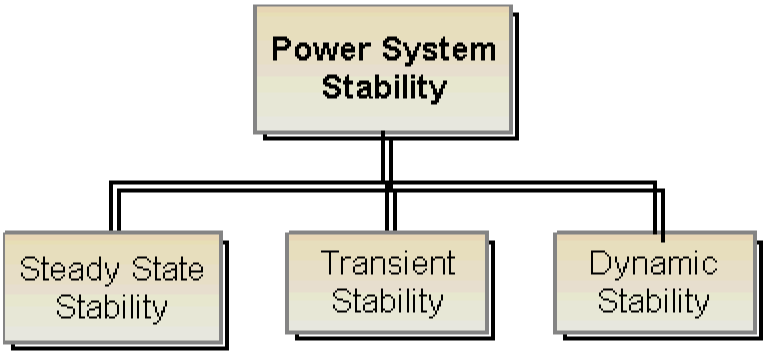
Introduction

The first electric power system was a dc system built by Edison in 1882. The subsequent power systems that were constructed in the late 19th century were all dc systems. However despite the initial popularity of dc systems by the turn of the 20th century ac systems started to outnumber them. The ac systems were though to be superior as ac machines were cheaper than their dc counterparts and more importantly ac voltages are easily transformable from one level to other using transformers. The early stability problems of ac systems were experienced in 1920 when insufficient damping caused spontaneous oscillations or hunting. These problems were solved using generator damper winding and the use of turbine-type prime movers.

The stability of a system refers to the ability of a system to return back to its steady state when subjected to a disturbance. As mentioned before, power is generated by synchronous generators that operate in synchronism with the rest of the system. A generator is synchronized with a bus when both of them have same frequency, voltage and phase sequence. We can thus define the power system stability as the ability of the power system to return to steady state without losing synchronism. Usually power system stability is categorized into **Steady State**, **Transient** and **Dynamic Stability.**



Steady State Stability studies are restricted to small and gradual changes in the system operating conditions. In this we basically concentrate on restricting the bus voltages close to their nominal values. We also ensure that phase angles between two buses are not too large and check for the overloading of the power equipment and transmission lines. These checks are usually done using power flow studies.

Transient Stability involves the study of the power system following a major disturbance. Following a large disturbance the synchronous alternator the machine power (load) angle changes due to sudden acceleration of the rotor shaft. The objective of the transient stability study is to ascertain whether the load angle returns to a steady value following the clearance of the disturbance.

The ability of a power system to maintain stability under continuous small disturbances is investigated under the name of **Dynamic Stability** (also known as small-signal stability). These small disturbances occur due random fluctuations in loads and generation levels. In an interconnected power system, these random variations can lead catastrophic failure as this may force the rotor angle to increase steadily.

In this chapter we shall discuss the transient stability aspect of a power system.

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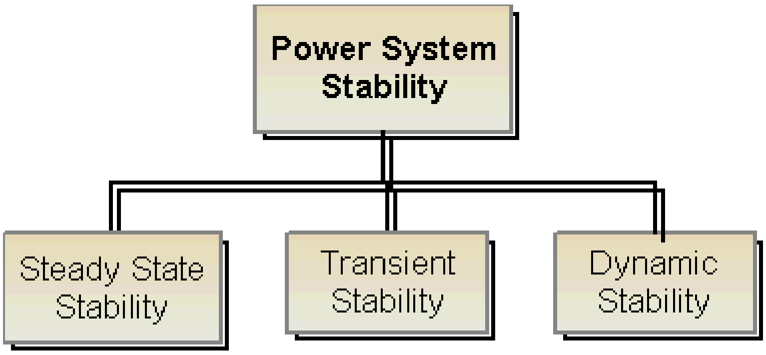

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Section I: Power-Angle Relationship

The power-angle relationship has been discussed in Section 2.4.3. In this section we shall consider this relation for a lumped parameter lossless transmission line. Consider the **single-machine-infinite-bus** (SMIB) system shown in Fig. 9.1. In this the reactance X includes the reactance of the transmission line and the synchronous reactance or the transient reactance of the generator. The sending end voltage is then the internal emf of the generator. Let the sending and receiving end voltages be given by

$$V_S = V_1 \angle \delta, \quad V_R = V_2 \angle 0° \tag{9.1}$$

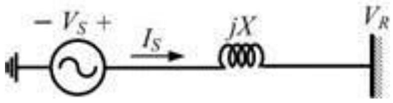


Fig. 9.1 An SMIB system.

