Introduction

Short circuits occur in power system due to various reasons like, equipment failure, lightning strikes, falling of branches or trees on the transmission lines, switching surges, insulation failures and other electrical or mechanical causes. All these are collectively called **faults** in power systems.

A fault usually results in high current flowing through the lines and if adequate protection is not taken, may result in damages in the power apparatus.

In this chapter we shall discuss the effects of **symmetrical faults** on the system. Here the term symmetrical fault refers to those conditions in which all three phases of a power system are grounded at the same point. For this reason the symmetrical faults sometimes are also called three-line-to-ground (3LG) faults.

Section I: Transients in R-L Circuits

Transients in R-L Circuits

In this section we shall consider transients in a circuit that contains a resistor and inductor (R - L circuit). Consider the circuit shown in Fig. 6.1 that contains an ideal source (v_s), a resistor (R), an inductor (L) and a switch (S). It is assumed that the switch is open and is closed at an instant of time t = 0. This implies that the current i is zero before the closing of the switch. We shall first discuss the effect of closing the switch on the line current (i) when the source is dc. Following this we shall study the effect when the source is ac and will show that the shape of the transient current changes with the changes in the phase of the source voltage waveform at the instant of closing the switch.

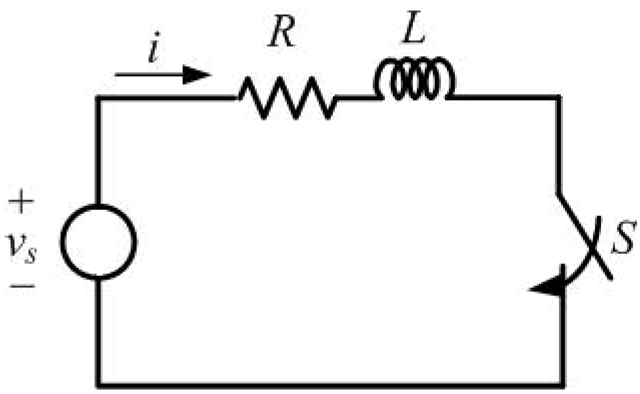


Fig 6.1 A Simple R - L Circuit

DC Source

Let us assume that the source voltage is dc and is given by $v_s = V_{dc}$. Then the line current is given by

$$\frac{di}{dt} = -\frac{R}{L}i + \frac{1}{L}v_s \tag{6.1}$$

the differential equation

$$i(t) = e^{-Rt/L}i(0) + \frac{1}{L}\int_0^t e^{-(R/L)(t-\tau)}v_s(\tau)d\tau \tag{6.2}$$

The solution of the above equation is written in the form

Since the initial current i (0) = 0 and since v_s (τ) = V_{dc} for 0 £ t < ¥ , we can rewrite the above

$$i(t) = \frac{V_{dc}}{R}\left(1 - e^{-Rt/L}\right) = \frac{V_{dc}}{R}\left(1 - e^{-t/T}\right) \tag{6.3}$$

equation as

where $T = L / R$ is the time constant of the circuit.

Let us assume R = 1Ω , L = 10 mH and V_{dc}= 100 V. Then the time response of the current is as shown in Fig. 6.2. It can be seen that the current reaches at steady state value of 100 A. The time constant of the circuit is 0.01 s. This is defined by the time in which the current *i* (*t*) reaches 63.2% of

$$\frac{di}{dt} = \frac{V_{dc}}{RT} e^{-t/T} \bigg|_{t=0} = \frac{V_{dc}}{RT} = 10^4 \tag{6.4}$$

its final value and is obtained by substituting *t* = *T*. Note that the slope of the curve is given by

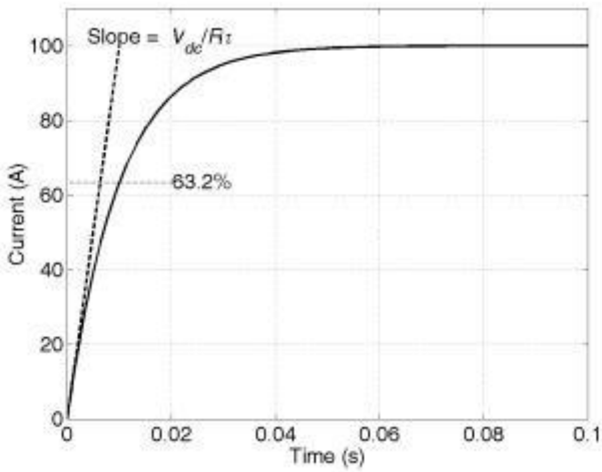


Fig 6.2 Current in the R-L circuit when the source is dc

AC Source

The current response remains unchanged when the voltage source is dc. This however is not the case when the circuit is excited by an ac source. Let us assume that the source voltage is now given

$$v_s = \sqrt{2}V_m \sin(\omega t + \alpha) \tag{6.5}$$

by

where *α* is the phase angle of the applied voltage. We shall show that the system response changes with a change in α .

$$i(t) = i_{ac}(t) + i_{dc}(t) \tag{6.6}$$

$$i_{ac} = \frac{\sqrt{2}V_m}{Z} \sin(\omega t + \alpha - \theta) \text{ A} \tag{6.7}$$

The solution of (6.2) for the source voltage given in (6.5) is

$$i_{dc} = \frac{\sqrt{2}V_m}{Z}\sin(\alpha-\theta)e^{-t/T} \text{ A} \tag{6.8}$$

$$Z = \sqrt{R^2 + (\omega L)^2}, \quad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

The system response for V_m = 100 V and α = 45° is shown in Fig. 6.3. In this figure both i_{ac} and i_{dc} are also shown. It can be seen that i_{ac} is the steady state waveform of the circuit, while i_{dc} dies out once the initial transient phase is over. Fig. 6.4 shows the response of the current for different values of *a* . Since the current is almost inductive, it can be seen that the transient is minimum when α = 90° , i.e., the circuit is switched on almost at the zero-crossing of the current. On the other hand, the transient is maximum when α = 0° , i.e., almost at the peak of the current.

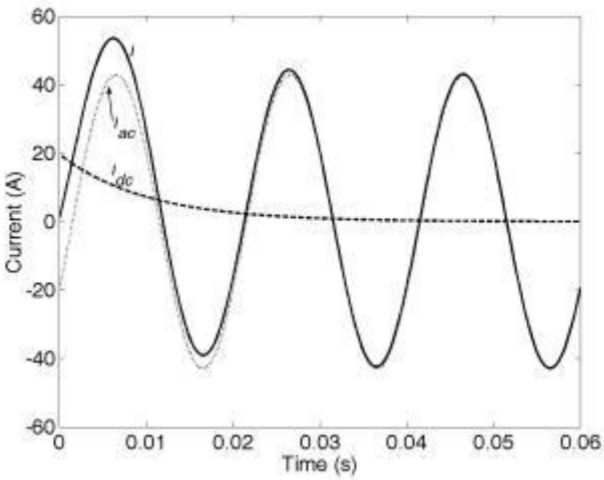


Fig 6.3 Transient in current and its ac and dc components at the instant of switch closing

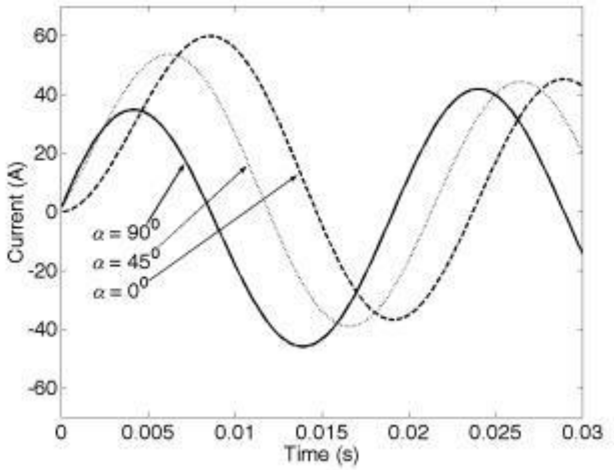


Fig 6.4 Transient in current for different values of α

AC Source

The current response remains unchanged when the voltage source is dc. This however is not the case when the circuit is excited by an ac source. Let us assume that the source voltage is now given

$$v_s = \sqrt{2}V_m \sin(\omega t + \alpha) \tag{6.5}$$

by

where α is the phase angle of the applied voltage. We shall show that the system response changes with a change in α .

$$i(t) = i_{ac}(t) + i_{dc}(t) \tag{6.6}$$

$$i_{ac} = \frac{\sqrt{2}V_m}{Z}\sin(\omega t + \alpha - \theta) \text{ A} \tag{6.7}$$

$$i_{dc} = \frac{\sqrt{2}V_m}{Z}\sin(\alpha - \theta)e^{-t/T} \text{ A} \tag{6.8}$$

The solution of (6.2) for the source voltage given in (6.5) is

$$Z = \sqrt{R^2 + (\omega L)^2}, \quad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

The system response for V_m = 100 V and α = 45° is shown in Fig. 6.3. In this figure both i_{ac} and i_{dc} are also shown. It can be seen that i_{ac} is the steady state waveform of the circuit, while i_{dc} dies out once the initial transient phase is over. Fig. 6.4 shows the response of the current for different values of *a* . Since the current is almost inductive, it can be seen that the transient is minimum when α = 90° , i.e., the circuit is switched on almost at the zero-crossing of the current. On the other hand, the transient is maximum when α = 0° , i.e., almost at the peak of the current.

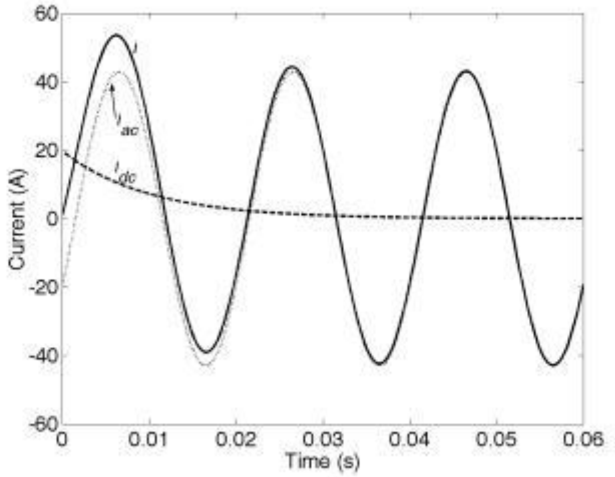


Fig 6.3 Transient in current and its ac and dc components at the instant of switch closing

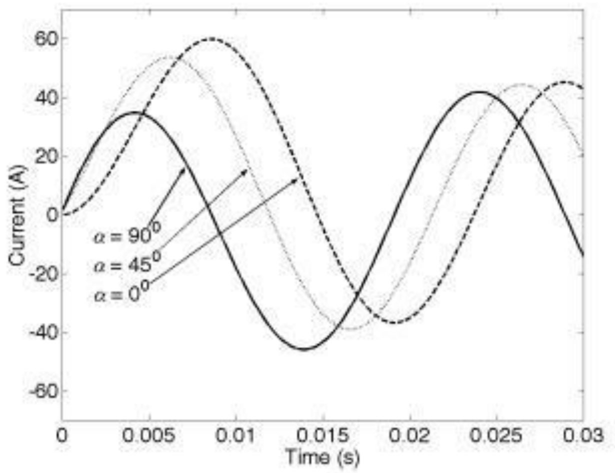


Fig 6.4 Transient in current for different values of α

AC Source

The current response remains unchanged when the voltage source is dc. This however is not the case when the circuit is excited by an ac source. Let us assume that the source voltage is now given by

$$v_s = \sqrt{2}V_m \sin(\omega t + \alpha) \tag{6.5}$$

where *α* is the phase angle of the applied voltage. We shall show that the system response changes with a change in α .

$$i(t) = i_{ac}(t) + i_{dc}(t) \tag{6.6}$$

$$i_{ac} = \frac{\sqrt{2}V_m}{Z}\sin(\omega t + \alpha - \theta) \text{ A} \tag{6.7}$$

$$i_{dc} = \frac{\sqrt{2}V_m}{Z}\sin(\alpha - \theta)e^{-t/T} \text{ A} \tag{6.8}$$

The solution of (6.2) for the source voltage given in (6.5) is

$$Z = \sqrt{R^2 + (\omega L)^2}, \quad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

The system response for V_m = 100 V and *α* = 45° is shown in Fig. 6.3. In this figure both i_{ac} and i_{dc} are also shown. It can be seen that i_{ac} is the steady state waveform of the circuit, while i_{dc} dies out once the initial transient phase is over. Fig. 6.4 shows the response of the current for different values of *a* . Since the current is almost inductive, it can be seen that the transient is minimum when α = 90° , i.e., the circuit is switched on almost at the zero-crossing of the current. On the other hand, the transient is maximum when *α* = 0° , i.e., almost at the peak of the current.

