frac{0.0493}{2.2164}\right) = 1.5486\text{ rad} = 88.73°$$

Now a frequently asked question is what does the critical clearing angle mean?

Since we are interested in finding out the maximum time that the circuit breakers may take for opening, we should be more concerned about the critical clearing time rather than clearing angle. Furthermore, notice that the clearing angle is independent of the generalized inertia constant H . Hence we can comment that the critical clearing angle in this case is true for any generator that has a d-axis transient reactance of 0.20 per unit. The critical clearing time, however, is dependent on H and will vary as this parameter varies.

To obtain a description for the critical clearing time, let us consider the period during which the fault occurs. We then have $P_e = 0$. We can therefore write from

$$\frac{d^2\delta}{dt^2} = \frac{\omega_s}{2H}P_m \tag{9.19}$$

Integrating the above equation with the initial acceleration being zero we get

$$\frac{d\delta}{dt} = \int_0^t \frac{\omega_s}{2H}P_m dt = \frac{\omega_s}{2H}P_m t$$

Further integration will lead to

$$\delta = \int_0^t \frac{\omega_s}{2H}P_m t\,dt = \frac{\omega_s}{4H}P_m t^2 + \delta_0$$

Replacing δ by δ_{cr} and t by t_{cr} in the above equation, we get the critical clearing time as

$$t_{cr} = \sqrt{\frac{4H}{\omega_s P_m}\left(\delta_{cr} - \delta_0\right)} \tag{9.20}$$

Example 9.4:

In Example 9.2, let us choose the system frequency as 50 Hz such that ω_s is 100π. Also let us choose H as 4 MJ/MVA. Then with δ_{cr} being 1.5486 rad, δ_0 being 0.4182 rad and P_m being 0.9 per unit, we get the following critical clearing time from (9.20)

$$t_{cr} = \sqrt{\frac{16}{90\pi}\left(1.5486 - 0.4182\right)} = 0.253 \text{ s}$$

To illustrate the response of the load angle δ , the swing equation is simulated in MATLAB. The swing equation of (9.14) is then expressed as

$$\begin{aligned} \frac{d\Delta\omega_r}{dt} &= \frac{1}{2H}\left(P_m - P_e\right) \\ \frac{d\delta}{dt} &= \omega_s \times \Delta\omega_r \end{aligned} \tag{9.21}$$

where $\Delta\omega_r$ is the deviation for the rotor speed from the synchronous speed ω_s . It is to be noted that the swing equation of (9.21) does not contain any damping. Usually a damping term, that is proportional to the machine speed $\Delta\omega_r$, is added with the accelerating power. Without the damping the load angle will exhibit a sustained oscillation even when the system remains stable when the fault cleared within the critical clearing time.

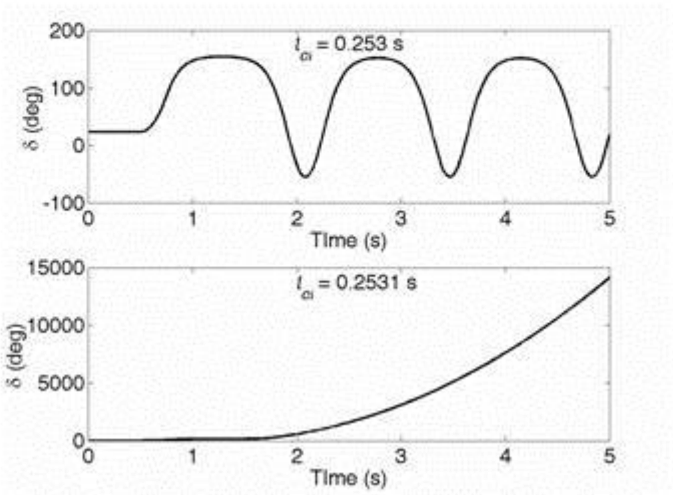


Fig. 9.4 Stable and unstable system response as a function of clearing time.

Fig. 9.4 depicts the response of the load angle δ for two different values of load angle. It is assumed that the fault occurs at 0.5 s when the system is operating in the steady state delivering 0.9 per unit power. The load angle during this time is constant at 23.96° . The load angle remains stable, albeit the sustained oscillation when the clearing time t_{cl} is 0.253 s. The clearing angle during this time is 88.72° . The system however becomes unstable when the clearing time 0.2531s and the load angle increases asymptotically. The clearing time in this case is 88.77° . This is called the **Loss of Synchronism**. It is to be noted that such increase in the load angle is not permissible and the protection device will isolate the generator from the system.

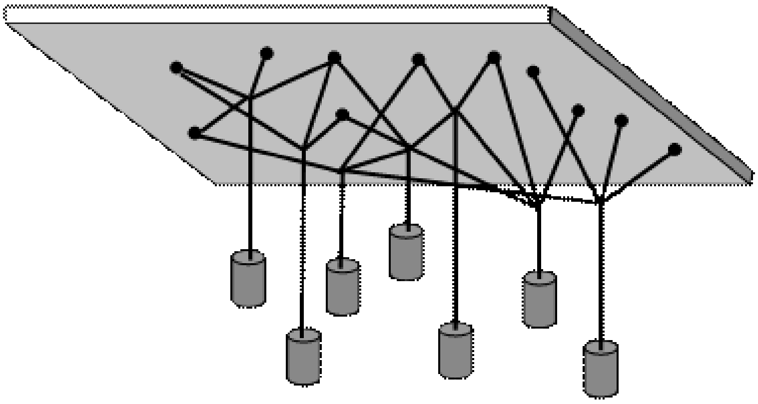
The clearing time of (8.20) is derived based on the assumption that the electrical power P_e becomes zero during the fault as in (8.19). This need not be the case always. In that even we have to resort to finding the clearing time using the numerical integration of the swing equation. See example 9.5 to illustrates the point.

Example 9.5

Section IV: Multimachine Stability

- **Oscillations in s Two Area System**

Consider Fig. 9.10, which depicts a number of weights that are suspended by elastic strings. The weights represent generators and the electric transmission lines being represented by the strings. Note that in a transmission system, each transmission line is loaded below its static stability limit. Similarly, when the mechanical system is in static steady state, each string is loaded below its break point. At this point one of the strings is suddenly cut. This will result in transient oscillations in the coupled strings and all the weights will wobble. In the best possible case, this may result in the coupled system settling down to a new steady state. On the other hand, in the worst possible scenario this may result in the breaking of one more additional string, resulting in a chain reaction in which more strings may break forcing a system collapse. In a similar way, in an interconnected electric power network, the tripping of a transmission line may cause a catastrophic failure in which a large number of generators are lost forcing a blackout in a large area.



Modern power systems are interconnected and operate close to their transient and steady state stability limits. In large interconnected systems, it is common to find a natural response of a group of closely coupled machines oscillating against other groups of machines. These oscillations have a frequency range of 0.1 Hz to 0.8 Hz. The lowest frequency mode involves all generators of the system. This oscillation groups the system into two parts - with generators in one part oscillating against those of the the other part. The higher frequency modes are usually localized with small groups oscillating against each other. Unfortunately, the inter-area oscillation can be initiated by a small disturbance in any part of the system. These small frequency oscillations fall under the category of dynamic stability and are analysed in linear domain through the liberalisation of the entire interconnected systems model.