$$I_S = \frac{V_1 \angle \delta - V_2}{jX} = \frac{V_1 \cos\delta - V_2 + jV_1 \sin\delta}{jX} \tag{9.2}$$

We then have

$$P_S + jQ_S = V_S I_S^* = V_1(\cos\delta + j\sin\delta)\frac{V_1\cos\delta - V_2 - jV_1\sin\delta}{-jX}$$

The sending end real power and reactive power are then given by

$$P_S + jQ_S = \frac{V_1V_2\sin\delta + j\left(V_1^2 - V_1V_2\cos\delta\right)}{X} \tag{9.3}$$

This is simplified to

Since the line is loss less, the real power dispatched from the sending end is equal to the real power

$$P_e = P_S = P_R = \frac{V_1V_2}{X}\sin\delta = P_{\max}\sin\delta \tag{9.4}$$

received at the receiving end. We can therefore write where $P_{max} = V_1 V_2 / X$ is the maximum power that can be transmitted over the transmission line. The power-angle curve is shown in Fig. 9.2. From this figure we can see that for a given power P_0 . There

$$\delta_0 = \sin^{-1}\left(\frac{P_0}{P_{max}}\right) \tag{9.5}$$

$$\delta_{max} = 180° - \delta_0$$

are two possible values of the angle δ - δ_0 and δ_{max} . The angles are given by

Example 9.1

Section II: Swing Equation

Let us consider a three-phase synchronous alternator that is driven by a prime mover. The equation of motion of the machine rotor is given by

$$J\frac{d^2\theta}{dt^2} = T_m - T_e = T_a \tag{9.6}$$

where

J	is the total moment of inertia of the rotor mass in kgm^2
T_m	is the mechanical torque supplied by the prime mover in N-m
T_e	is the electrical torque output of the alternator in N-m
θ	is the angular position of the rotor in rad

Neglecting the losses, the difference between the mechanical and electrical torque gives the net accelerating torque T_a . In the steady state, the electrical torque is equal to the mechanical torque, and hence the accelerating power will be zero. During this period the rotor will move at **synchronous speed** ω_s in rad/s.

The angular position θ is measured with a stationary reference frame. To represent it with respect to the synchronously rotating frame, we define

$$\theta = \omega_s t + \delta \tag{9.7}$$

where δ is the angular position in rad with respect to the synchronously rotating reference frame.

Taking the time derivative of the above equation we get

$$I_S = \frac{V_1\angle\delta - V_2}{jX} = \frac{V_1\cos\delta - V_2 + jV_1\sin\delta}{jX} \tag{9.8}$$

Defining the angular speed of the rotor as

we can write (9.8) as

$$\omega_r = \frac{d\theta}{dt}$$

$$\omega_r - \omega_s = \frac{d\delta}{dt} \tag{9.9}$$

We can therefore conclude that the rotor angular speed is equal to the synchronous speed only when *dδ / dt* is equal to zero. We can therefore term *dδ / dt* as the error in speed. Taking derivative of (9.8), we can then rewrite (9.6) as

$$J\frac{d^2\delta}{dt^2} = T_m - T_e = T_a \tag{9.10}$$

Multiplying both side of (9.11) by ω_m we get

$$J\omega_r\frac{d^2\delta}{dt^2} = P_m - P_e = P_a \tag{9.11}$$

where P_m , P_e and P_a respectively are the mechanical, electrical and accelerating power in MW.

We now define a normalized inertia constant as

$$H = \frac{\text{Stored kinetic energy at synchronou s speed in mega - joules}}{\text{Generator MVA rating}} = \frac{J\omega_s^2}{2S_{rated}} \tag{9.12}$$

Substituting (9.12) in (9.10) we get

$$2H\frac{S_{rated}}{\omega_s^2}\omega_r\frac{d^2\delta}{dt^2} = P_m - P_e = P_a \tag{9.13}$$

In steady state, the machine angular speed is equal to the synchronous speed and hence we can replace ω_r in the above equation by ω_s. Note that in (9.13) P_m , P_e and P_a are given in MW. Therefore dividing them by the generator MVA rating S_{rated} we can get these quantities in per unit. Hence dividing both sides of (9.13) by S_{rated} we get

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e = P_a \quad \text{per unit} \tag{9.14}$$

Equation (7.14) describes the behaviour of the rotor dynamics and hence is known as the swing equation. The angle δ is the angle of the internal emf of the generator and it dictates the amount of power that can be transferred. This angle is therefore called the **load angle** .