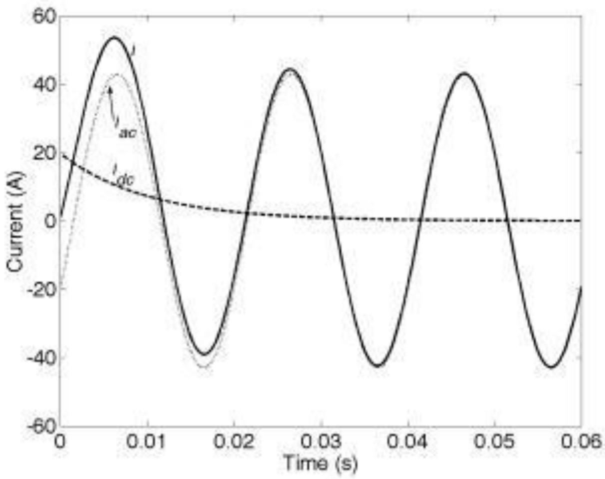


Fig 6.3 Transient in current and its ac and dc components at the instant of switch closing

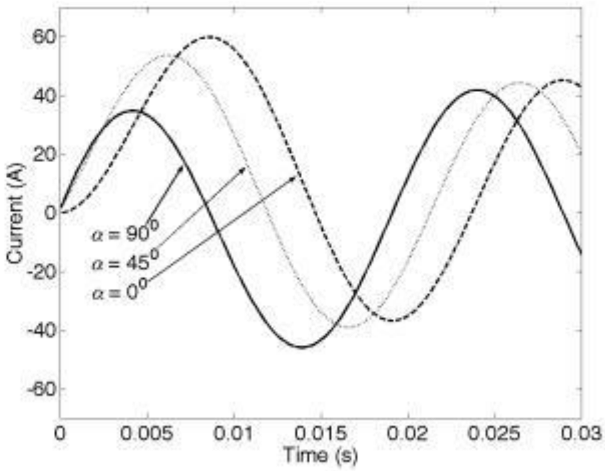


Fig 6.4 Transient in current for different values of α

AC Source

The current response remains unchanged when the voltage source is dc. This however is not the case when the circuit is excited by an ac source. Let us assume that the source voltage is now given

$$v_s = \sqrt{2}V_m \sin\left(\omega t + \alpha\right) \tag{6.5}$$

by

where α is the phase angle of the applied voltage. We shall show that the system response changes with a change in α .

The solution of (6.2) for the source voltage given in (6.5) is

$$i(t) = i_{ac}(t) + i_{dc}(t) \tag{6.6}$$

$$i_{ac} = \frac{\sqrt{2}V_m}{Z}\sin(\omega t + \alpha - \theta) \text{ A} \tag{6.7}$$

$$i_{dc} = \frac{\sqrt{2}V_m}{Z}\sin(\alpha - \theta)e^{-t/T} \text{ A} \tag{6.8}$$

$$Z = \sqrt{R^2 + (\omega L)^2}, \quad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

The system response for V_m = 100 V and α = 45° is shown in Fig. 6.3. In this figure both i_{ac} and i_{dc} are also shown. It can be seen that i_{ac} is the steady state waveform of the circuit, while i_{dc} dies out once the initial transient phase is over. Fig. 6.4 shows the response of the current for different values of *a* . Since the current is almost inductive, it can be seen that the transient is minimum when α = 90° , i.e., the circuit is switched on almost at the zero-crossing of the current. On the other hand, the transient is maximum when α = 0° , i.e., almost at the peak of the current.

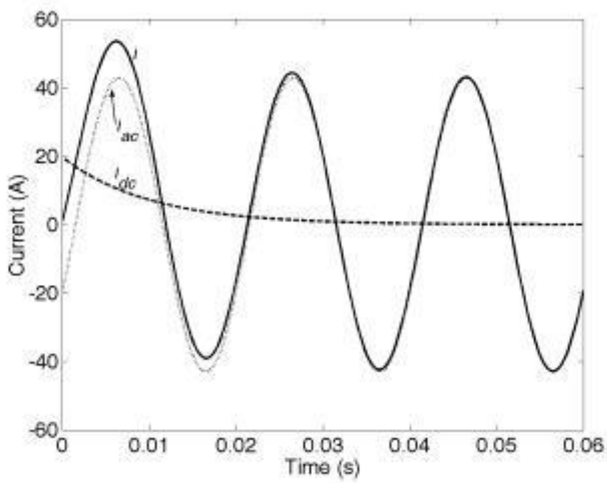


Fig 6.3 Transient in current and its ac and dc components at the instant of switch closing

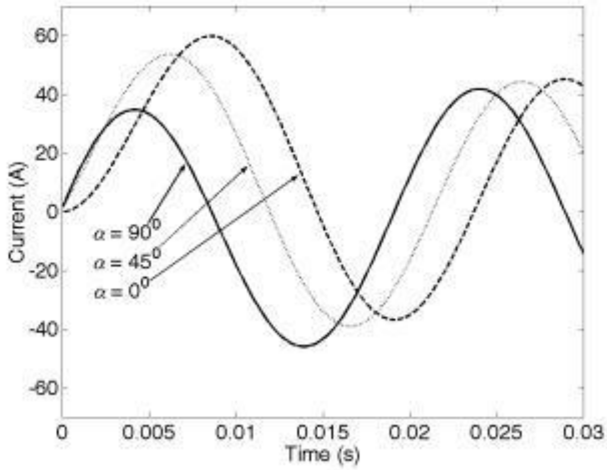


Fig 6.4 Transient in current for different values of α

Fault in an AC Circuit

Now consider the single-phase circuit of Fig. 6.5 where V_s = 240 V (rms), the system frequency is 50 Hz, R = 0.864 Ω, L = 11 mH (ωL = 3.46 Ω) and the load is R-L comprising of an 8.64 W resistor and a 49.5 mH inductor (ωL = 15.55Ω). With the system operating in the steady state, the switch S is suddenly closed creating a short circuit. The current (i) waveform is shown in Fig. 6.6. The current phasor before the short circuit occurs is

$$I = \frac{240}{9.504 + j19.01} = 5.05 - j10.10 = 11.29\angle -63.43^\circ \text{ A}$$

This means that the pre-fault current has a peak value of 15.97 A.

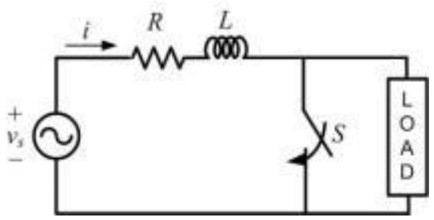


Fig. 6.5 A single-phase circuit in which a source supplies a load through a source impedance.

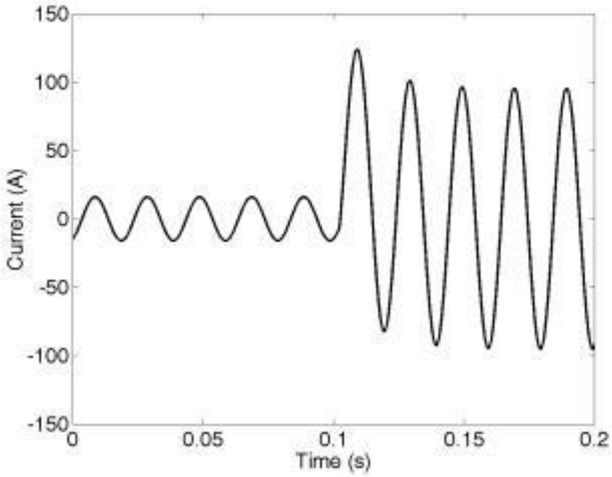


Fig 6.6 The current waveform of the circuit of Fig 6.5 before and after the closing of the switch S

Once the fault occurs and the system is allowed to reach the steady state, the current phasor is given by

$$I = \frac{240}{0.864 + j3.46} = 16.34 - j65.36 = 67.37\angle -75.96^\circ$$

This current has a peak value of 95.28 A. However it can be seen that the current rises suddenly and the first peak following the fault is 124 A which is about 30% higher than the post-fault steady-state value. Also note that the peak value of the current will vary with the instant of the occurrence of the fault. However the peak value of the current is nearly 8 times the pre-fault current value in this case. In general, depending on the ratio of source and load impedances, the faulted current may shoot up anywhere between 10 and 20 times the pre-fault current.

Short Circuit in an Unloaded Synchronous Generator

Fig. 6.7 shows a typical response of the armature current when a three-phase symmetrical short circuit occurs at the terminals of an unloaded synchronous generator.

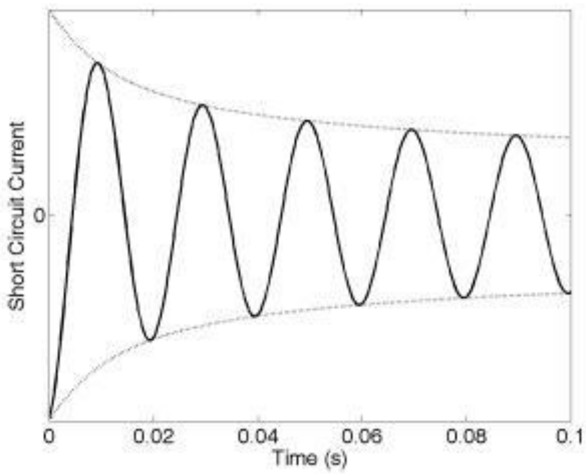


Fig. 6.7 Armature current of a synchronous generator as a short circuit occurs at its terminals.

It is assumed that there is no dc offset in the armature current. The magnitude of the current decreases exponentially from a high initial value. The instantaneous expression for the fault current is

$$i_f(t) = \sqrt{2}V_t\left[\left(\frac{1}{X_d''}-\frac{1}{X_d'}\right)e^{-t/T_d''}+\left(\frac{1}{X_d'}-\frac{1}{X_d}\right)e^{-t/T_d'}+\frac{1}{X_d}\right]\sin\left(\omega t+\alpha-\pi/2\right) \tag{6.9}$$

given by

where V_t is the magnitude of the terminal voltage, α is its phase angle and

X_d'' is the direct axis subtransient reactance

X_d' is the direct axis transient reactance

X_d is the direct axis synchronous reactance

with $X_d'' < X_d' < X_d$. The time constants are

T_d'' is the direct axis subtransient time constant

T_d' is the direct axis transient time constant

In the expression of (6.9) we have neglected the effect of the armature resistance hence α = π/2. Let

$$I_f(0) = I_f'' = \frac{V_t}{X_d''} \tag{6.10}$$

us assume that the fault occurs at time t = 0. From (6.9) we get the rms value of the current as

which is called the **subtransient fault current**. The duration of the subtransient current is dictated by the time constant T_d'' . As the time progresses and $T_d'' < t < T_d'$, the first exponential term of (6.9) will start decaying and will eventually vanish. However since t is still nearly equal to zero, we have the

$$I_f' = \frac{V_t}{X_d'} \tag{6.11}$$

following rms value of the current

This is called the **transient fault current**. Now as the time progress further and the second exponential term also decays, we get the following rms value of the current for the sinusoidal steady

$$I_f = \frac{V_t}{X_d} \tag{6.12}$$

state

In addition to the ac, the fault currents will also contain the dc offset. Note that a symmetrical fault occurs when three different phases are in three different locations in the ac cycle. Therefore the dc

$$i_{dc}^{\max} = \sqrt{2} I_f'' e^{-t/T_a}$$

offsets in the three phases are different. The maximum value of the dc offset is given by

Section III: Symmetrical Fault in a Power System

- **Calculation of Fault Current Using Impedance Diagram**
- **Calculation of Fault Current Using Z_{bus} Matrix**

Calculation of Fault Current Using Impedance Diagram

Let us first illustrate the calculation of the fault current using the impedance diagram with the help of the following examples.