Inter-area oscillations manifest wherever the power system is heavily interconnected. The oscillations, unless damped, can lead to grid failure and total system collapse. Low frequency oscillations in the range of 0.04 Hz to 0.06 Hz were observed in the Pacific North West region as early as 1950. Improper speed governor control of hydro units created these oscillations. The Northern and Southern regions of WSCC were interconnected by a 230 kV line in 1964. Immediately the system experienced a 0.1 Hz oscillation resulting in over 100 instances of opening of the tie line in the first nine months of operation. Some system damping was provided through the modification in the hydro turbine governors.

A 500 kV pacific intertie and another ± 400 kV HVDC system was commissioned in 1968. This raised the frequency of oscillation from 0.1 Hz to 0.33 Hz and these oscillations could no longer be controlled through governor action alone. In late 1980's a new intertie joined the WSCC system to Alberta and British Columbia in Canada . As a result of this interconnection, the two different oscillation frequencies manifested - one at 0.29 Hz and the other at 0.45 Hz.

Ontario Hydro is one of the largest utilities in North America . Due to the vast and sparsely populated topology of Canada , the operating span of Ontario hydro is over 1000 km from East to West and from North to South. The Ontario Hydro system is connected to the neighbouring Canadian provinces and the North Western region of the United States . In 1959 Ontario Hydro was connected to Michigan in the South and Quebec Hydro in the East. As a result of this connection, a 0.25 Hz oscillation was observed and a result of this it was decided to remove the tie with Quebec and retain the tie to Michigan . The Western portion of Ontario was connected to neighbouring Manitoba in 1956 and then Manitoba was connected to its neighbour Saskatchewan in 1960. This resulted in oscillation in the frequency range 0.35 Hz to 0.45 Hz often tripping the tie. As a result of this, Ontario Hydro decided to commission power system stabilizers for all their generating units since early 1960's. It has also sponsored extensive research in this area.

Through research it was established that the action of automatic voltage regulators caused these oscillations. An automatic voltage regulator (AVR) regulates the generator terminal voltage and also helps in the enhancement of transient stability by reducing the peak of the first swing following any disturbance. However, its high gain contributed to negative damping to the system. The knowledge of this relation resulted in the commissioning of power system stabilizers. It was observed that these oscillations were results of the periodic interchange of kinetic energy between the generator rotors. A power system stabilizer (PSS) provides a negative feedback of the changes in rotor kinetic energy when it is connected to the excitation system thereby providing damping to these small oscillations. The PSS has been a subject of extensive research. The team of Dr. P. Kundur, then with Ontario Hydro, and his co-workers has done extensive research in the area of PSS tuning and its characteristics. Through their vast experience and extensive research, they reported the enhancement of inter-area and local modes through PSS reported in. Since a power system is piece-wise linear, its system characteristics changes with operating point. Therefore an adaptive controller that can tune with the changes in the system has been developed and reported in. It was shown that the adaptive PSS is effective in damping large as well as small disturbances.

The power flow between generators, as evident from (9.4), is dependent on the angle between those generators. The stable operating point of the power system is where the generated power at each station is matched by the electrical power sent out from that station. When there is a mismatch between electrical power out and the generated mechanical shaft power, the generator will accelerate at a rate determined by the power mismatch and the machine inertia as given in (9.14).

Oscillations in s Two Area System

Consider the simple power system shown in Fig. 9.11 in which two machines are operating. Let us assume that starting with the initial angles δ_1 and δ_2 with respect to some reference at nominal frequency, machine 1 accelerates while machine 2 decelerates from this nominal frequency. We then have

$$\frac{2H_1}{\omega_s}\ddot{\delta}_1 = P_{m1} - P_{e1}$$
$$\frac{2H_2}{\omega_s}\ddot{\delta}_2 = P_{m2} - P_{e2} \tag{9.25}$$

where the subscripts 1 and 2 refer to machines 1 and 2 respectively. Let us assume that the transmission line is loss less. Then in the simple case where the power from machine 1 flows to machine 2, we get

$$P_{e1} = -P_{e2} = \frac{V_1V_2}{X}\sin(\delta_1 - \delta_2) = \frac{V_1V_2}{X}\sin\,\delta_{12} \tag{9.26}$$

where $\delta_{12} = \delta_1 - \delta_2$.

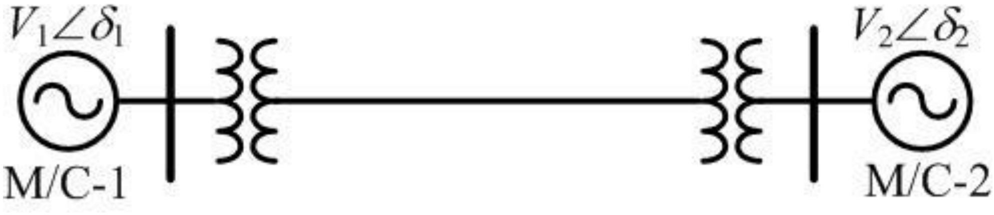


Fig. 9.11 Single-line diagram of a two-machine power system.

Now since the system is lossless, (9.26) will also imply that P_{m1} = - P_{m2} . This means that in the steady state, the power generated at machine 1 is absorbed through machine 2. Combining (9.25) and (9.26) we get

$$\frac{2H_1}{\omega_s}\ddot{\delta}_1 - \frac{2H_2}{\omega_s}\ddot{\delta}_2 = 2P_{m1} - P_{e1} + P_{e2} = 2P_{m1} - \frac{2V_1V_2}{X}\sin\delta_{12} \qquad (9.27)$$

Let us now assume that $H_1 = H_2 = H$, $V_1 = V_2 = 1.0$ per unit and $P_{m1} = 0$. We then get from (9.27)

$$\ddot{\delta}_{12} = -\omega^2 \sin\delta_{12} \qquad (9.28)$$

where the oscillation frequency ω is given by

$$\omega = \sqrt{\omega_s / HX} \qquad (9.29)$$

Thus the weighted difference of angles will approximate simple harmonic motion for small changes in δ_{12} and the frequency will decrease for an increase in inertia H or impedance X . Another aspect can be seen by adding the system to give

$$H_1\ddot{\delta}_1 + H_2\ddot{\delta}_2 = P_{m1} + P_{m2} = 0 \qquad (9.30)$$

Thus the overall acceleration of the machine group will depend on the overall balance between power generated and consumed. Usually there are governors on the generators to reduce generated power if the system frequency increases.

Introduction

The two major problems that the modern power systems are facing are voltage and angle stabilities. There are various approaches to overcome the problem of stability arising due to small signal oscillations in an interconnected power system. As mentioned in the previous chapter, installing power system stabilizers with generator excitation control system provides damping to these oscillations. However, with the advancement in the power electronic technology, various reactive power control equipment are increasingly used in power transmission systems.

A power network is mostly reactive. A synchronous generator usually generates active power that is specified by the mechanical power input. The reactive power supplied by the generator is dictated by the network and load requirements. A generator usually does not have any control over it. However the lack of reactive power can cause voltage collapse in a system. It is therefore important to supply/absorb excess reactive power to/from the network. Shunt compensation is one possible approach of providing reactive power support.