Example 9.2

Section III: Equal Area Criterion

The real power transmitted over a lossless line is given by (9.4). Now consider the situation in which the synchronous machine is operating in steady state delivering a power P_e equal to P_m when there is a fault occurs in the system. Opening up of the circuit breakers in the faulted section subsequently clears the fault. The circuit breakers take about 5/6 cycles to open and the subsequent post-fault transient last for another few cycles. The input power, on the other hand, is supplied by a prime mover that is usually driven by a steam turbine. The time constant of the turbine mass system is of the order of few seconds, while the electrical system time constant is in milliseconds. Therefore, for all practical purpose, the mechanical power is remains constant during this period when the electrical transients occur. The transient stability study therefore concentrates on the ability of the power system to recover from the fault and deliver the constant power P_m with a possible new load angle δ .

Consider the power angle curve shown in Fig. 9.3. Suppose the system of Fig. 9.1 is operating in the steady state delivering a power of P_m at an angle of δ_0 when due to malfunction of the line, circuit breakers open reducing the real power transferred to zero. Since P_m remains constant, the accelerating power P_a becomes equal to P_m . The difference in the power gives rise to the rate of change of stored kinetic energy in the rotor masses. Thus the rotor will accelerate under the constant influence of non-zero accelerating power and hence the load angle will increase. Now suppose the circuit breaker re-closes at an angle δ_c. The power will then revert back to the normal operating curve. At that point, the electrical power will be more than the mechanical power and the accelerating power will be negative. This will cause the machine decelerate. However, due to the inertia of the rotor masses, the load angle will still keep on increasing. The increase in this angle may eventually stop and the rotor may start decelerating, otherwise the system will lose synchronism.

Note that

$$\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = 2\left(\frac{d\delta}{dt}\right)\left(\frac{d^2\delta}{dt^2}\right)$$

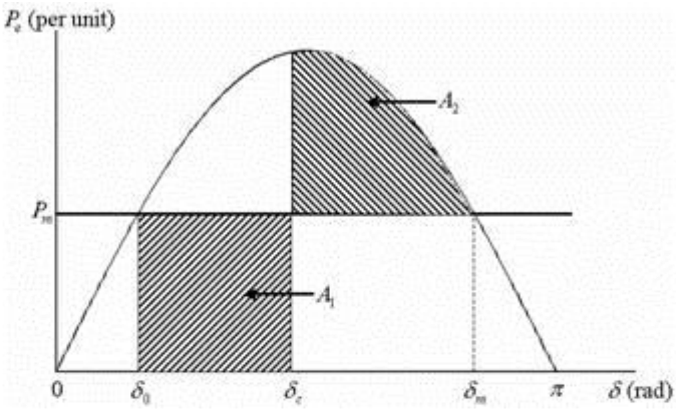


Fig. 9.3 Power-angle curve for equal area criterion.

$$\frac{H}{\omega_s}\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = \left(P_m - P_e\right)\frac{d\delta}{dt}$$

Hence multiplying both sides of (9.14) by $d\delta/dt$ and rearranging we get

Multiplying both sides of the above equation by *dt* and then integrating between two arbitrary angles

$$\frac{H}{\omega_s}\left(\frac{d\delta}{dt}\right)^2\Bigg|_{\delta_0}^{\delta_c} = \int_{\delta_0}^{\delta_c}\left(P_m - P_e\right)d\delta \tag{9.15}$$