Example 6.1

Consider the power system of Fig. 6.8 in which a synchronous generator supplies a synchronous motor. The motor is operating at rated voltage and rated MVA while drawing a load current at a power factor of 0.9 (lagging) when a three phase symmetrical short circuit occurs at its terminals. We shall calculate the fault current that flow from both the generator and the motor.

We shall choose a base of 50 MVA, 20 kV in the circuit of the generator. Then the motor synchronous reactance is given by

$$X_m'' = 0.2 \times \frac{50}{25} = 0.4$$ per unit

Also the base impedance in the circuit of the transmission line is

$$Z_{base} = \frac{66^2}{50} = 87.12$$ Ω

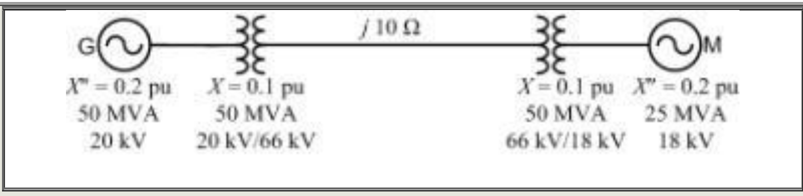


Fig. 6.8 A generator supplying a motor load though a transmission line.

Therefore the impedance of the transmission line is

$$X_{line} = j\frac{10}{87.12} = j0.1148$$ per unit

The impedance diagram for the circuit is shown in Fig. 6.9 in which the switch *S* indicates the fault.

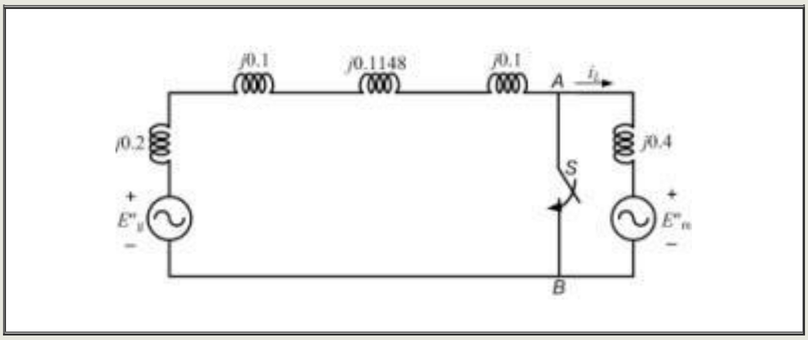


Fig. 6.9 Impedance diagram of the circuit of Fig. 6.8.

The motor draws a load current at rated voltage and rated MVA with 0.9 lagging power factor. Therefore

$$i_L = 1\angle -\cos^{-1}(0.9) = 0.9 - j0.4359$$ per unit

Then the subtransient voltages of the motor and the generator are

$$E''_m = 1.0 - j0.4 \times i_L = 0.8256 - j0.36$$ per unit

$$E''_g = 1.0 + j0.5148 \times i_L = 1.2244 + j0.4633$$ per unit

Hence the subtransient fault currents fed by the motor and the generator are

$$I''_m = \frac{E''_m}{j0.4} = -0.9 - j2.0641$$ per unit

$$I''_g = \frac{E''_g}{j0.5148} = 0.9 - j2.3784$$ per unit

and the total current flowing to the fault is

$$I_f'' = I_g'' + I_m'' = -j4.4425 \text{ per unit}$$

Note that the base current in the circuit of the motor is

$$I_{base} = \frac{50 \times 10^3}{\sqrt{3} \times 18} = 1603.8 \text{ A}$$

Therefore while the load current was 1603.8 A, the fault current is 7124.7 A.

Example 6.2

We shall now solve the above problem differently. The Thevenin impedance at the circuit between the terminals *A* and *B* of the circuit of Fig. 6.9 is the parallel combination of the impedances *j* 0.4 and *j* 0.5148. This is then given as

$$Z_{th} = j\frac{0.4 \times 0.5148}{0.4 + 0.5148} = j0.2251 \text{ per unit}$$

Since voltage at the motor terminals before the fault is 1.0 per unit, the fault current is

$$I_f'' = \frac{1.0}{Z_{th}} = -j4.4425 \text{ per unit}$$

If we neglect the pre-fault current flowing through the circuit, then fault current fed by the motor and the generator can be determined using the current divider principle, i.e.,

$$I_{m0}'' = \frac{I_f''}{j0.9148} \times j0.5148 = -j2.5 \text{ per unit}$$

$$I_{g0}'' = \frac{I_f''}{j0.9148} \times j0.4 = -j1.9425 \text{ per unit}$$

If, on the other hand, the pre-fault current is not neglected, then the fault current supplied by the motor and the generator are

$$I_m'' = I_{m0}'' - I_L = -0.9 - j2.0641 \text{ per unit}$$

$$I_g'' = I_{g0}'' + I_L = 0.9 - j2.3784 \text{ per unit}$$

Calculation of Fault Current Using Z_{bus} Matrix

Consider the circuit of Fig. 3.3 which is redrawn as shown in Fig. 6.10.

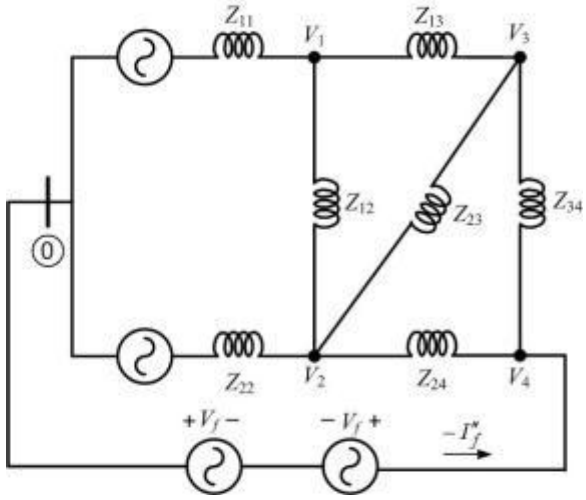


Fig. 6.10 Network depicting a symmetrical fault at bus-4.

We assume that a symmetrical fault has occurred in bus-4 such that it is now connected to the reference bus. Let us assume that the pre-fault voltage at this bus is V_f . To denote that bus-4 is short circuit, we add two voltage sources V_f and - V_f together in series between bus-4 and the reference bus. Also note that the subtransient fault current I''_f flows from bus-4 to the reference bus. This implies that a current that is equal to - I''_f is injected into bus-4. This current, which is due to the source - V_f will flow through the various branches of the network and will cause a change in the bus voltages. Assuming that the two sources and V_f are short circuited. Then - V_f is the only source left in the network that injects a current - I''_f into bus-4. The voltages of the different nodes that are caused by

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ -V_f \end{bmatrix} = Z_{bus} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -I''_f \end{bmatrix} \tag{6.14}$$

the voltage - V_f and the current - I''_f are then given by

where the prefix Δ indicates the changes in the bus voltages due to the current - I''_f .

$$V_f = Z_{44} I''_f \tag{6.15}$$

From the fourth row of (6.14) we can write

Combining (6.14) and (6.15) we get

$$\Delta V_i = -Z_{i4} I_f'' = -\frac{Z_{i4}}{Z_{44}} V_f, \quad i = 1,2,3 \tag{6.16}$$

We further assume that the system is unloaded before the fault occurs and that the magnitude and phase angles of all the generator internal emfs are the same. Then there will be no current circulating anywhere in the network and the bus voltages of all the nodes before the fault will be same and equal

$$V_i = V_f + \Delta V_i = \left(1 - \frac{Z_{i4}}{Z_{44}}\right) V_f, \quad i = 1, \cdots, 4 \tag{6.17}$$

to V_f . Then the new altered bus voltages due to the fault will be given from (6.16) by

Example 6.3

Section IV: Circuit Breaker Selection

A typical circuit breaker operating time is given in Fig. 6.11. Once the fault occurs, the protective devices get activated. A certain amount of time elapses before the protective relays determine that there is overcurrent in the circuit and initiate trip command. This time is called the **detection time**. The contacts of the circuit breakers are held together by spring mechanism and, with the trip command, the spring mechanism releases the contacts. When two current carrying contacts part, a voltage instantly appears at the contacts and a large voltage gradient appears in the medium between the two contacts. This voltage gradient ionizes the medium thereby maintaining the flow of current. This current generates extreme heat and light that is called electric arc. Different mechanisms are used for elongating the arc such that it can be cooled and extinguished. Therefore the circuit breaker has to withstand fault current from the instant of initiation of the fault to the time the arc is extinguished.

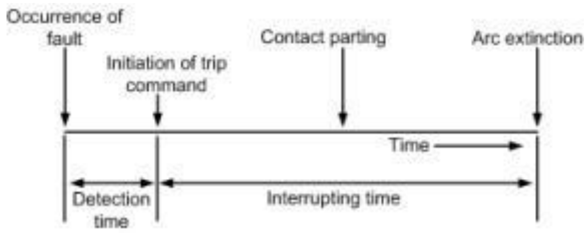


Fig. 6.11 Typical circuit breaker operating time.

Two factors are of utmost importance for the selection of circuit breakers. These are:

- The maximum instantaneous current that a breaker must withstand and
- The total current when the breaker contacts part.

In this chapter we have discussed the calculation of symmetrical subtransient fault current in a network. However the instantaneous current following a fault will also contain the dc component. In a high power circuit breaker selection, the subtransient current is multiplied by a factor of 1.6 to determine the rms value of the current the circuit breaker must withstand. This current is called the

momentary current . The **interrupting current** of a circuit breaker is lower than the momentary current and will depend upon the speed of the circuit breaker. The interrupting current may be asymmetrical since some dc component may still continue to decay.

Breakers are usually classified by their nominal voltage, continuous current rating, rated maximum voltage, *K* -factor which is the voltage range factor, rated short circuit current at maximum voltage and operating time. The *K* -factor is the ratio of rated maximum voltage to the lower limit of the range of the operating voltage. The maximum symmetrical interrupting current of a circuit breaker is given by *K* times the rated short circuit current.

Overview

An unbalanced three-phase system can be resolved into three balanced systems in the sinusoidal steady state. This method of resolving an unbalanced system into three balanced phasor system has been proposed by C. L. Fortescue. This method is called **resolving symmetrical components** of the original phasors or simply **symmetrical components**.

In this chapter we shall discuss symmetrical components transformation and then will present how unbalanced components like Y- or Δ -connected loads, transformers, generators and transmission lines can be resolved into symmetrical components. We can then combine all these components together to form what are called **sequence networks** .

Section I: Symmetrical Components

Symmetrical Components

A system of three unbalanced phasors can be resolved in the following three symmetrical components:

- Positive Sequence: A balanced three-phase system with the same phase sequence as the original sequence.

- Negative sequence: A balanced three-phase system with the opposite phase sequence as the original sequence.
- Zero Sequence: Three phasors that are equal in magnitude and phase.

Fig. 7.1 depicts a set of three unbalanced phasors that are resolved into the three sequence components mentioned above. In this the original set of three phasors are denoted by V_a , V_b and V_c , while their positive, negative and zero sequence components are denoted by the subscripts 1, 2 and 0 respectively. This implies that the positive, negative and zero sequence components of phase-a are denoted by V_{a1} , V_{a2} and V_{a0} respectively. Note that just like the voltage phasors given in Fig. 7.1 we can also resolve three unbalanced current phasors into three symmetrical components.