A device that is connected in parallel with a transmission line is called a **shunt compensator** , while a device that is connected in series with the transmission line is called a *series compensator* . These are referred to as compensators since they compensate for the reactive power in the ac system. We shall assume that the shunt compensator is always connected at the midpoint of transmission system, while the series compensator can be connected at any point in the line. We shall demonstrate that such connections in an SMIB power system improves

- voltage profile
- power-angle characteristics
- stability margin
- damping to power oscillations

A **static var compensator (SVC)** is the first generation shunt compensator. It has been around since 1960s. In the beginning it was used for load compensation such as to provide var support for large industrial loads, for flicker mitigation etc. However with the advancement of semiconductor technology, the SVC started appearing in the transmission systems in 1970s. Today a large number of SVCs are connected to many transmission systems all over the world. An SVC is constructed using the thyristor technology and therefore does not have gate turn off capability.

With the advancement in the power electronic technology, the application of a gate turn off thyristor (GTO) to high power application became commercially feasible. With this the second generation shunt compensator device was conceptualized and constructed. These devices use synchronous voltage sources for generating or absorbing reactive power. A synchronous voltage source (SVS) is constructed using a voltage source converter (VSC). Such a shunt compensating device is called **static compensator or STATCOM** . A STATCOM usually contains an SVS that is driven from a dc storage capacitor and the SVS is connected to the ac system bus through an interface transformer. The transformer steps the ac system voltage down such that the voltage rating of the SVS switches are within specified limit. Furthermore, the leakage reactance of the transformer plays a very significant role in the operation of the STATCOM.

Like the SVC, **a thyristor controlled series compensator (TCSC)** is a thyristor based series compensator that connects a **thyristor controlled reactor (TCR)** in parallel with a fixed capacitor. By varying the firing angle of the anti-parallel thyristors that are connected in series with a reactor in the TCR, the fundamental frequency inductive reactance of the TCR can be changed. This effects a change in the reactance of the TCSC and it can be controlled to produce either inductive or capacitive reactance.

Alternatively a **static synchronous series compensator or SSSC** can be used for series compensation. An SSSC is an SVS based all GTO based device which contains a VSC. The VSC is driven by a dc capacitor. The output of the VSC is connected to a three-phase transformer. The other end of the transformer is connected in series with the transmission line. Unlike the TCSC, which changes the impedance of the line, an SSSC injects a voltage in the line in quadrature with the line current. By making the SSSC voltage to lead or lag the line current by 90 ° , the SSSC can emulate the behavior of an inductance or capacitance.

In this chapter, we shall discuss the ideal behavior of these compensating devices. For simplicity we shall consider the ideal models and broadly discuss the advantages of series and shunt compensation.

Section I: Ideal Shunt Compensator

- **Improving Volatage Profile**
- **Improving Power-Angle Characteristics**

- **Improving Stability Margin**
- **Improving Damping to Power Oscillations**

The ideal shunt compensator is an ideal current source. We call this an ideal shunt compensator because we assume that it only supplies reactive power and no real power to the system. It is needless to say that this assumption is not valid for practical systems. However, for an introduction, the assumption is more than adequate. We shall investigate the behavior of the compensator when connected in the middle of a transmission line. This is shown in Fig. 10.1, where the shunt compensator, represented by an ideal current source, is placed in the middle of a lossless transmission line. We shall demonstrate that such a configuration improves the four points that are mentioned above.

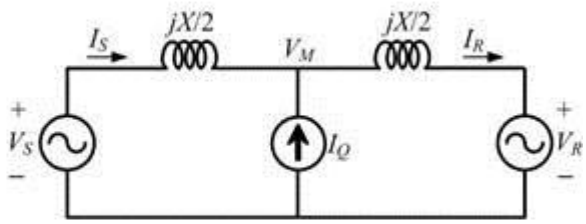


Fig 10.1 Schematic diagram of an ideal, midpoint shunt compensation

Improving Voltage Profile

Let the sending and receiving voltages be given by $V\angle\delta$ and $V\angle 0°$ respectively. The ideal shunt compensator is expected to regulate the midpoint voltage to

$$V_M = V\angle(\delta/2) \tag{10.1}$$

against any variation in the compensator current. The voltage current characteristic of the compensator is shown in Fig. 10.2. This ideal behavior however is not feasible in practical systems where we get a slight droop in the voltage characteristic. This will be discussed later.

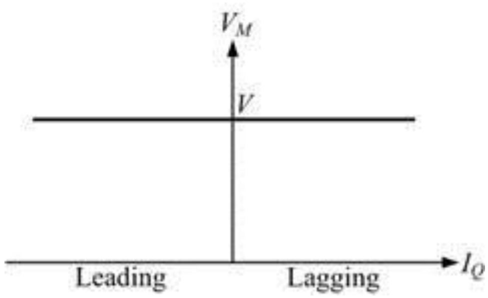


Fig. 10.2 Voltage-current characteristic of an ideal shunt compensator.

Under the assumption that the shunt compensator regulates the midpoint voltage tightly as given by (10.1), we can write the following expressions for the sending and receiving end currents

$$I_S = \frac{V\angle\delta - V\angle(\delta/2)}{jX/2} \tag{10.2}$$

$$I_R = \frac{V\angle(\delta/2) - V}{jX/2} \tag{10.3}$$

Again from Fig. 10.1 we write

$$I_S + I_Q = I_R \tag{10.4}$$

Combing (10.2)-(10.4) and solving we get

$$I_Q = -j\frac{4V}{X}\{1-\cos(\delta/2)\}\angle(\delta/2) \tag{10.5}$$

We thus have to generate a current that is in phase with the midpoint voltage and has a magnitude of ($4V$ / X_L){1 - cos(δ /2)}. The apparent power injected by the shunt compensator to the ac bus is then

$$P_Q + jQ_Q = V_M I_Q^* = -j\frac{4V^2}{X}\{1-\cos(\delta/2)\} \tag{10.6}$$

Since the real part of the injected power is zero, we conclude that the ideal shunt compensator injects only reactive power to the ac system and no real power.

Improving Power-Angle Characteristics

The apparent power supplied by the source is given by

$$\begin{aligned} P_S + jQ_S = V_S I_S^* &= V\angle\delta\left[\frac{V\angle-\delta - V\angle-(\delta/2)}{-jX/2}\right] = \frac{V^2 - V\angle(\delta/2)}{-jX/2} \\ &= \frac{2V^2\sin(\delta/2)}{X} + j\frac{2V^2\{1-\cos(\delta/2)\}}{X} \end{aligned} \tag{10.7}$$

Similarly the apparent power delivered at the receiving end is

$$P_R + jQ_R = V_R I_R^* = V\left[\frac{V\angle-(\delta/2)-V}{-jX/2}\right]$$
$$= \frac{2V^2\sin(\delta/2)}{X} + j\frac{2V^2\{\cos(\delta/2)-1\}}{X} \tag{10.8}$$

Hence the real power transmitted over the line is given by

$$P_e = P_S = P_R = \frac{2V^2}{X}\sin(\delta/2) \tag{10.9}$$

Combining (10.6)-(10.8), we find the reactive power consumed by the line as

$$Q_e = Q_S + Q_Q - Q_R = \frac{8V^2}{X}\{1-\cos(\delta/2)\} \tag{10.10}$$

The power-angle characteristics of the shunt compensated line are shown in Fig. 10.3. In this figure $P_{max} = V^2/X$ is chosen as the power base.