δ_0 and δ_c we get

Now suppose the generator is at rest at δ_0. We then have $d\delta / dt = 0$. Once a fault occurs, the machine starts accelerating. Once the fault is cleared, the machine keeps on accelerating before it reaches its peak at δ_c , at which point we again have $d\delta / dt = 0$. Thus the area of accelerating is

$$A_1 = \int_{\delta_0}^{\delta_c} (P_m - P_e) d\delta = 0 \tag{9.16}$$

given from (9.15) as

In a similar way, we can define the area of deceleration. In Fig. 9.3, the area of acceleration is given

$$A_2 = \int_{\delta_c}^{\delta_m} (P_e - P_m) d\delta = 0$$

by A_1 while the area of deceleration is given by A_2 . This is given by

Contd... Equal Area Criterion

Now consider the case when the line is reclosed at δ_c such that the area of acceleration is larger than the area of deceleration, i.e., $A_1 > A_2$. The generator load angle will then cross the point δ_m , beyond which the electrical power will be less than the mechanical power forcing the accelerating power to be positive. The generator will therefore start accelerating before is slows down completely and will eventually become unstable. If, on the other hand, $A_1 < A_2$, i.e., the decelerating area is larger than the accelerating area, the machine will decelerate completely before accelerating again. The rotor inertia will force the subsequent acceleration and deceleration areas to be smaller than the first ones and the machine will eventually attain the steady state. If the two areas are equal, i.e., $A_1 = A_2$, then the accelerating area is equal to decelerating area and this is defines the **boundary of the stability limit.** The clearing angle δ_c for this mode is called the **Critical Clearing Angle** and is denoted by δ_{cr}.

$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_e) d\delta = \int_{\delta_{cr}}^{\delta_m} (P_e - P_m) d\delta \tag{9.18}$$

We then get from Fig. 9.3 by substituting $\delta_c = \delta_{cr}$

We can calculate the critical clearing angle from the ab move equation. Since the critical clearing angle depends on the equality of the areas, this is called the **equal area criterion**.

Example 9.3: Consider the system of Example 9.1. Let us assume that the system is operating with $P_m = P_e = 0.9$ per unit when a circuit breaker opens inadvertently isolating the generator from the infinite bus. During this period the real power transferred becomes zero. From Example 9.1 we have calculated δ_0 =

23.96 ° = 0.4182 rad and the maximum power transferred as

$$P_{max} = \frac{1.1082 \times 1}{0.5} = 2.2164 \text{ per unit}$$

We have to find the critical clearing angle.

From (9.15) the accelerating area is computed as by note that $P_e = 0$ during this time. This is then given by

$$A_1 = \int_{0.4182}^{\delta_{cr}} 0.9\, d\delta = 0.9\delta_{cr} - 0.9 \times 0.4182 = 0.9\delta_{cr} - 0.3764$$

To calculate the decelerating area we note that $\delta_m = \pi - 0.4182 = 2.7234$ rad. This area is computed by noting that $P_e = 2.2164 \sin(\delta)$ during this time. Therefore

$$\begin{aligned} A_2 &= \int_{\delta_{cr}}^{2.7234} (2.2164 \sin \delta - 0.9)\, d\delta \\ &= -2.2164 \times \cos(2.7234) + 2.2164 \cos(\delta_{cr}) - 0.9 \times 2.7234 + 0.9\delta_{cr} \\ &= 2.2164 \cos(\delta_{cr}) + 0.9\delta_{cr} - 0.4257 \end{aligned}$$

Equating $A_1 = A_2$ and rearranging we get

$$\delta_{cr} = \cos^{-1}\left(\frac{0.0493}{2.2164}\right) = 1.5486 \text{ rad} = 88.73°$$

Now a frequently asked question is what does the critical clearing angle mean?

Since we are interested in finding out the maximum time that the circuit breakers may take for opening, we should be more concerned about the critical clearing time rather than clearing angle. Furthermore, notice that the clearing angle is independent of the generalized inertia constant H . Hence we can comment that the critical clearing angle in this case is true for any generator that has a d-axis transient reactance of 0.20 per unit. The critical clearing time, however, is dependent on H and will vary as this parameter varies.