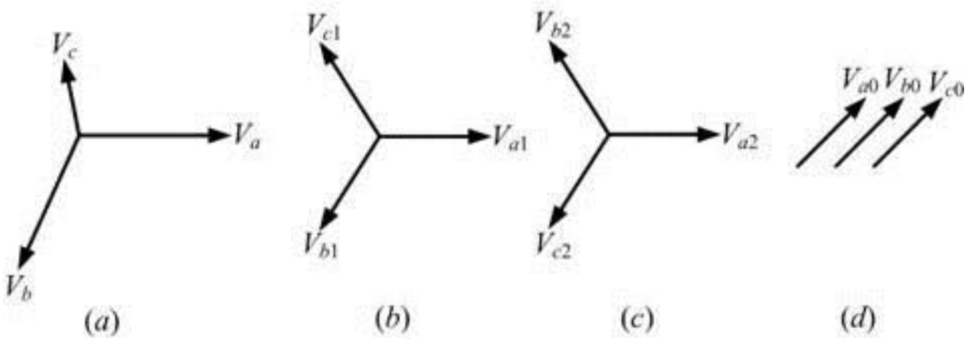


Fig. 7.1 Representation of (a) an unbalanced network, its (b) positive sequence, (c) negative sequence and (d) zero sequence.

Symmetrical Component Transformation

Before we discuss the symmetrical component transformation, let us first define the a -operator. This

$$a = e^{j120^0} = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \tag{7.1}$$

has been given in (1.34) and is reproduced below.

$$\begin{aligned} a^2 &= e^{j240^0} = -\frac{1}{2} - j\frac{\sqrt{3}}{2} = a^* \\ a^3 &= e^{j360^0} = 1 \\ a^4 &= e^{j480^0} = e^{j360^0}e^{j120^0} = a \\ a^5 &= e^{j600^0} = e^{j360^0}e^{j240^0} = a^2 \text{ and so on} \end{aligned} \tag{7.2}$$

Note that for the above operator the following relations hold

$$1 + a + a^2 = 1 - \frac{1}{2} + j\frac{\sqrt{3}}{2} - \frac{1}{2} - j\frac{\sqrt{3}}{2} = 0 \tag{7.3}$$

Also note that we have

$$V_{b1} = a^2 V_{a1} \text{ and } V_{c1} = aV_{a1} \tag{7.4}$$

Using the a -operator we can write from Fig. 7.1 (b)

$$V_{b2} = aV_{a2} \text{ and } V_{c2} = a^2 V_{a2} \tag{7.5}$$

Similarly from Fig. 7.1 (c) we get

$$V_{a0} = V_{b0} = V_{c0} \tag{7.6}$$

Finally from Fig. 7.1 (d) we get

$$V_a = V_{a0} + V_{a1} + V_{a2} \tag{7.7}$$

$$V_b = V_{a0} + a^2 V_{a1} + aV_{a2} = V_{b0} + V_{b1} + V_{b2} \tag{7.8}$$

$$V_c = V_{a0} + aV_{a1} + a^2 V_{a2} = V_{c0} + V_{c1} + V_{c2} \tag{7.9}$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \tag{7.10}$$

Therefore,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{7.11}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{7.12}$$

The symmetrical component transformation matrix is then given by

Defining the vectors V_{a012} and V_{abc} as

$$V_{a012} = \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}, \quad V_{abc} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$V_{a012} = CV_{abc}$$

we can write (7.4) as

$$C = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \tag{7.13}$$

where *C* is the symmetrical component transformation matrix and is given by

The original phasor components can be obtained from the inverse symmetrical component

$$V_{abc} = C^{-1}V_{a012} \tag{7.14}$$

transformation, i.e.,

Finally, if we define a set of unbalanced current phasors as I_{abc} and their symmetrical components as

$$\begin{gathered} I_{a012} = CI_{abc} \\ I_{abc} = C^{-1}I_{a012} \end{gathered} \tag{7.15}$$

I_{a012} , we can then define

Example 7.1

Example 7.2

Real and Reactive Power

$$P_{abc} + jQ_{abc} = V_a I_a^* + V_b I_b^* + V_c I_c^* = V_{abc}^T I_{abc}^* \tag{7.16}$$

The three-phase power in the original unbalanced system is given by

$$P_{abc} + jQ_{abc} = V_{a012}^T C^{-T} C^{-1*} I_{a012}^* \tag{7.17}$$

where I^* is the complex conjugate of the vector I . Now from (7.10) and (7.15) we get

$$C^{-T}C^{-1*} = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

From (7.11) we get

$$P_{abc} + jQ_{abc} = 3\left(V_{a0}I_{a0}^* + V_{a1}I_{a1}^* + V_{a2}I_{a2}^*\right) \tag{7.18}$$

Therefore from (7.17) we get

We then find that the complex power is three times the summation of the complex power of the three phase sequences.

Example 7.3

Real and Reactive Power

$$P_{abc} + jQ_{abc} = V_a I_a^* + V_b I_b^* + V_c I_c^* = V_{abc}^T I_{abc}^* \tag{7.16}$$

The three-phase power in the original unbalanced system is given by

$$P_{abc} + jQ_{abc} = V_{a012}^T C^{-T} C^{-1*} I_{a012}^* \tag{7.17}$$

where I^* is the complex conjugate of the vector I . Now from (7.10) and (7.15) we get

$$C^{-T} C^{-1*} = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

From (7.11) we get

$$P_{abc} + jQ_{abc} = 3\left(V_{a0} I_{a0}^* + V_{a1} I_{a1}^* + V_{a2} I_{a2}^*\right) \tag{7.18}$$

Therefore from (7.17) we get

We then find that the complex power is three times the summation of the complex power of the three phase sequences.

Example 7.3

Section II: Sequence Circuits for Loads

In this section we shall construct sequence circuits for both Y and Δ -connected loads separately.

- **Sequence Circuit for a Y-Connected Load**
- **Sequence Circuit for a Δ-Connected Load**

Sequence Circuit for a Y-Connected Load

Consider the balanced Y-connected load that is shown in Fig. 7.2. The neutral point (*n*) of the windings are grounded through an impedance Z_n . The load in each phase is denoted by Z_Y . Let us consider phase-a of the load. The voltage between line and ground is denoted by V_a , the line-to-neutral voltage is denoted by V_{an} and voltage between the neutral and ground is denoted by V_n . The

$$\begin{aligned} I_n &= I_a + I_b + I_c \\ &= 3I_{a0} + \left(I_{a1} + I_{b1} + I_{c1}\right) + \left(I_{a2} + I_{b2} + I_{c2}\right) = 3I_{a0} \end{aligned} \tag{7.22}$$

neutral current is then

Therefore there will not be any positive or negative sequence current flowing out of the neutral point.

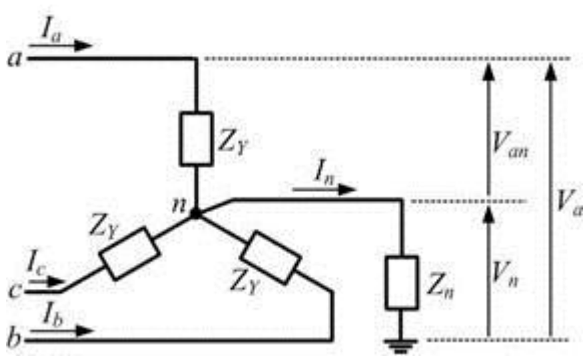


Fig. 7.2 Schematic diagram of a balanced Y-connected load.

$$V_n = 3Z_n I_{a0} \tag{7.23}$$

The voltage drop between the neutral and ground is

$$V_a = V_{an} + V_n = V_{an} + 3Z_n I_{a0} \tag{7.24}$$

Now

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} + \begin{bmatrix} V_n \\ V_n \\ V_n \end{bmatrix} = Z_Y \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + 3Z_n I_{a0} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{7.25}$$

We can write similar expression for the other two phases. We can therefore write

Pre-multiplying both sides of the above equation by the matrix *C* and using (7.8) we get

$$V_{a012} = Z_Y I_{a012} + 3Z_n I_{a0} C \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{7.26}$$

Now since

$$C \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = Z_Y \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} + 3Z_n \begin{bmatrix} I_{a0} \\ 0 \\ 0 \end{bmatrix} \quad (7.27)$$

We get from (7.26)

We then find that the zero, positive and negative sequence voltages only depend on their respective sequence component currents. The sequence component equivalent circuits are shown in Fig. 7.3. While the positive and negative sequence impedances are both equal to Z_Y , the zero sequence

$$Z_0 = Z_Y + 3Z_n \quad (7.28)$$

impedance is equal to

If the neutral is grounded directly (i.e., Z_n = 0), then $Z_0 = Z_Y$. On the other hand, if the neutral is kept floating (i.e., $Z_n = \infty$), then there will not be any zero sequence current flowing in the circuit at all.

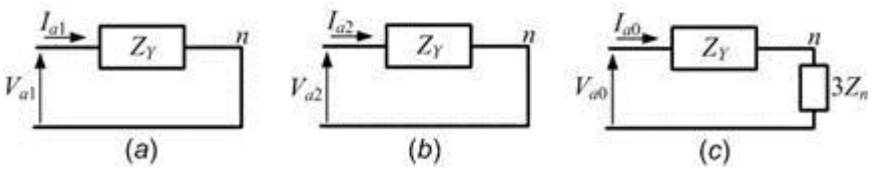


Fig. 7.3 Sequence circuits of Y-connected load: (a) positive, (b) negative and (c) zero sequence

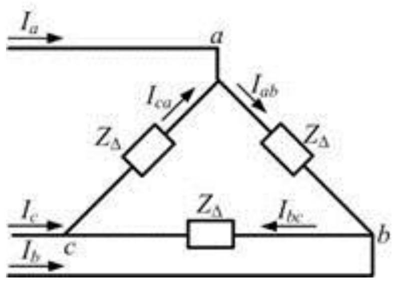
Sequence Circuit for a Δ -Connected Load

Consider the balanced Δ -connected load shown in Fig. 7.4 in which the load in each phase is

$$\begin{aligned} V_{ab} &= Z_\Delta I_{ab} \\ V_{bc} &= Z_\Delta I_{bc} \\ V_{ca} &= Z_\Delta I_{ca} \end{aligned} \quad (7.29)$$

denoted by Z_Δ . The line-to-line voltages are given by

Adding these three voltages we get



$$V_{ab} + V_{bc} + V_{ca} = Z_\Delta (I_{ab} + I_{bc} + I_{ca}) \tag{7.30}$$

Fig. 7.4 Schematic diagram of a balanced Δ -connected load.

Denoting the zero sequence component V_{ab} , V_{bc} and V_{ca} as V_{ab0} and that of I_{ab} , I_{bc} and I_{ca} as I_{ab0} we

$$V_{ab0} = Z_\Delta I_{ab0} \tag{7.31}$$

can rewrite (7.30) as

$$V_{ab} + V_{bc} + V_{ca} = V_a - V_b + V_b - V_c + V_c - V_a = 0$$

Again since

We find from (7.31) V_{ab0} = I_{ab0} = 0. Hence a Δ -connected load with no mutual coupling has not any zero sequence circulating current. Note that the positive and negative sequence impedance for this load will be equal to Z_Δ .

Example 7.4

Section III: Sequence Circuits for Synchronous Generator

The three-phase equivalent circuit of a synchronous generator is shown in Fig. 1.16. This is redrawn in Fig. 7.6 with the neutral point grounded through a reactor with impedance Z_n . The neutral current is

$$I_n = I_a + I_b + I_c \tag{7.32}$$

then given by

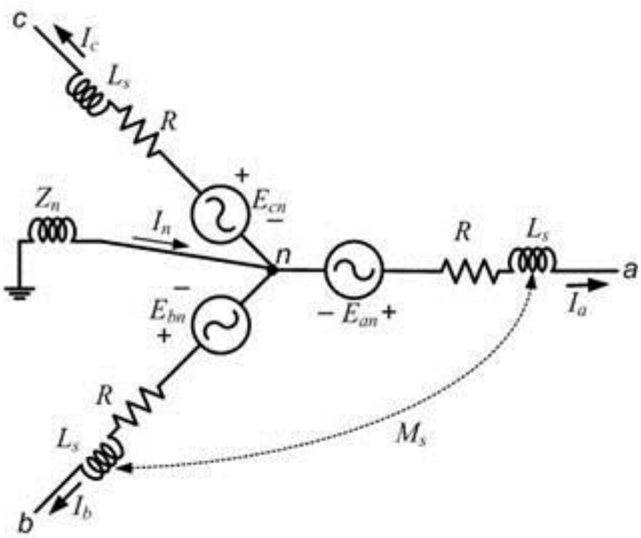


Fig. 7.6 Equivalent circuit of a synchronous generator with grounded neutral.