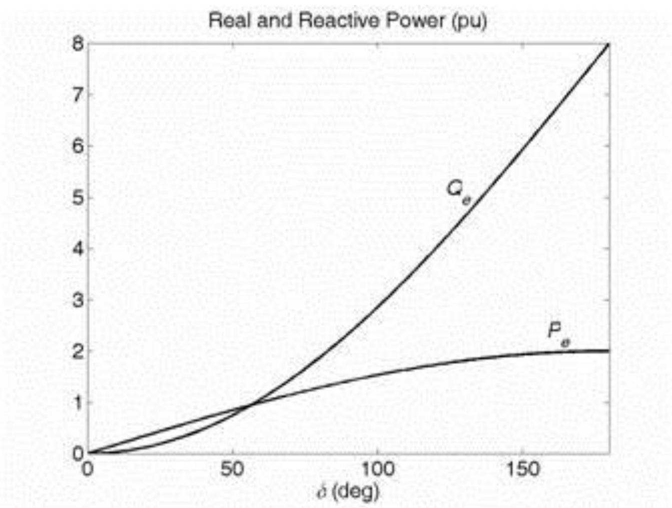


Fig. 10.3 Power-angle characteristics of ideal shunt compensated line.

Fig. 10.3 depicts P_e - δ and Q_Q - δ characteristics. It can be seen from fig 10.4 that for a real power transfer of 1 per unit, a reactive power injection of roughly 0.5359 per unit will be required from the shunt compensator if the midpoint voltage is regulated as per (10.1). Similarly for increasing the real power transmitted to 2 per unit, the shunt compensator has to inject 4 per unit of reactive power. This will obviously increase the device rating and may not be practical. Therefore power transfer enhancement using midpoint shunt compensation may not be feasible from the device rating point of view.

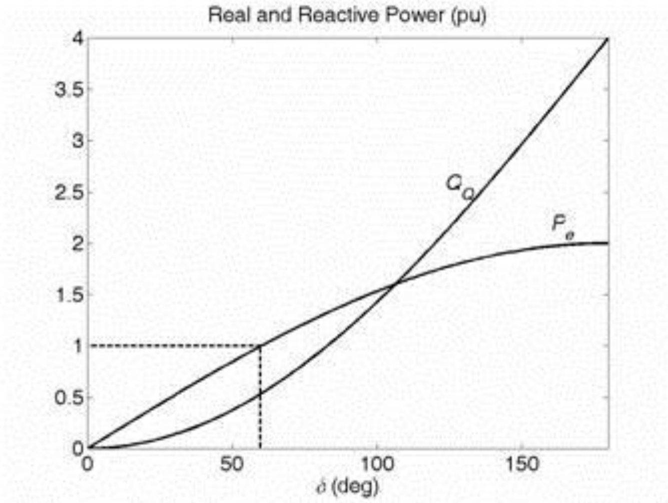


Fig. 10.4 Variations in transmitted real power and reactive power injection by the shunt compensator with load angle for perfect midpoint voltage regulation.

Let us now relax the condition that the midpoint voltage is regulated to 1.0 per unit. We then obtain some very interesting plots as shown in Fig. 10.5. In this figure, the x-axis shows the reactive power available from the shunt device, while the y-axis shows the maximum power that can be transferred over the line without violating the voltage constraint. There are three different P-Q relationships given for three midpoint voltage constraints. For a reactive power injection of 0.5 per unit, the power transfer can be increased from about 0.97 per unit to 1.17 per unit by lowering the midpoint voltage to 0.9 per unit. For a reactive power injection greater than 2.0 per unit, the best power transfer capability is obtained for V_M = 1.0 per unit. Thus there will be no benefit in reducing the voltage constraint when the shunt device is capable of injecting a large amount of reactive power. In practice, the level to which the midpoint voltage can be regulated depends on the rating of the installed shunt device as well the power being transferred.

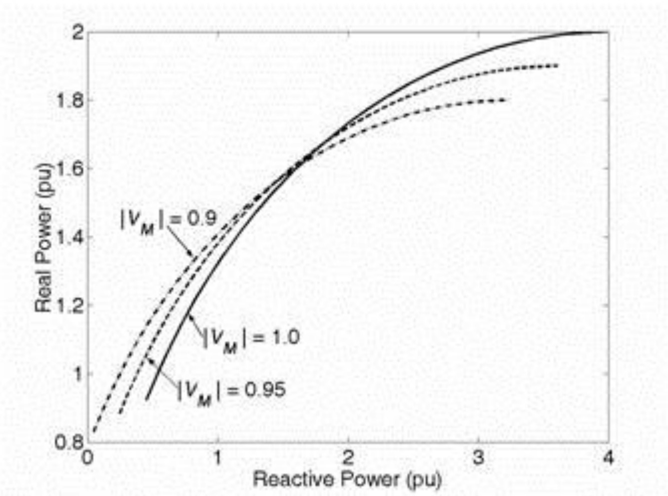
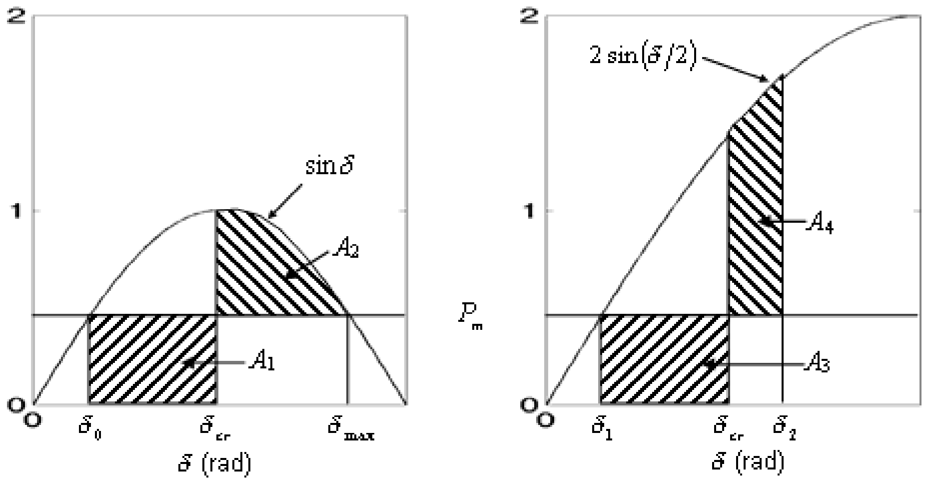


Fig. 10.5 Power transfer versus shunt reactive injection under midpoint voltage constraint.

Improving Stability Margin

This is a consequence of the improvement in the power angle characteristics and is one of the major benefits of using midpoint shunt compensation. As mentioned before, the stability margin of the system pertains to the regions of acceleration and deceleration in the power-angle curve. We shall use this concept to delineate the advantage of mid point shunt compensation.

Consider the power angle curves shown in Fig. 10.6.