To obtain a description for the critical clearing time, let us consider the period during which the fault

$$\frac{d^2\delta}{dt^2} = \frac{\omega_s}{2H} P_m \tag{9.19}$$

occurs. We then have $P_e = 0$. We can therefore write from

Integrating the above equation with the initial acceleration being zero we get

$$\frac{d\delta}{dt} = \int_0^t \frac{\omega_s}{2H} P_m dt = \frac{\omega_s}{2H} P_m t$$

$$\delta = \int_0^t \frac{\omega_s}{2H} P_m t\, dt = \frac{\omega_s}{4H} P_m t^2 + \delta_0$$

Further integration will lead to

$$t_{cr} = \sqrt{\frac{4H}{\omega_s P_m}\left(\delta_{cr} - \delta_0\right)} \qquad (9.20)$$

Replacing δ by δ_{cr} and t by t_{cr} in the above equation, we get the critical clearing time as

Example 9.4:

<u>In Example 9.2</u>, let us choose the system frequency as 50 Hz such that ω_s is 100π. Also let us choose H as 4 MJ/MVA. Then with δ_{cr} being 1.5486 rad, δ_0 being 0.4182 rad and P_m being 0.9 per unit, we get the following critical clearing time from (9.20)

$$t_{cr} = \sqrt{\frac{16}{90\pi}\left(1.5486 - 0.4182\right)} = 0.253$$ s

To illustrate the response of the load angle δ , the swing equation is simulated in MATLAB. The swing

$$\frac{d\Delta\omega_r}{dt} = \frac{1}{2H}\left(P_m - P_e\right)$$
$$\frac{d\delta}{dt} = \omega_s \times \Delta\omega_r \qquad (9.21)$$

equation of (9.14) is then expressed as

where $\Delta\omega_r$ is the deviation for the rotor speed from the synchronous speed ω_s . It is to be noted that the swing equation of (9.21) does not contain any damping. Usually a damping term, that is proportional to the machine speed $\Delta\omega_r$, is added with the accelerating power. Without the damping the load angle will exhibit a sustained oscillation even when the system remains stable when the fault cleared within the critical clearing time.

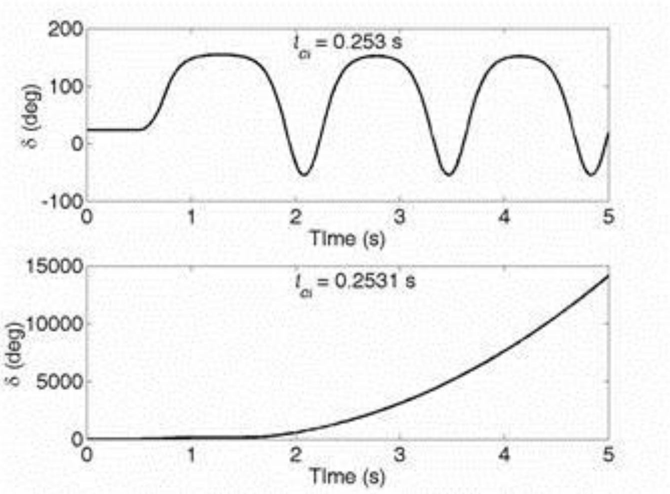


Fig. 9.4 Stable and unstable system response as a function of clearing time.

Fig. 9.4 depicts the response of the load angle δ for two different values of load angle. It is assumed that the fault occurs at 0.5 s when the system is operating in the steady state delivering 0.9 per unit power. The load angle during this time is constant at 23.96° . The load angle remains stable, albeit the sustained oscillation when the clearing time t_{cl} is 0.253 s. The clearing angle during this time is 88.72° . The system however becomes unstable when the clearing time 0.2531s and the load angle increases asymptotically. The clearing time in this case is 88.77° . This is called the **Loss of Synchronism**. It is to be noted that such increase in the load angle is not permissible and the protection device will isolate the generator from the system.

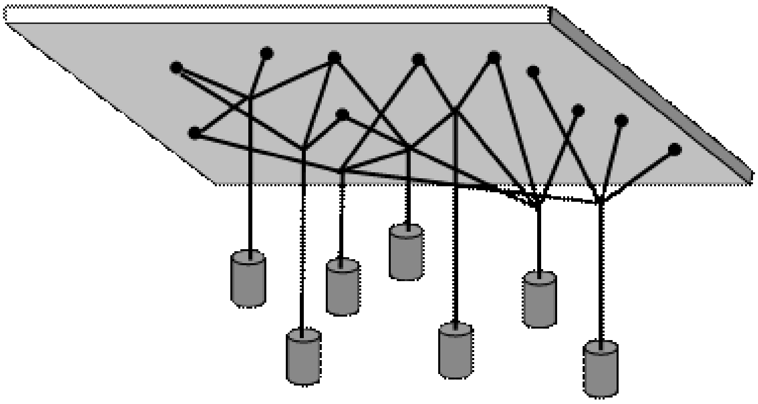
The clearing time of (8.20) is derived based on the assumption that the electrical power P_e becomes zero during the fault as in (8.19). This need not be the case always. In that even we have to resort to finding the clearing time using the numerical integration of the swing equation. See example 9.5 to illustrates the point.