The derivation of Section 1.3 assumes balanced operation which implies $I_a + I_b + I_c = 0$. As per (7.32) this assumption is not valid any more. Therefore with respect to this figure we can write for phase-a

$$\begin{aligned} V_{an} &= -(R + j\omega L_s)I_a + j\omega M_s(I_b + I_c) + E_{an} \\ &= -(R + j\omega L_s + j\omega M_s)I_a + j\omega M_s(I_a + I_b + I_c) + E_{an} \end{aligned} \tag{7.33}$$

voltage as

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = -[R + j\omega(L_s + M_s)]\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + j\omega M_s \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + \begin{bmatrix} E_{an} \\ E_{bn} \\ E_{cn} \end{bmatrix} \tag{7.34}$$

Similar expressions can also be written for the other two phases. We therefore have

$$\begin{bmatrix} V_{an0} \\ V_{an1} \\ V_{an2} \end{bmatrix} = -\left[R + j\omega\left(L_s + M_s\right)\right]\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} + j\omega M_s C\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}C^{-1}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} + \begin{bmatrix} E_{an0} \\ E_{an1} \\ E_{an2} \end{bmatrix} \tag{7.35}$$

Pre-multiplying both sides of (7.34) by the transformation matrix *C* we get

Since the synchronous generator is operated to supply only balanced voltages we can assume that

$$\begin{bmatrix} V_{an0} \\ V_{an1} \\ V_{an2} \end{bmatrix} = -\left[R + j\omega\left(L_s + M_s\right)\right]\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} + j\omega M_s\begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} + \begin{bmatrix} 0 \\ E_{an} \\ 0 \end{bmatrix} \tag{7.36}$$

$E_{an0} = E_{an2} = 0$ and $E_{an1} = E_{an}$. We can therefore modify (7.35) as

$$V_{an0} = -\left[R + j\omega\left(L_s - 2M_s\right)\right]I_{a0} = -Z_{g0}I_{a0} \tag{7.37}$$

$$V_{an1} = -\left[R + j\omega\left(L_s + M_s\right)\right]I_{a1} + E_{an} = E_{an} - Z_1 I_{a1} \tag{7.38}$$

$$V_{an2} = -\left[R + j\omega\left(L_s + M_s\right)\right]I_{a2} = -Z_2 I_{a2} \tag{7.39}$$

We can separate the terms of (7.36) as

Furthermore we have seen for a Y-connected load that $V_{a1} = V_{an1}$, $V_{a2} = V_{an2}$ since the neutral current does not affect these voltages. However $V_{a0} = V_{an0} + V_n$. Also we know that $V_n = - 3Z_nI_{a0}$. We

$$V_{a0} = -(Z_{g0} + 3Z_n)I_{a0} = -Z_0 I_{a0} \qquad (7.40)$$

can therefore rewrite (7.37) as

The sequence diagrams for a synchronous generator are shown in Fig. 7.7.

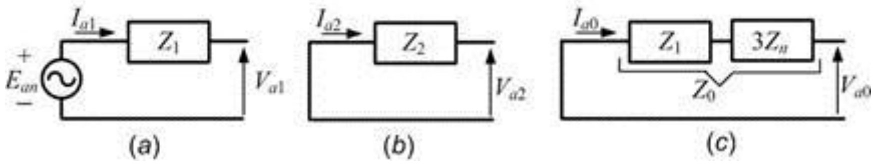


Fig. 7.7 Sequence circuits of synchronous generator: (a) positive, (b) negative and (c) zero sequence

Section IV: Sequence Circuits for Symmetrical Transmission Line

The schematic diagram of a transmission line is shown in Fig. 7.8. In this diagram the self impedance of the three phases are denoted by Z_{aa} , Z_{bb} and Z_{cc} while that of the neutral wire is denoted by Z_{nn} .

$$Z_{aa} = Z_{bb} = Z_{cc}$$

Let us assume that the self impedances of the conductors to be the same, i.e.,

Since the transmission line is assumed to be symmetric, we further assume that the mutual inductances between the conductors are the same and so are the mutual inductances between the

$$Z_{ab} = Z_{bc} = Z_{ca}$$

$$Z_{an} = Z_{bn} = Z_{cn}$$

conductors and the neutral, i.e.,

The directions of the currents flowing through the lines are indicated in Fig. 7.8 and the voltages between the different conductors are as indicated.

Fig. 7.8 Lumped parameter representation of a symmetrical transmission line.

$$V_{an} = V_{aa'} + V_{a'n'} + V_{n'n} = V_{aa'} + V_{a'n'} - V_{nn'} \tag{7.41}$$

Applying Kirchoff's voltage law we get

$$V_{aa'} = Z_{aa} I_a + Z_{ab}\left(I_b + I_c\right) + Z_{an} I_n \tag{7.42}$$

Again

$$V_{nn'} = Z_{nn} I_a + Z_{an}\left(I_a + I_b + I_c\right) \tag{7.43}$$

$$V_{an} - V_{a'n'} = \left(Z_{aa} - Z_{an}\right)I_a + \left(Z_{ab} - Z_{an}\right)\left(I_b + I_c\right) + \left(Z_{an} - Z_{nn}\right)I_n \tag{7.44}$$

Substituting (7.42) and (7.43) in (7.41) we get

Since the neutral provides a return path for the currents I_a , I_b and I_c , we can write

$$I_n = -\left(I_a + I_b + I_c\right) \tag{7.45}$$

$$V_{an} - V_{a'n'} = \left(Z_{aa} + Z_{nn} - 2Z_{an}\right)I_a + \left(Z_{ab} + Z_{nn} - 2Z_{an}\right)\left(I_b + I_c\right) \tag{7.46}$$

Therefore substituting (7.45) in (7.44) we get the following equation for phase-a of the circuit

$$Z_s = Z_{aa} + Z_{nn} - 2Z_{an} \text{ and } Z_m = Z_{ab} + Z_{nn} - 2Z_{an}$$

Denoting

$$V_{an} - V_{a'n'} = Z_s I_a + Z_m\left(I_b + I_c\right) \tag{7.47}$$

(7.46) can be rewritten as

Since (7.47) does not explicitly include the neutral conductor we can define the voltage drop across

$$V_{aa'} = V_{an} - V_{a'n'} \tag{7.48}$$

the phase-a conductor as

$$V_{aa'} = Z_s I_a + Z_m\left(I_b + I_c\right) \tag{7.49}$$

Combining (7.47) and (7.48) we get

$$\begin{bmatrix} V_{aa'} \\ V_{bb'} \\ V_{cc'} \end{bmatrix} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{7.50}$$

Similar expression can also be written for the other two phases. We therefore get

$$V_{aa'012} = C \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} C^{-1} I_{a012} \tag{7.51}$$

Pre-multiplying both sides of (7.50) by the transformation matrix *C* we get

Now

$$\begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} C^{-1} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= \begin{bmatrix} Z_s + 2Z_m & Z_s - Z_m & Z_s - Z_m \\ Z_s + 2Z_m & a^2 Z_s + (1+a)Z_m & aZ_s + (1+a^2)Z_m \\ Z_s + 2Z_m & aZ_s + (1+a^2)Z_m & a^2 Z_s + (1+a)Z_m \end{bmatrix}$$

$$C\begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} C^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} Z_s + 2Z_m & Z_s - Z_m & Z_s - Z_m \\ Z_s + 2Z_m & a^2 Z_s + (1+a)Z_m & aZ_s + (1+a^2)Z_m \\ Z_s + 2Z_m & aZ_s + (1+a^2)Z_m & a^2 Z_s + (1+a)Z_m \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3Z_s + 6Z_m & 0 & 0 \\ 0 & 3Z_s - 3Z_m & 0 \\ 0 & 0 & 3Z_s - 3Z_m \end{bmatrix}$$

Hence

$$\begin{bmatrix} V_{aa'0} \\ V_{aa'1} \\ V_{aa'2} \end{bmatrix} = \begin{bmatrix} Z_s + 2Z_m & & \\ & Z_s - Z_m & \\ & & Z_s - Z_m \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \tag{7.52}$$

Therefore from (7.51) we get

The positive, negative and zero sequence equivalent circuits of the transmission line are shown in

$$Z_1 = Z_2 = Z_s - Z_m = Z_{aa} - Z_{ab}$$

$$Z_0 = Z_s + 2Z_m = Z_{aa} + 2Z_{ab} + 3Z_{nn} - 6Z_{an}$$

Fig. 7.9 where the sequence impedances are

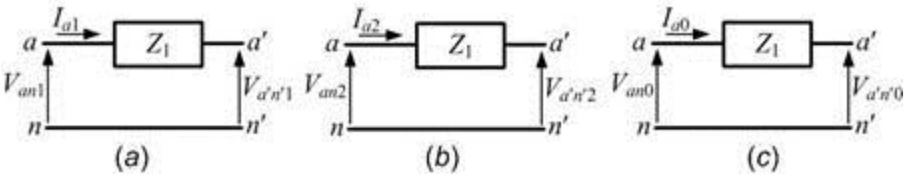


Fig. 7.9 Sequence circuits of symmetrical transmission line: (a) positive, (b) negative and (c) zero sequence.

Section V: Sequence Circuits for Transformers

- <u>**Y-Y Connected Transformer**</u>
- <u>**Δ - Δ Connected Transformer**</u>
- <u>**Y- Δ Connected Transformer**</u>

In this section we shall discuss the sequence circuits of transformers. As we have seen earlier that the sequence circuits are different for Y- and Δ -connected loads, the sequence circuits are also different for Y and Δ connected transformers. We shall therefore treat different transformer connections separately.

-Y Connected Transformer

Fig. 7.10 shows the schematic diagram of a Y-Y connected transformer in which both the neutrals are grounded. The primary and secondary side quantities are denoted by subscripts in uppercase letters and lowercase letters respectively. The turns ratio of the transformer is given by α = N_1 : N_2 .