The curve of Fig. 10.6 (a) is for an uncompensated system, while that of Fig. 10.6 (b) for the compensated system. Both these curves are drawn assuming that the base power is V^2/X . Let us assume that the uncompensated system is operating on steady state delivering an electrical power equal to P_m with a load angle of δ_0 when a three-phase fault occurs that forces the real power to zero. To obtain the critical clearing angle for the uncompensated system is δ_{cr} , we equate the accelerating area A_1 with the decelerating area A_2 , where

$$A_1 = \int_{\delta_0}^{\delta_{cr}} P_m dt = P_m(\delta_{cr} - \delta_0)$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (\sin\ \delta - P_m) dt = (\cos \delta_{cr} - \cos \delta_{max}) - P_m(\delta_{max} - \delta_{cr})$$

with $\delta_{max} = \pi - \delta_0$. Equating the areas we obtain the value of δ_{cr} as

$$\delta_{cr} = \cos^{-1}[P_m(\delta_{max} - \delta_0) + \cos \delta_{max}] \tag{10.11}$$

Let us now consider that the midpoint shunt compensated system is working with the same mechanical power input P_m. The operating angle in this case is δ_1 and the maximum power that can be transferred in this case is 2 per unit. Let the fault be cleared at the same clearing angle δ_{cr} as before. Then equating areas A_3 and A_4 in Fig. 10.6 (b) we get δ_2, where

$$A_3 = \int_{\delta_1}^{\delta_{cr}} P_m\, dt = P_m\left(\delta_{cr} - \delta_1\right)$$

$$A_4 = \int_{\delta_{cr}}^{\delta_2} \left[2\sin\left(\delta/2\right) - P_m\right] dt = 4\left[\cos\left(\delta_{cr}/2\right) - \cos\left(\delta_2/2\right)\right] - P_m\left(\delta_2 - \delta_{cr}\right)$$

Example 10.1

Improving Damping to Power Oscillations

The swing equation of a synchronous machine is given by (9.14). For any variation in the electrical quantities, the mechanical power input remains constant. Assuming that the magnitude of the midpoint voltage of the system is controllable by the shunt compensating device, the accelerating power in (9.14) becomes a function of two independent variables, | V_M | and δ . Again since the mechanical power is constant, its perturbation with the independent variables is zero. We then get the following small perturbation expression of the swing equation

$$\frac{2H}{\omega}\frac{d^2\Delta\delta}{dt^2} + \frac{\partial P_e}{\partial|V_M|}\Delta|V_M| + \frac{\partial P_e}{\partial\delta}\Delta\delta = 0 \tag{10.12}$$

where Δ indicates a perturbation around the nominal values.

If the mid point voltage is regulated at a constant magnitude, Δ| V_M | will be equal to zero. Hence the above equation will reduce to

$$\frac{2H}{\omega}\frac{d^2\Delta\delta}{dt^2} + \frac{\partial P_e}{\partial\delta}\Delta\delta = 0 \tag{10.13}$$

The 2nd order differential equation given in (10.13) can be written in the Laplace domain by neglecting the initial conditions as

$$\left(\frac{2H}{\omega}s^2 + \frac{\partial P_e}{\partial\delta}\right)\Delta\delta(s) = 0 \qquad \text{(10.14}$$

The roots of the above equation are located on the imaginary axis of the s-plane at locations ± $j\ \omega_m$ where

$$\omega_m = \sqrt{(\omega/2H)(\partial P_e/\partial\delta)}$$

This implies that the load angle will oscillate with a constant frequency of ω_m . Obviously, this solution is not acceptable. Thus in order to provide damping, the mid point voltage must be varied according to in sympathy with the rate of change in $\Delta\delta$. We can then write

$$\Delta|V_M| = K_M \frac{d\Delta\delta}{dt} \tag{10.15}$$

where K_M is a proportional gain. Substituting (10.15) in (10.12) we get

$$\frac{2H}{\omega}\frac{d^2\Delta\delta}{dt^2} + \frac{\partial P_e}{\partial|V_M|}K_M\frac{d\Delta\delta}{dt} + \frac{\partial P_e}{\partial\delta}\Delta\delta = 0 \tag{10.16}$$

Provided that K_M is positive definite, the introduction of the control action (10.15) ensures that the roots of the second order equation will have negative real parts. Therefore through the feedback, damping to power swings can be provided by placing the poles of the above equation to provide the necessary damping ratio and undamped natural frequency of oscillations.

Example 10.2

Section II: Ideal Series Compensator

- **Impact of Series Compensator on Voltage Profile**
- **Improving Power-Angle Characteristics**
- **An Alternate Method of Voltage Injection**
- **Improving Stability Margin**
- **Comparisons of the Two Modes of Operation**
- **Power Flow Control and Power Swing Damping**

Ideal Series Compensator

Let us assume that the series compensator is represented by an ideal voltage source. This is shown in Fig. 10.10. Let us further assume that the series compensator is ideal, i.e., it only supplies reactive power and no real power to the system. It is needless to say that this assumption is not valid for practical systems. However, for an introduction, the assumption is more than adequate. It is to be noted that, unlike the shunt compensator, the location of the series compensator is not crucial, and it can be placed anywhere along the transmission line.

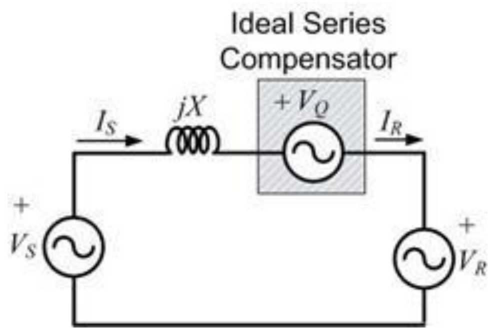


Fig. 10.10 Schematic diagram of an ideal series compensated system.

Impact of Series Compensator on Voltage Profile

In the equivalent schematic diagram of a series compensated power system is shown in Fig. 10.10, the receiving end current is equal to the sending end current, i.e., $I_S = I_R$. The series voltage V_Q is injected in such a way that the magnitude of the injected voltage is made proportional to that of the line current. Furthermore, the phase of the voltage is forced to be in quadrature with the line current. We then have

$$V_Q = \lambda I_S e^{\mp j90^{\circ}} \tag{10.20}$$

The ratio λ/X is called the **compensation level** and is often expressed in percentage. This compensation level is usually measured with respect to the transmission line reactance. For example, we shall refer the compensation level as 50% when $\lambda = X/2$. In the analysis presented below, we assume that the injected voltage lags the line current. The implication of the voltage leading the current will be discussed later.

Applying KVL we get

$$V_S - V_Q - V_R = jXI_S \quad \Rightarrow \quad V_S - V_R = \mp j\lambda I_S + jXI_S$$

Assuming $V_S = V < \delta$ and $V_R = V < 0°$, we get the following expression for the line current

$$I_S = \frac{V\angle\delta - V}{j(X \mp \lambda)} \tag{10.21}$$

When we choose $V_Q = \lambda\, I_S\, e^{-j90^{\circ}}$, the line current equation becomes

$$I_S = \frac{V\angle\delta - V}{j(X - \lambda)}$$

Thus we see that λ is subtracted from X . This choice of the sign corresponds to the voltage source acting as a pure capacitor. Hence we call this as the **capacitive mode of operation** . In contrast, if we choose $V_Q = \lambda I_S\, e^{+j90^{\circ}}$, λ is added to X , and this mode is referred to as the **inductive mode of**

operation . Since this voltage injection using (10.20) add λ to or subtract λ from the line reactance, we shall refer it as voltage injection in **constant reactance mode**. We shall consider the implication of series voltage injection on the transmission line voltage through the following example.