Example 9.5

Section IV: Multimachine Stability

- <u>**Oscillations in s Two Area System**</u>

Consider Fig. 9.10, which depicts a number of weights that are suspended by elastic strings. The weights represent generators and the electric transmission lines being represented by the strings. Note that in a transmission system, each transmission line is loaded below its static stability limit. Similarly, when the mechanical system is in static steady state, each string is loaded below its break point. At this point one of the strings is suddenly cut. This will result in transient oscillations in the coupled strings and all the weights will wobble. In the best possible case, this may result in the coupled system settling down to a new steady state. On the other hand, in the worst possible scenario this may result in the breaking of one more additional string, resulting in a chain reaction in which more strings may break forcing a system collapse. In a similar way, in an interconnected electric power network, the tripping of a transmission line may cause a catastrophic failure in which a large number of generators are lost forcing a blackout in a large area.



Modern power systems are interconnected and operate close to their transient and steady state stability limits. In large interconnected systems, it is common to find a natural response of a group of closely coupled machines oscillating against other groups of machines. These oscillations have a frequency range of 0.1 Hz to 0.8 Hz. The lowest frequency mode involves all generators of the system. This oscillation groups the system into two parts - with generators in one part oscillating against those of the the other part. The higher frequency modes are usually localized with small groups oscillating against each other. Unfortunately, the inter-area oscillation can be initiated by a small disturbance in any part of the system. These small frequency oscillations fall under the category of dynamic stability and are analysed in linear domain through the liberalisation of the entire interconnected systems model.

Inter-area oscillations manifest wherever the power system is heavily interconnected. The oscillations, unless damped, can lead to grid failure and total system collapse. Low frequency oscillations in the range of 0.04 Hz to 0.06 Hz were observed in the Pacific North West region as early as 1950. Improper speed governor control of hydro units created these oscillations. The Northern and Southern regions of WSCC were interconnected by a 230 kV line in 1964. Immediately the system experienced a 0.1 Hz oscillation resulting in over 100 instances of opening of the tie line in the first nine months of operation. Some system damping was provided through the modification in the hydro turbine governors.

A 500 kV pacific intertie and another ± 400 kV HVDC system was commissioned in 1968. This raised the frequency of oscillation from 0.1 Hz to 0.33 Hz and these oscillations could no longer be controlled through governor action alone. In late 1980's a new intertie joined the WSCC system to Alberta and British Columbia in Canada . As a result of this interconnection, the two different oscillation frequencies manifested - one at 0.29 Hz and the other at 0.45 Hz.

Ontario Hydro is one of the largest utilities in North America . Due to the vast and sparsely populated topology of Canada , the operating span of Ontario hydro is over 1000 km from East to West and from North to South. The Ontario Hydro system is connected to the neighbouring Canadian provinces and the North Western region of the United States . In 1959 Ontario Hydro was connected to Michigan in the South and Quebec Hydro in the East. As a result of this connection, a 0.25 Hz oscillation was observed and a result of this it was decided to remove the tie with Quebec and retain the tie to Michigan . The Western portion of Ontario was connected to neighbouring Manitoba in 1956 and then Manitoba was connected to its neighbour Saskatchewan in 1960. This resulted in oscillation in the frequency range 0.35 Hz to 0.45 Hz often tripping the tie. As a result of this, Ontario Hydro decided to commission power system stabilizers for all their generating units since early 1960's. It has also sponsored extensive research in this area.