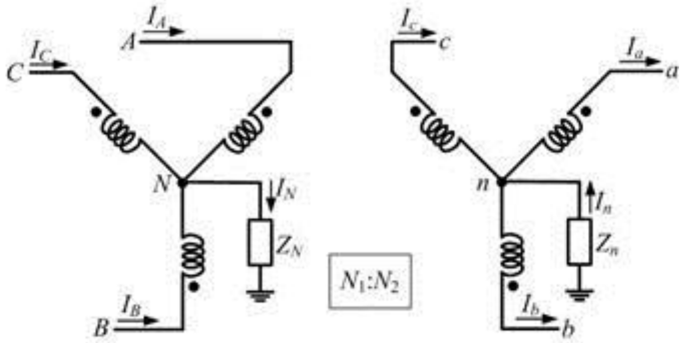


Fig. 7.10 Schematic diagram of a grounded neutral Y-Y connected transformer.

$$V_A = V_{AN} + V_N = V_{AN} + 3Z_N I_{A0}$$

The voltage of phase-a of the primary side is

Expanding V_A and V_{AN} in terms of their positive, negative and zero sequence components, the above

$$V_{A0} + V_{A1} + V_{A2} = V_{AN0} + V_{AN1} + V_{AN2} + 3Z_N I_{A0} \tag{7.53}$$

equation can be rewritten as

Noting that the direction of the neutral current I_n is opposite to that of I_N , we can write an equation

$$V_{a0} + V_{a1} + V_{a2} = V_{an0} + V_{an1} + V_{an2} - 3Z_n I_{a0} \tag{7.54}$$

similar to that of (7.53) for the secondary side as

$$\alpha = \frac{N_1}{N_2} = \frac{V_{AN}}{V_{an}} \Rightarrow V_{an} = \frac{V_{AN}}{\alpha}$$

$$N_1 I_A = N_2 I_a \Rightarrow I_a = \alpha I_A$$

Now since the turns ratio of the transformer is α = N_1 : N_2 we can write

$$V_{a0} + V_{a1} + V_{a2} = \frac{1}{\alpha}\left(V_{AN0} + V_{AN1} + V_{AN2}\right) - 3Z_n \alpha I_{A0}$$

Substituting in (7.54) we get

$$\alpha\left(V_{a0} + V_{a1} + V_{a2}\right) = V_{AN0} + V_{AN1} + V_{AN2} - 3Z_n \alpha^2 I_{A0} \tag{7.55}$$

Multiplying both sides of the above equation by *a* results in

$$\alpha\left(V_{a0}+V_{a1}+V_{a2}\right)=V_{A0}+V_{A1}+V_{A2}-3\left(Z_N+Z_n\alpha^2\right)I_{A0} \tag{7.56}$$

Finally combining (7.53) with (7.55) we get

$$\alpha V_{a1}=\frac{N_1}{N_2}V_{a1}=V_{A1} \tag{7.57}$$

$$\alpha V_{a2}=\frac{N_1}{N_2}V_{a2}=V_{A2} \tag{7.58}$$

$$\alpha V_{a0}=\frac{N_1}{N_2}V_{a0}=V_{A0}-3\left[Z_N+\left(N_1/N_2\right)^2 Z_n\right]I_{A0} \tag{7.59}$$

Separating out the positive, negative and zero sequence components we can write

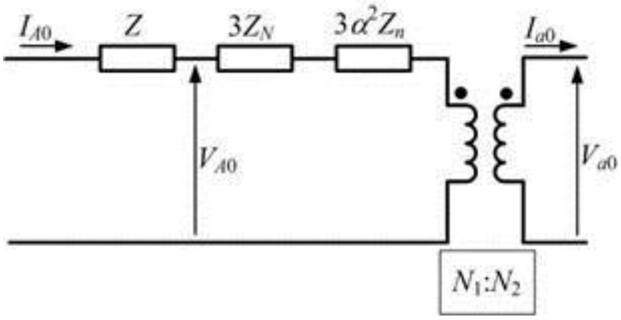


Fig. 7.11 Zero sequence equivalent circuit of grounded neutral Y-Y connected transformer.

From (7.57) and (7.58) we see that the positive and negative sequence relations are the same as that we have used for representing transformer circuits given in Fig. 1.18. Hence the positive and negative sequence impedances are the same as the transformer leakage impedance Z . The zero sequence equivalent circuit is shown in Fig. 7.11.

$$Z_0 = Z + 3Z_N + 3(N_1/N_2)^2 Z_n \tag{7.60}$$

The total zero sequence impedance is given by

The zero sequence diagram of the grounded neutral Y-Y connected transformer is shown in Fig. 7.12 (a) in which the impedance Z_0 is as given in (7.60). If both the neutrals are solidly grounded, i.e., Z_n = Z_N = 0, then Z_0 is equal to Z . The single line diagram is still the same as that shown in Fig. 7.12 (a). If however one of the two neutrals or both neutrals are ungrounded, then we have either $Z_n = \infty$ or $Z_N = \infty$ or both. The zero sequence diagram is then as shown in Fig. 7.12 (b) where the value of Z_0 will depend on which neutral is kept ungrounded.

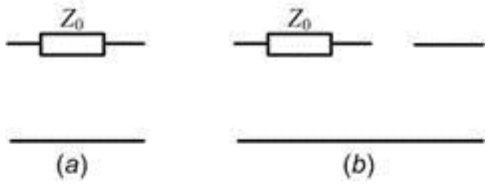


Fig. 7.12 Zero sequence diagram of (a) grounded neutral and (b) ungrounded neutral Y-Y connected transformer.

Δ - Δ Connected Transformer

The schematic diagram of a Δ - Δ connected transformer is shown in Fig. 7.13. Now we have

$$\begin{aligned} V_{AB} &= V_A - V_B \\ &= V_{A0} + V_{A1} + V_{A2} - V_{B0} - V_{B1} - V_{B2} = V_{AB1} + V_{AB2} \end{aligned} \tag{7.61}$$

Again

$$V_{AB} = \frac{N_1}{N_2} V_{ab} = \alpha V_{ab}$$

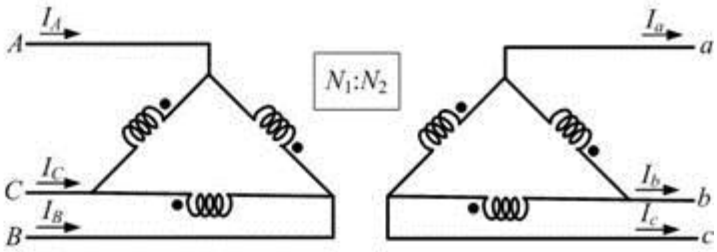


Fig. 7.13 Schematic diagram of a Δ - Δ connected transformer.

Therefore from (7.61) we get

$$V_{AB} = V_{AB1} + V_{AB2} = \alpha\left(V_{ab1} + V_{ab2}\right) \tag{7.62}$$

The sequence components of the line-to-line voltage V_{AB} can be written in terms of the sequence com ponents of the line-to-neutral voltage as

$$V_{AB1} = \sqrt{3}V_{AN1}\angle 30° \tag{7.63}$$

$$V_{AB2} = \sqrt{3}V_{AN2}\angle -30° \tag{7.64}$$

Therefore combining (7.62)-(7.64) we get

$$\sqrt{3}V_{AN1}\angle 30° + \sqrt{3}V_{AN2}\angle -30° = \alpha\left(\sqrt{3}V_{an1}\angle 30° + \sqrt{3}V_{an2}\angle -30°\right) \tag{7.65}$$

Hence we get

$$V_{AN1} = \alpha V_{an1} \text{ and } V_{AN2} = \alpha V_{an2} \tag{7.66}$$

Thus the positive and negative sequence equivalent circuits are represented by a series impedance that is equal to the leakage impedance of the transformer. Since the Δ -connected winding does not provide any path for the zero sequence current to flow we have

$I_{A0} = I_{a0} = 0$

However the zero sequence current can sometimes circulate within the Δ windings. We can then draw the zero sequence equivalent circuit as shown in Fig. 7.14.

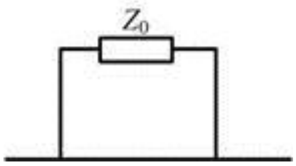


Fig. 7.14 Zero sequence diagram of Δ - Δ connected transformer.

- Δ Connected Transformer

The schematic diagram of a Y- Δ connected transformer is shown in Fig. 7.15. It is assumed that the Y-connected side is grounded with the impedance Z_N . Even though the zero sequence current in the primary Y-connected side has a path to the ground, the zero sequence current flowing in the Δ -connected secondary winding has no path to flow in the line. Hence we have I_{a0} = 0. However the circulating zero sequence current in the Δ winding magnetically balances the zero sequence current of the primary winding.

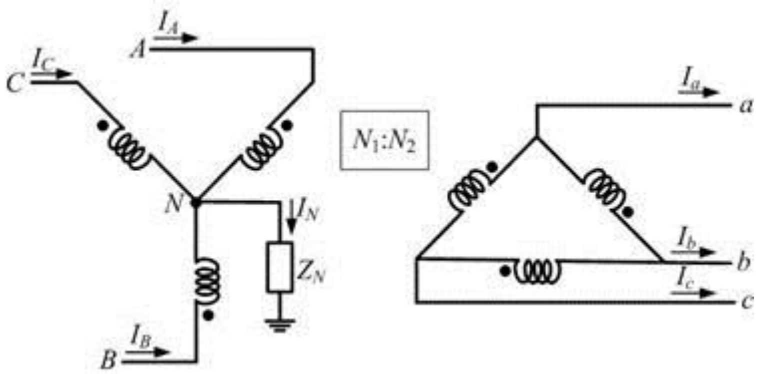


Fig. 7.15 Schematic diagram of a Y- Δ connected transformer.

$$V_{AN} = \frac{N_1}{N_2} V_{ab} = \alpha V_{ab}$$

The voltage in phase-a of both sides of the transformer is related by

Also we know that

$$V_A = V_{AN} + V_N$$

$$V_{A0} + V_{A1} + V_{A2} = V_{AN0} + V_{AN1} + V_{AN2} + 3Z_N I_{A0} \\ = \alpha\left(V_{ab0} + V_{ab1} + V_{ab2}\right) + 3Z_N I_{A0} \tag{7.67}$$

We therefore have

$$V_{A0} - 3Z_N I_{A0} = \alpha V_{ab0} = 0 \tag{7.68}$$

Separating zero, positive and negative sequence components we can write

$$V_{A1} = \alpha V_{ab1} = \sqrt{3}\alpha V_{a1}\angle 30^\circ \tag{7.69}$$

$$V_{A2} = \alpha V_{ab2} = \sqrt{3}\alpha V_{a2}\angle -30^\circ \tag{7.70}$$

The positive sequence equivalent circuit is shown in Fig. 7.16 (a). The negative sequence circuit is the same as that of the positive sequence circuit except for the phase shift in the induced emf. This is shown in Fig. 7.16 (b). The zero sequence equivalent circuit is shown in Fig. 7.16 (c) where $Z_0 = Z + 3Z_N$. Note that the primary and the secondary sides are not connected and hence there is an open circuit between them. However since the zero sequence current flows through primary windings, a return path is provided through the ground. If however, the neutral in the primary side is not grounded, i.e., $Z_N = \infty$, then the zero sequence current cannot flow in the primary side as well. The sequence diagram is then as shown in Fig. 7.16 (d) where $Z_0 = Z$.

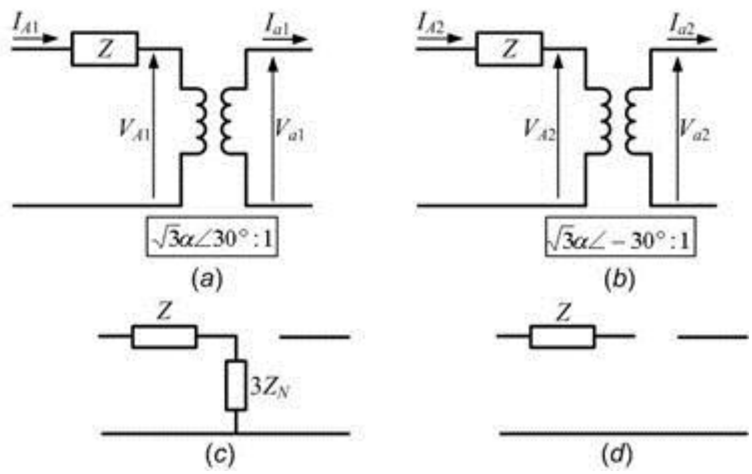


Fig. 7.16 Sequence diagram of a Y- Δ connected transformer: (a) positive sequence, (b) negative sequence, (c) zero sequence with grounded Y-connection and (d) zero sequence with ungrounded Y-connection.

Section VI: Sequence Networks

The sequence circuits developed in the previous sections are combined to form the sequence networks. The sequence networks for the positive, negative and zero sequences are formed separately by combining the sequence circuits of all the individual elements. Certain assumptions are made while forming the sequence networks. These are listed below.

1. Apart from synchronous machines, the network is made of static elements.
2. The voltage drop caused by the current in a particular sequence depends only on the impedance of that part of the network.
3. The positive and negative sequence impedances are equal for all static circuit components, while the zero sequence component need not be the same as them. Furthermore subtransient positive and negative sequence impedances of a synchronous machine are equal.
4. Voltage sources are connected to the positive sequence circuits of the rotating machines.
5. No positive or negative sequence current flows between neutral and ground.