Example 10.3

Improving Power-Angle Characteristics

Noting that the sending end apparent power is $V_S I_S^*$, we can write

$$P_S + jQ_S = V_S I_S^* = V\angle\delta\left[\frac{V\angle-\delta - V}{-j(X\mp\lambda)}\right] = \frac{V^2 - V^2\angle\delta}{-j(X\mp\lambda)}$$
$$= \frac{V^2\sin\delta}{X\mp\lambda} + j\frac{V^2(1-\cos\delta)}{X\mp\lambda} \qquad (10.22)$$

Similarly the receiving end apparent power is given by

$$P_R + jQ_R = V_R I_S^* = V\left[\frac{V\angle-\delta - V}{-j(X\mp\lambda)}\right]$$
$$= \frac{V^2\sin\delta}{X\mp\lambda} + j\frac{V^2(\cos\delta - 1)}{X\mp\lambda} \qquad (10.23)$$

Hence the real power transmitted over the line is given by

$$P_S = P_R = P_e = \frac{V^2}{X\mp\lambda}\sin\delta \qquad (10.24)$$

The power-angle characteristics of a series compensated power system are given in Fig. 10.12. In this figure the base power is chosen as V^2 / X . Three curves are shown, of which the curve P_0 is the **power-angle curve** when the line is not compensated. Curves which have maximum powers greater than the base power pertain to capacitive mode of operation. On the other hand, all curves the inductive mode of operation will have maximum values less than 1. For example, in Fig. 10.12, the curve P_1 is for capacitive mode and the curve P_2 is for inductive mode of operation.

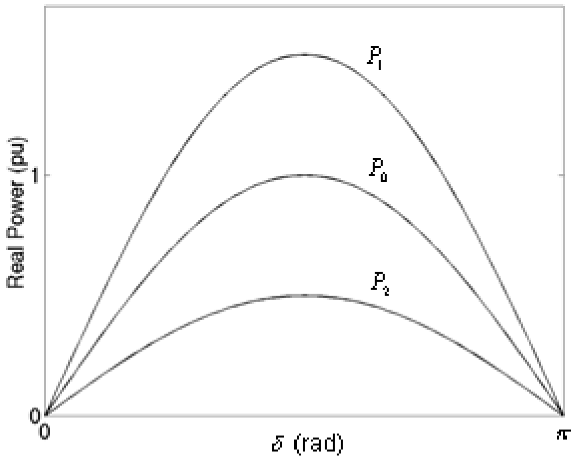


Fig. 10.12 Power-angle characteristics in constant reactance mode.

Let us now have a look at the reactive power. For simplicity let us restrict our attention to capacitive mode of operation only as this represents the normal mode of operation in which the power transfer over the line is enhanced. From (10.20) and (10.21) we get the reactive power supplied by the compensator as

$$Q_Q = V_Q I_S^* = -j\lambda \frac{V\angle\delta - V}{j(X-\lambda)} \times \frac{V\angle -\delta - V}{-j(X-\lambda)}$$

Solving the above equation we get

$$Q_Q = -j\frac{2\lambda V^2}{(X-\lambda)^2}\left(1-\cos\delta\right) \tag{10.25}$$

In Fig. 10.13, the reactive power injected by the series compensator is plotted against the maximum power transfer as the compensation level changes from 10% to 60%. As the compensation level increases, the maximum power transfer also increases. However, at the same time, the reactive injection requirement from the series compensator also increases. It is interesting to note that at 50% compensation level, the reactive power injection requirement from a series compensator is same that from shunt compensator that is regulating the midpoint voltage to 1.0 per unit.

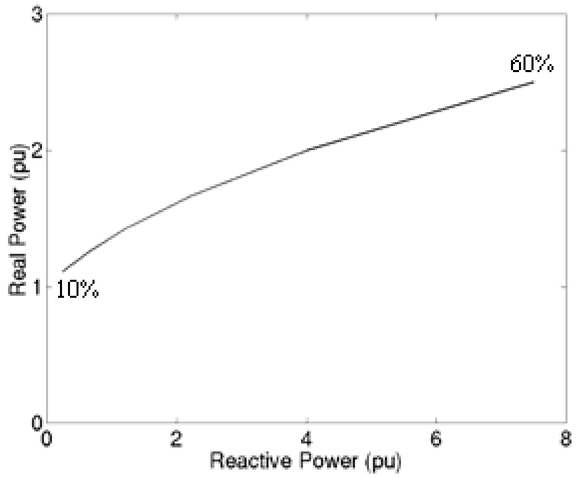


Fig. 10.13 Reactive power injection by a series compensator versus maximum power transfer as the level of compensation changes in constant reactance mode.

Improving Power-Angle Characteristics

Noting that the sending end apparent power is $V_S\, I_S{}^*$, we can write

$$\begin{aligned} P_S + jQ_S = V_S I_S^* = V\angle\delta\left[\frac{V\angle-\delta - V}{-j(X \mp \lambda)}\right] = \frac{V^2 - V^2\angle\delta}{-j(X \mp \lambda)} \\ = \frac{V^2 \sin\delta}{X \mp \lambda} + j\frac{V^2(1-\cos\delta)}{X \mp \lambda} \end{aligned} \qquad (10.22)$$

Similarly the receiving end apparent power is given by

$$\begin{aligned} P_R + jQ_R = V_R I_S^* = V\left[\frac{V\angle-\delta - V}{-j(X \mp \lambda)}\right] \\ = \frac{V^2 \sin\delta}{X \mp \lambda} + j\frac{V^2(\cos\delta - 1)}{X \mp \lambda} \end{aligned} \qquad (10.23)$$

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The power-angle characteristics of a series compensated power system are given in Fig. 10.12. In this figure the base power is chosen as V^2 / X. Three curves are shown, of which the curve P_0 is the **power-angle curve** when the line is not compensated. Curves which have maximum powers greater than the base power pertain to capacitive mode of operation. On the other hand, all curves the inductive mode of operation will have maximum values less than 1. For example, in Fig. 10.12, the curve P_1 is for capacitive mode and the curve P_2 is for inductive mode of operation.

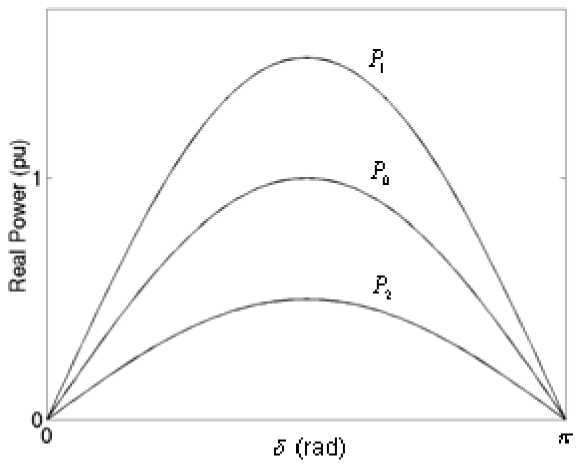

Fig. 10.12 Power-angle characteristics in constant reactance mode.

Let us now have a look at the reactive power. For simplicity let us restrict our attention to capacitive mode of operation only as this represents the normal mode of operation in which the power transfer over the line is enhanced. From (10.20) and (10.21) we get the reactive power supplied by the compensator as

$$Q_Q = V_Q I_S^* = -j\lambda \frac{V\angle\delta - V}{j(X-\lambda)} \times \frac{V\angle-\delta - V}{-j(X-\lambda)}$$

Solving the above equation we get

$$Q_Q = -j \frac{2\lambda V^2}{(X-\lambda)^2} (1 - \cos\delta) \tag{10.25}$$

In Fig. 10.13, the reactive power injected by the series compensator is plotted against the maximum power transfer as the compensation level changes from 10% to 60%. As the compensation level increases, the maximum power transfer also increases. However, at the same time, the reactive injection requirement from the series compensator also increases. It is interesting to note that at 50% compensation level, the reactive power injection requirement from a series compensator is same that from shunt compensator that is regulating the midpoint voltage to 1.0 per unit.