Through research it was established that the action of automatic voltage regulators caused these oscillations. An automatic voltage regulator (AVR) regulates the generator terminal voltage and also helps in the enhancement of transient stability by reducing the peak of the first swing following any disturbance. However, its high gain contributed to negative damping to the system. The knowledge of this relation resulted in the commissioning of power system stabilizers. It was observed that these oscillations were results of the periodic interchange of kinetic energy between the generator rotors. A power system stabilizer (PSS) provides a negative feedback of the changes in rotor kinetic energy when it is connected to the excitation system thereby providing damping to these small oscillations. The PSS has been a subject of extensive research. The team of Dr. P. Kundur, then with Ontario Hydro, and his co-workers has done extensive research in the area of PSS tuning and its characteristics. Through their vast experience and extensive research, they reported the enhancement of inter-area and local modes through PSS reported in. Since a power system is piece-wise linear, its system characteristics changes with operating point. Therefore an adaptive controller that can tune with the changes in the system has been developed and reported in. It was shown that the adaptive PSS is effective in damping large as well as small disturbances.

The power flow between generators, as evident from (9.4), is dependent on the angle between those generators. The stable operating point of the power system is where the generated power at each station is matched by the electrical power sent out from that station. When there is a mismatch between electrical power out and the generated mechanical shaft power, the generator will accelerate at a rate determined by the power mismatch and the machine inertia as given in (9.14).

Oscillations in s Two Area System

Consider the simple power system shown in Fig. 9.11 in which two machines are operating. Let us assume that starting with the initial angles δ_1 and δ_2 with respect to some reference at nominal frequency, machine 1 accelerates while machine 2 decelerates from this nominal frequency. We then

$$\frac{2H_1}{\omega_s}\ddot{\delta}_1 = P_{m1} - P_{e1}$$
$$\frac{2H_2}{\omega_s}\ddot{\delta}_2 = P_{m2} - P_{e2} \qquad (9.25)$$

have

where the subscripts 1 and 2 refer to machines 1 and 2 respectively. Let us assume that the transmission line is loss less. Then in the simple case where the power from machine 1 flows to

$$P_{e1} = -P_{e2} = \frac{V_1V_2}{X}\sin\left(\delta_1 - \delta_2\right) = \frac{V_1V_2}{X}\sin\,\delta_{12} \qquad (9.26)$$

machine 2, we get

where $\delta_{12} = \delta_1 - \delta_2$.

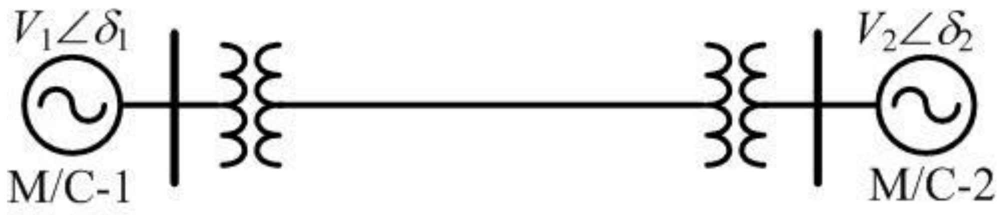


Fig. 9.11 Single-line diagram of a two-machine power system.

Now since the system is lossless, (9.26) will also imply that P_{m1} = - P_{m2} . This means that in the steady state, the power generated at machine 1 is absorbed through machine 2. Combining (9.25) and (9.26)

$$\frac{2H_1}{\omega_s}\ddot{\delta}_1 - \frac{2H_2}{\omega_s}\ddot{\delta}_2 = 2P_{m1} - P_{e1} + P_{e2} = 2P_{m1} - \frac{2V_1V_2}{X}\sin\,\delta_{12} \tag{9.27}$$

we get

$$\ddot{\delta}_{12} = -\omega^2\,\sin\,\delta_{12} \tag{9.28}$$

Let us now assume that $H_1 = H_2 = H$, $V_1 = V_2 = 1.0$ per unit and $P_{m1} = 0$. We then get from (9.27)

$$\omega = \sqrt{\omega_s / HX} \tag{9.29}$$

where the oscillation frequency ω is given by

Thus the weighted difference of angles will approximate simple harmonic motion for small changes in δ_{12} and the frequency will decrease for an increase in inertia H or impedance X . Another aspect can

$$H_1\ddot{\delta}_1 + H_2\ddot{\delta}_2 = P_{m1} + P_{m2} = 0 \tag{9.30}$$

be seen by adding the system to give

Thus the overall acceleration of the machine group will depend on the overall balance between power generated and consumed. Usually there are governors on the generators to reduce generated power if the system frequency increases.