Example 7.5

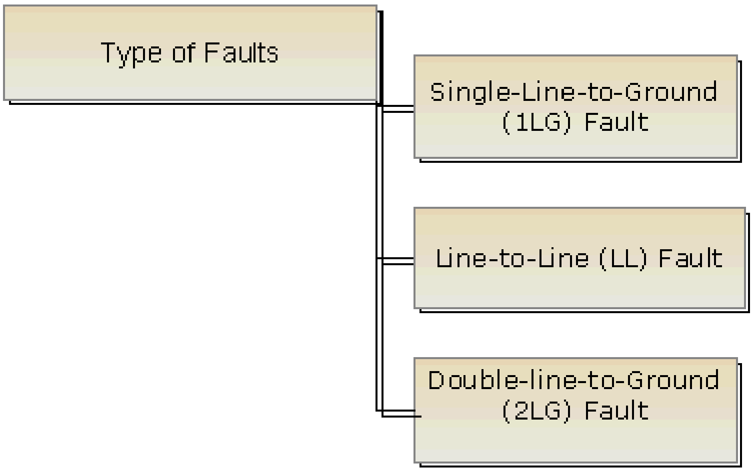
Introduction

- **Single-Line to Ground Fault**
- **Line-to-Line Fault**
- **Double-Line -to-Ground Fault**
- **Fault Current Computation using Sequence Networks**

Introduction

The sequence circuits and the sequence networks developed in the previous chapter will now be used for finding out fault current during unsymmetrical faults.

Three Types of Faults



Calculation of fault currents

Let us make the following assumptions:

- The power system is balanced before the fault occurs such that of the three sequence networks only the positive sequence network is active. Also as the fault occurs, the sequence networks are connected only through the fault location.
- The fault current is negligible such that the pre-fault positive sequence voltages are same at all nodes and at the fault location.
- All the network resistances and line charging capacitances are negligible.
- All loads are passive except the rotating loads which are represented by synchronous machines.

Based on the assumptions stated above, the faulted network will be as shown in Fig. 8.1 where the voltage at the faulted point will be denoted by V_f and current in the three faulted phases are I_{fa} , I_{fb} and I_{fc} .

We shall now discuss how the three sequence networks are connected when the three types of faults discussed above occur.

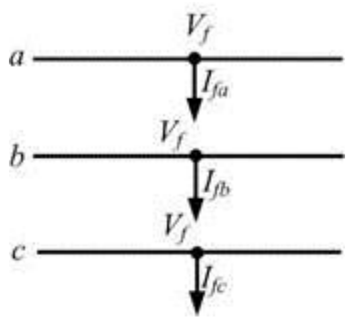


Fig. 8.1 Representation of a faulted segment.

Single-Line-to-Ground Fault

Let a 1LG fault has occurred at node *k* of a network. The faulted segment is then as shown in Fig. 8.2 where it is assumed that phase-a has touched the ground through an impedance Z_f . Since the

$$I_{fb} = I_{fc} = 0 \tag{8.1}$$

system is unloaded before the occurrence of the fault we have

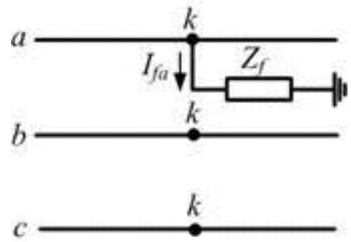


Fig. 8.2 Representation of 1LG fault.

$$V_{ka} = Z_f I_{fa} \tag{8.2}$$

Also the phase-a voltage at the fault point is given by

$$I_{fa012} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_{fa} \\ 0 \\ 0 \end{bmatrix} \tag{8.3}$$

From (8.1) we can write

$$I_{fa0} = I_{fa1} = I_{fa2} = \frac{I_{fa}}{3} \tag{8.4}$$

Solving (8.3) we get

This implies that the three sequence currents are in series for the 1LG fault. Let us denote the zero, positive and negative sequence Thevenin impedance at the faulted point as Z_{kk0} , Z_{kk1} and Z_{kk2} respectively. Also since the Thevenin voltage at the faulted phase is V_f we get three sequence circuits

$$\begin{aligned} V_{ka0} &= -Z_{kk0} I_{fa0} \\ V_{ka1} &= V_f - Z_{kk1} I_{fa1} \\ V_{ka2} &= -Z_{kk2} I_{fa2} \end{aligned} \tag{8.5}$$

that are similar to the ones shown in Fig. 7.7. We can then write

$$\begin{aligned} V_{ka} &= V_{ka0} + V_{ka1} + V_{ka2} \\ &= V_f - \left(Z_{kk0} + Z_{kk1} + Z_{kk2}\right) I_{fa0} \end{aligned} \tag{8.6}$$

Then from (8.4) and (8.5) we can write

$$V_{ka} = Z_f I_{fa} = Z_f \left(I_{fa0} + I_{fa1} + I_{fa2}\right) = 3 Z_f I_{fa0}$$

Again since

$$I_{fa0} = \frac{V_f}{Z_{kk0} + Z_{kk1} + Z_{kk2} + 3Z_f} \tag{8.7}$$

We get from (8.6)

The Thevenin equivalent of the sequence network is shown in Fig. 8.3.

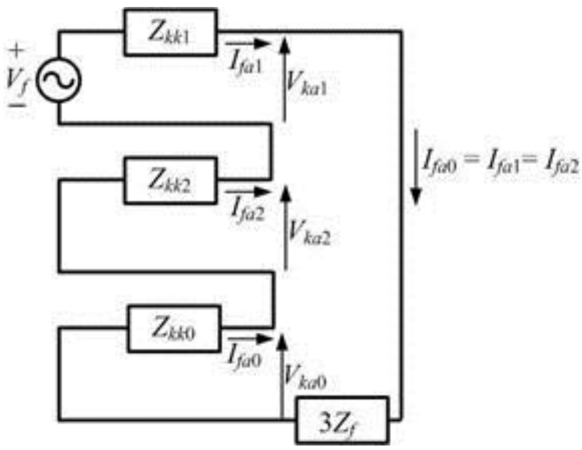


Fig. 8.3 Thevenin equivalent of a 1LG fault.

Line-to-Line Fault

The faulted segment for an L-L fault is shown in Fig. 8.5 where it is assumed that the fault has occurred at node *k* of the network. In this the phases b and c got shorted through the impedance Z_f . Since the system is unloaded before the occurrence of the fault we have

$$I_{fa} = 0 \tag{8.8}$$

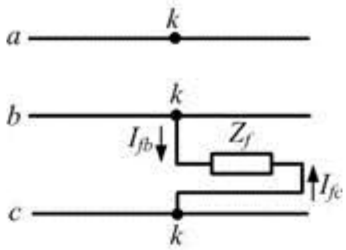


Fig. 8.5 Representation of L-L fault.

Also since phases b and c are shorted we have

$$I_{fb} = -I_{fc} \tag{8.9}$$

Therefore from (8.8) and (8.9) we have

$$I_{fa012} = C\begin{bmatrix} 0 \\ I_{fb} \\ -I_{fb} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 0 \\ \left(a-a^2\right)I_{fb} \\ \left(a^2-a\right)I_{fb} \end{bmatrix} \tag{8.10}$$

We can then summarize from (8.10)

$$\begin{aligned} I_{fa0} &= 0 \\ I_{fa1} &= -I_{fa2} \end{aligned} \tag{8.11}$$

Therefore no zero sequence current is injected into the network at bus k and hence the zero sequence remains a dead network for an L-L fault. The positive and negative sequence currents are negative of each other.

Now from Fig. 8.5 we get the following expression for the voltage at the faulted point

$$V_{kb} - V_{kc} = Z_f I_{fb} \tag{8.12}$$

Again

$$\begin{aligned} V_{kb} - V_{kc} &= V_{kb0} + V_{kb1} + V_{kb2} - V_{kc0} - V_{kc1} - V_{kc2} \\ &= \left(V_{kb1} - V_{kc1}\right) + \left(V_{kb2} - V_{kc2}\right) \\ &= \left(a^2 - a\right)V_{ka1} + \left(a - a^2\right)V_{ka2} \\ &= \left(a^2 - a\right)\left(V_{ka1} - V_{ka2}\right) \end{aligned} \tag{8.13}$$

Moreover since $I_{fa0} = I_{fb0} = 0$ and $I_{fa1} = -I_{fb2}$, we can write

$$I_{fb} = I_{fb1} + I_{fb2} = a^2 I_{fa1} + a I_{fb2} = \left(a^2 - a\right)I_{fa1} \tag{8.14}$$

Therefore combining (8.12) - (8.14) we get

$$V_{ka1} - V_{ka2} = Z_f I_{fa1} \tag{8.15}$$

Equations (8.12) and (8.15) indicate that the positive and negative sequence networks are in parallel. The sequence network is then as shown in Fig. 8.6. From this network we get

$$I_{fa1} = -I_{fa2} = \frac{V_f}{Z_{kk1} + Z_{kk2} + Z_f} \tag{8.16}$$

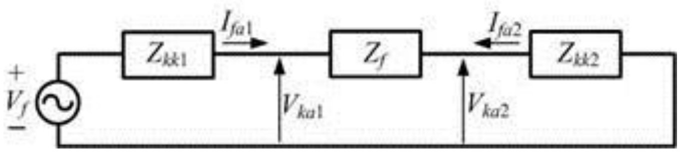


Fig. 8.6 Thevenin equivalent of an LL fault.

Example 8.2

Double- Line -to Ground Fault

The faulted segment for a 2LG fault is shown in Fig. 8.7 where it is assumed that the fault has occurred at node *k* of the network. In this the phases b and c got shorted through the impedance Z_f to the ground. Since the system is unloaded before the occurrence of the fault we have the same

$$I_{fa0} = \frac{1}{3}\left(I_{fa} + I_{fb} + I_{fc}\right) = \frac{1}{3}\left(I_{fb} + I_{fc}\right)$$
$$\Rightarrow 3I_{fa0} = I_{fb} + I_{fc} \tag{8.17}$$

condition as (8.8) for the phase-a current. Therefore

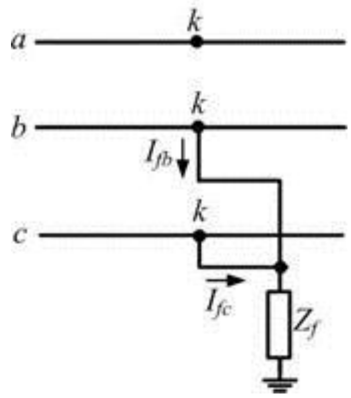


Fig. 8.7 Representation of 2LG fault.

$$V_{kb} = V_{kc} = Z_f\left(I_b + I_c\right) = 3Z_f I_{fa0} \tag{8.18}$$

Also voltages of phases b and c are given by

$$V_{ka012} = C\begin{bmatrix} V_{ka} \\ V_{kb} \\ V_{kb} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} V_{ka} + 2V_{kb} \\ V_{ka} + \left(a + a^2\right)V_{kb} \\ V_{ka} + \left(a + a^2\right)V_{kb} \end{bmatrix} \tag{8.19}$$

Therefore

$$V_{ka1} = V_{ka2} \tag{8.20}$$

We thus get the following two equations from (8.19)

$$3V_{ka0} = V_{ka} + 2V_{kb} = V_{ka0} + V_{ka1} + V_{ka2} + 2V_{kb} \tag{8.21}$$

$$V_{ka1} = V_{ka2} = V_{ka0} - 3Z_f I_{fa0} \tag{8.22}$$

Substituting (8.18) and (8.20) in (8.21) and rearranging we get

$$I_{fa0} + I_{fa1} + I_{fa2} = 0 \tag{8.23}$$

Also since $I_{fa} = 0$ we have

$$I_{fa1} = \frac{V_f}{Z_{kk1} + Z_{kk2} \| (Z_{kk0} + 3Z_f)} = \frac{V_f}{Z_{kk1} + \dfrac{Z_{kk2}(Z_{kk0} + 3Z_f)}{Z_{kk2} + Z_{kk0} + 3Z_f}} \tag{8.24}$$

The Thevenin equivalent circuit for 2LG fault is shown in Fig. 8.8. From this figure we get

$$I_{fa0} = -I_{fa1}\left(\frac{Z_{kk2}}{Z_{kk2} + Z_{kk0} + 3Z_f}\right) \tag{8.25}$$

The zero and negative sequence currents can be obtained using the current divider principle as

$$I_{fa2} = -I_{fa1}\left(\frac{Z_{kk0} + 3Z_f}{Z_{kk2} + Z_{kk0} + 3Z_f}\right) \tag{8.26}$$

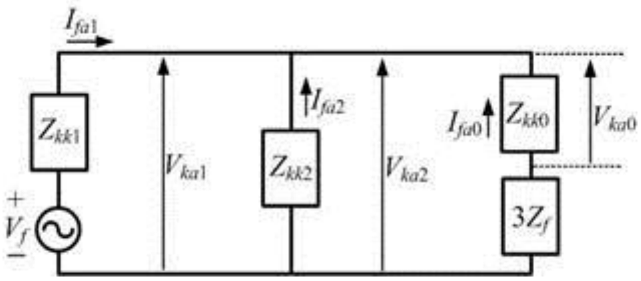


Fig. 8.8 Thevenin equivalent of a 2LG fault.