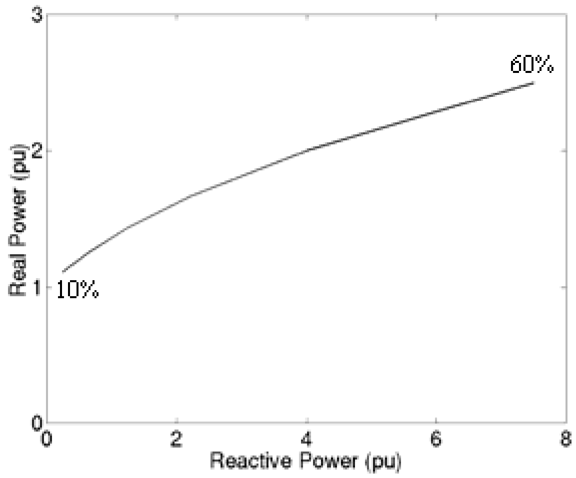

Fig. 10.13 Reactive power injection by a series compensator versus maximum power transfer as the level of compensation changes in constant reactance mode.

Improving Power-Angle Characteristics

Noting that the sending end apparent power is $V_S\, I_S{}^*$, we can write

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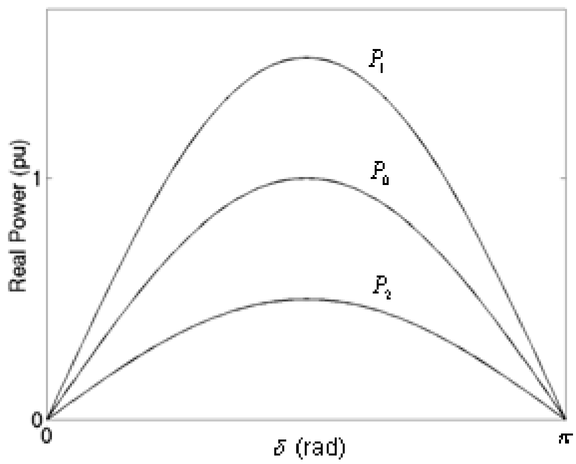

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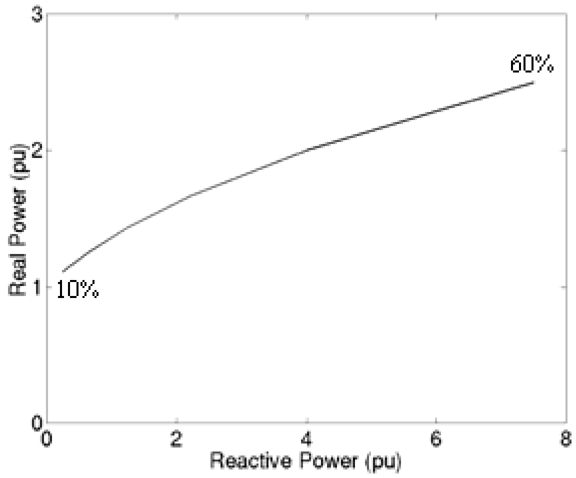

Fig. 10.13 Reactive power injection by a series compensator versus maximum power transfer as the level of compensation changes in constant reactance mode.

Improving Power-Angle Characteristics

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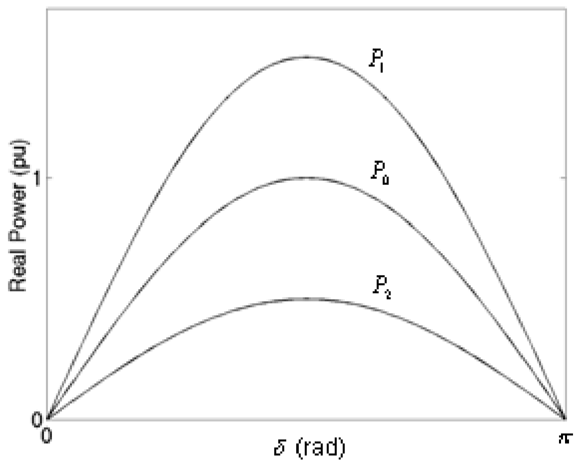

Fig. 10.12 Power-angle characteristics in constant reactance mode.

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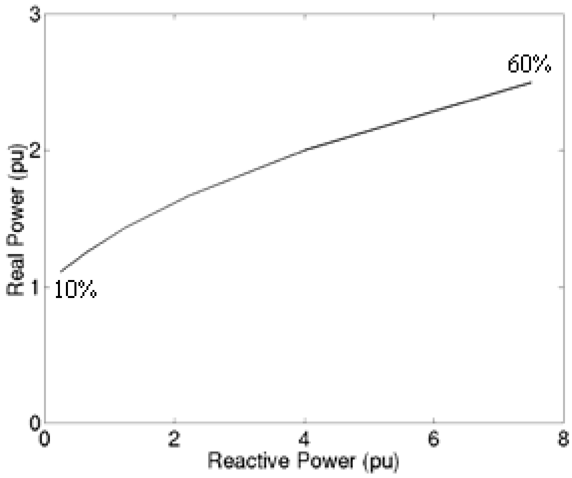


Fig. 10.13 Reactive power injection by a series compensator versus maximum power transfer as the level of compensation changes in constant reactance mode.

An Alternate Method of Voltage Injection

So far we have assumed that the series compensator injects a voltage that is in quadrature with the line current and its magnitude is proportional to the magnitude of the line current. A set of very interesting equations can be obtained if the last assumption about the magnitude is relaxed. The injected voltage is then given by

$$V_Q = \lambda \frac{\vec{I}_S}{|I_S|} e^{\mp j90^\circ} \tag{10.26}$$

We can then write the above equation as

$$\frac{V_Q}{I_S} = \frac{\lambda}{|I_S|} e^{\mp j90^\circ} = \mp j X_Q \tag{10.27}$$

i.e., the voltage source in quadrature with the current is represented as a pure reactance that is either inductive or capacitive. Since in this form we injected a constant voltage in quadrature with the line current, we shall refer this as **constant voltage injection mode**. The total equivalent inductance of the line is then

$$X_{eq} = X \mp X_Q$$

Defining $V_S = V < \delta$ and $V_R < 0°$, we can then write the power transfer equation as

$$P_e = \frac{V^2}{X_{eq}} \sin\ \delta = \frac{V^2}{X(1 \mp X_Q / X)} \sin\ \delta$$

Since $|V_Q| / |I_S| = X_Q$, we can modify the above equation as

$$P_e = \frac{V^2}{X(1 \mp |V_Q| / |I_S| X)} \sin\ \delta \tag{10.28}$$

Consider the phasor diagram of Fig. 10.14 (a), which is for capacitive operation of the series compensator. From this diagram we get

$$|I_S|X = |V_Q| + 2V\sin(\delta/2)$$

Similarly from the inductive operation phasor diagram shown in Fig. 10.14 (b), we get

$$|I_S|X = -|V_Q| + 2V\sin(\delta/2)$$

Substituting the above two equations in (10.28) and rearranging we get

$$\begin{aligned} P_e &= \frac{V^2}{X}\sin\delta\frac{|I_S|X}{|I_S|X \mp |V_Q|} = \frac{V^2}{X}\sin\delta\frac{\pm|V_Q| + 2V\sin(\delta/2)}{\pm|V_Q| + 2V\sin(\delta/2) \mp |V_Q|} \\ &= \frac{V^2}{X}\sin\delta\frac{\pm|V_Q| + 2V\sin(\delta/2)}{2V\sin(\delta/2)} = \frac{V^2}{X}\sin\delta \pm \frac{V}{X}|V_Q|\cos(\delta/2) \end{aligned} \tag{10.29}$$

where the positive sign is for capacitive operation.

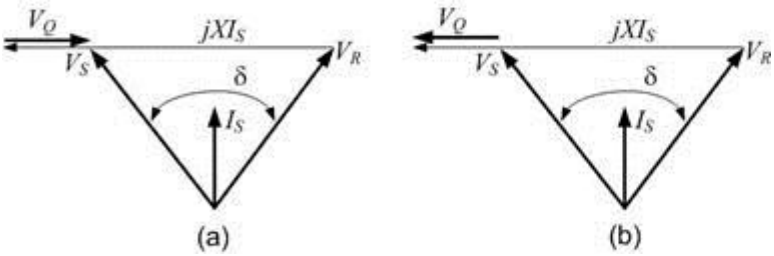


Fig. 10.14 Phasor diagram of series compensated system: (a) capacitive operation and (b) inductive operation.

Contd...An Alternate Method of Voltage Injection

The power-angle characteristics of this particular series connection are given in Fig. 10.15. In this figure the base power is chosen as V^2/X . Three curves are shown, of which the curve P_0 is the **power-angle curve** when the line is not compensated. Curves which have maximum powers greater than the base power pertain to capacitive mode of operation. On the other hand, all curves the inductive mode of operation will have maximum values less than 1. For example, in Fig. 10.15, the curve P_1 is for capacitive mode and the curve P_2 is for inductive mode of operation.

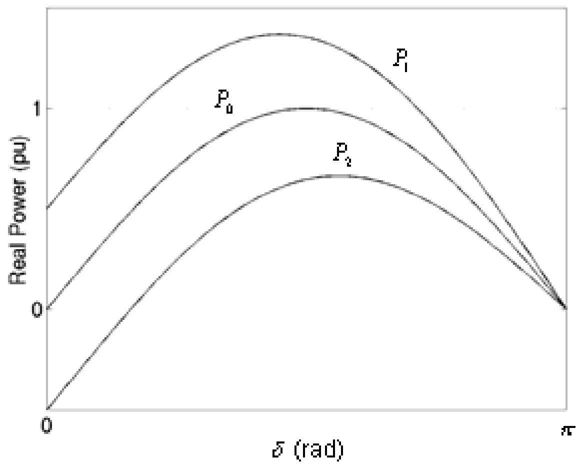


Fig. 10.15 Power-angle characteristics for constant voltage mode.

The reactive power supplied by the compensator in this case will be

$$Q_Q = |V_Q||I_S| \tag{10.30}$$

Improving Stability Margin

From the power-angle curves of Figs. 10.13 and 10.15 it can be seen that the same amount of power can be transmitted over a capacitive compensated line at a lower load angle than an uncompensated system. Furthermore, an increase in the height in the power-angle curve means that a larger amount of decelerating area is available for a compensated system. Thus improvement in stability margin for a capacitive series compensated system over an uncompensated system is obvious.

Comparisons of the Two Modes of Operation

As a comparison between the two different modes of voltage injection, let us first consider the constant reactance mode of voltage injection with a compensation level of 50%. Choosing V^2 / X as the base power, the power-angle characteristic reaches a maximum of 2.0 per unit at a load angle π / 2. Now | V_Q | in constant voltage mode is chosen such that the real power is 2.0 per unit at a load angle of π / 2. This is accomplished using (10.29) where we get

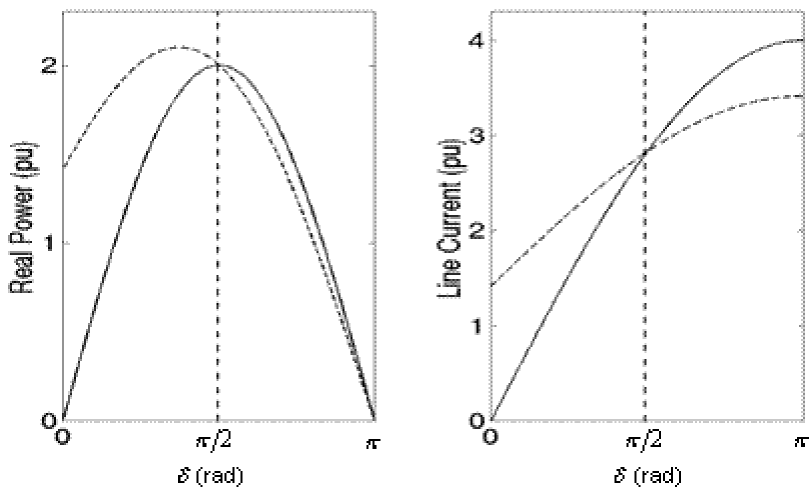
$$|V_Q| = \frac{2 - \sin 90^\circ}{\cos 45^\circ} = 1.4142 \text{ per unit}$$

The power-angle characteristics of the two different modes are now drawn in Fig. 10.16 (a). It can be seen that the two curves match at π / 2. However, the maximum power for constant voltage case is about 2.1 per unit and occurs at an angle of 67° .

Fig. 10.16 (b) depicts the line current for the two cases. It can be seen that the increase in line current in either case is monotonic. This is not surprising for the case of constant reactance mode since as the load angle increases, both real power and line currents increase. Now consider the case of constant voltage control. When the load angle moves backwards from $\pi/2$ to 67° , the power moves from 2.0 per unit to its peak value of 2.1 per unit. The line current during this stage decreases from about 2.83 to 2.50 per unit. Thus, even though the power through the line increases, the line current decreases.

Power Flow Control and Power Swing Damping

One of the major advantages of series compensation is that through its use real power flow over transmission corridors can be effectively controlled. Consider, for example, the SMIB system shown in Fig. 10.17 in which the generator and infinite bus are connected through a double circuit transmission line, labeled line-1 and line-2. Of the two transmission lines, line-2 is compensated by a series compensator. The compensator then can be utilized to regulate power flow over the entire system.



(a) (b)

Fig. 10.16 Power-angle and line current-angle characteristics of the two different methods of voltage injection: solid line showing constant reactance mode and dashed line showing constant voltage mode.

For example, let us consider that the system is operating in the steady state delivering a power of P_{m0} at a load angle of δ_0 . Lines 1 and 2 are then sending power P_{e1} and P_{e2} respectively, such that $P_{m0} = P_{e1} + P_{e2}$. The mechanical power input suddenly goes up to P_{m1} . There are two ways of controlling the power in this situation:

- **Regulating Control**: Channeling the increase in power through line-1. In this case the series compensator maintains the power flow over line-2 at P_{e2} . The load angle in this case goes up in sympathy with the increase in P_{e1} .
- **Tracking Control:** Channeling the increase in power through line-2. In this case the series compensator helps in maintaining the power flow over line-1 at P_{e1} while holding the load angle to δ_0 .

Let us illustrate these two aspects with the help of a numerical example.

Example 10.4