Example 8.3

FAULT CURRENT COMPUTATION USING SEQUENCE NETWORKS

In this section we shall demonstrate the use of sequence networks in the calculation of fault currents using sequence network through some examples.

Example 8.4

Consider the network shown in Fig. 8.10. The system parameters are given below

enerator G : 50 MVA, 20 kV, $X'' = X_1 = X_2 = 20\%$, $X_0 = 7.5\%$

lotor M : 40 MVA, 20 kV, $X'' = X_1 = X_2 = 20\%$, $X_0 = 10\%$, $X_n = 5\%$

ransformer T_1 : 50 MVA, 20 kV Δ /110 kVY, $X = 10\%$

ransformer T_2 : 50 MVA, 20 kV Δ /110 kVY, $X = 10\%$

ransmission line: $X_1 = X_2 = 24.2\ \Omega$, $X_0 = 60.5\ \Omega$

We shall find the fault current for when a (a) 1LG, (b) LL and (c) 2LG fault occurs at bus-2.

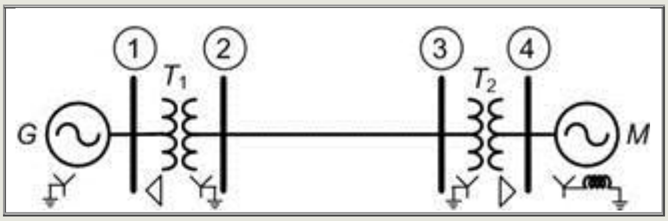


Fig. 8.10 Radial power system of Example 8.4.

Let us choose a base in the circuit of the generator. Then the per unit impedances of the generator are:

$$X_{G1} = X_{G2} = 0.2, \quad X_{G0} = 0.075$$

The per unit impedances of the two transformers are

$$X_{T1} = X_{T2} = 0.1$$

The MVA base of the motor is 40, while the base MVA of the total circuit is 50. Therefore the per unit impedances of the motor are

$$X_{M1} = X_{M2} = 0.2 \times \frac{50}{40} = 0.25, \quad X_{M0} = 0.1 \times \frac{50}{40} = 0.125, \quad X_n = 0.05 \times \frac{50}{40} = 0.0625$$

For the transmission line

$$Z_{base} = \frac{110^2}{50} = 242\,\Omega$$

Therefore

$$X_{L1} = X_{L2} = \frac{24.2}{242} = 0.1, \quad X_{L0} = \frac{60.5}{242} = 0.25$$

Let us neglect the phase shift associated with the Y/ Δ transformers. Then the positive, negative and zero sequence networks are as shown in Figs. 8.11-8.13.

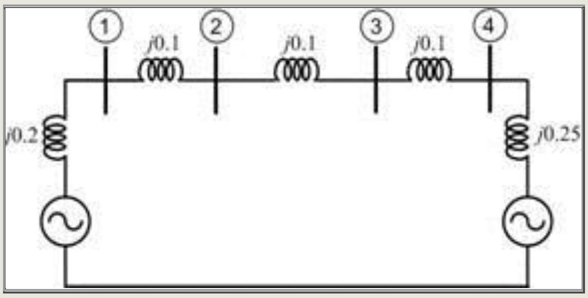


Fig. 8.11 Positive sequence network of the power system of Fig. 8.10.

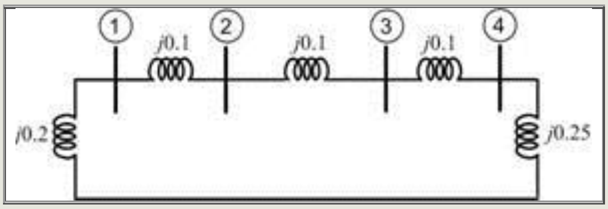


Fig. 8.12 Negative sequence network of the power system of Fig. 8.10.

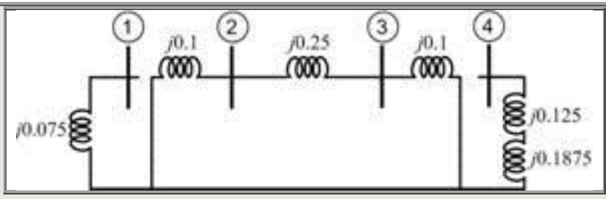


Fig. 8.13 Zero sequence network of the power system of Fig. 8.10.

From Figs. 8.11 and 8.12 we get the following Y_{bus} matrix for both positive and negative sequences

$$Y_{bus1} = Y_{bus2} = j\begin{bmatrix} -15 & 10 & 0 & 0 \\ 10 & -20 & 10 & 0 \\ 0 & 10 & -20 & 10 \\ 0 & 0 & 10 & 14 \end{bmatrix}$$

Inverting the above matrix we get the following Z_{bus} matrix

$$Z_{bus1} = Z_{bus2} = j\begin{bmatrix} 0.1467 & 0.1200 & 0.0933 & 0.0667 \\ 0.1200 & 0.1800 & 0.1400 & 0.1000 \\ 0.0933 & 0.1400 & 0.1867 & 0.1333 \\ 0.0667 & 0.1000 & 0.1333 & 0.1667 \end{bmatrix}$$

Again from Fig. 8.13 we get the following Y_{bus} matrix for the zero sequence

$$Y_{bus0} = j\begin{bmatrix} -13.3333 & 0 & 0 & 0 \\ 0 & -14 & 4 & 0 \\ 0 & 4 & -14 & 0 \\ 0 & 0 & 0 & -3.2 \end{bmatrix}$$

Inverting the above matrix we get

$$Z_{bus0} = j\begin{bmatrix} 0.075 & 0 & 0 & 0 \\ 0 & 0.0778 & 0.0222 & 0 \\ 0 & 0.0222 & 0.0778 & 0 \\ 0 & 0 & 0 & 0.3125 \end{bmatrix}$$

Hence for a fault in bus-2, we have the following Thevenin impedances

$$Z_1 = Z_2 = j0.18, \quad Z_0 = j0.0778$$

Alternatively we find from Figs. 8.11 and 8.12 that

$$Z_1 = Z_2 = j0.3 \| j0.45 = j0.18$$

$Z_0 = j0.1 \| j0.35 = j0.0778$

(a) Single-Line-to-Ground Fault : Let a bolted 1LG fault occurs at bus-2 when the system is unloaded with bus voltages being 1.0 per unit. Then from (8.7) we get

$$I_{fa0} = I_{fa1} = I_{fa2} = \frac{1}{j(2 \times 0.18 + 0.0778)} = -j2.2841$$ per unit

Also from (8.4) we get

$$I_{fa} = 3I_{fa0} = -j6.8524$$ per unit

Also $I_{fb} = I_{fc} = 0$. From (8.5) we get the sequence components of the voltages as

$$V_{2a0} = -j0.0778 I_{fa0} = -0.1777$$
$$V_{2a1} = 1 - j0.18 I_{fa1} = 0.5889$$
$$V_{2a2} = -j0.18 I_{fa2} = -0.4111$$

Therefore the voltages at the faulted bus are

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = C^{-1} \begin{bmatrix} V_{2a0} \\ V_{2a1} \\ V_{2a2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.9061\angle -107.11^\circ \\ 0.9061\angle 107.11^\circ \end{bmatrix}$$

(b) Line-to-Line Fault : For a bolted LL fault, we can write from (8.16)

$$I_{fa1} = -I_{fa2} = \frac{1}{j2 \times 0.18} = -j2.7778$$ per unit

Then the fault currents are

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = C^{-1} \begin{bmatrix} 0 \\ I_{fa1} \\ I_{fa2} \end{bmatrix} = \begin{bmatrix} 0 \\ -4.8113 \\ 4.8113 \end{bmatrix}$$

Finally the sequence components of bus-2 voltages are

$$V_{2a0} = 0$$
$$V_{2a1} = 1 - j0.18 I_{fa1} = 0.5$$
$$V_{2a2} = -j0.18 I_{fa2} = 0.5$$

Hence faulted bus voltages are

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = C^{-1} \begin{bmatrix} V_{2a0} \\ V_{2a1} \\ V_{2a2} \end{bmatrix} = \begin{bmatrix} 1.0 \\ -0.5 \\ -0.5 \end{bmatrix}$$

(c) Double-Line-to-Ground Fault : Let us assumes that a bolted 2LG fault occurs at bus-2. Then

$$Z_{eq} = j0.18 \| j0.0778 = j0.0543$$

Hence from (8.24) we get the positive sequence current as

$$I_{fa1} = \frac{1}{j0.18 + Z_{eq}} = -j4.2676$$ per unit

The zero and negative sequence currents are then computed from (8.25) and (8.26) as

$$I_{fa0} = -I_{fa1} \frac{j0.18}{j(0.18 + 0.0778)} = j2.9797$$ per unit

$$I_{fa2} = -I_{fa1} \frac{j0.0778}{j(0.18 + 0.0778)} = j1.2879$$ per unit

Therefore the fault currents flowing in the line are

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = C^{-1} \begin{bmatrix} I_{fa0} \\ I_{fa1} \\ I_{fa2} \end{bmatrix} = \begin{bmatrix} 0 \\ 6.657\angle 137.11^\circ \\ 6.657\angle 42.89^\circ \end{bmatrix}$$

Furthermore the sequence components of bus-2 voltages are

$$V_{2a0} = -j0.0778 I_{fa0} = 0.2318$$
$$V_{2a1} = 1 - j0.18 I_{fa1} = 0.2318$$
$$V_{2a2} = -j0.18 I_{fa2} = 0.2318$$

Therefore voltages at the faulted bus are

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = C^{-1} \begin{bmatrix} V_{2a0} \\ V_{2a1} \\ V_{2a2} \end{bmatrix} = \begin{bmatrix} 0.6954 \\ 0 \\ 0 \end{bmatrix}$$

Example 8.5

Let us now assume that a 2LG fault has occurred in bus-4 instead of the one in bus-2. Therefore

$$X_1 = X_2 = j0.1667, \quad X_0 = j0.3125$$

Also we have

$$Z_{eq} = j0.1667 \| j0.3125 = j0.1087$$

Hence

$$I_{fa1} = \frac{1}{j0.1667 + Z_{eq}} = -j3.631$$ per unit

Also

$$I_{fa0} = -I_{fa1} \frac{j0.1667}{j(0.1667 + 0.3125)} = j1.2631$$ per unit

$$I_{fa2} = -I_{fa1} \frac{j0.3125}{j(0.1667 + 0.3125)} = j2.3678$$ per unit

Therefore the fault currents flowing in the line are

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = C^{-1} \begin{bmatrix} I_{fa0} \\ I_{fa1} \\ I_{fa2} \end{bmatrix} = \begin{bmatrix} 0 \\ 5.5298\angle 159.96° \\ 5.5298\angle 20.04° \end{bmatrix}$$

We shall now compute the currents contributed by the generator and the motor to the fault. Let us denote the current flowing to the fault from the generator side by I_g , while that flowing from the motor by I_m . Then from Fig. 8.11 using the current divider principle, the positive sequence currents contributed by the two buses are

$$I_{ga1} = I_{fa1} \times \frac{j0.25}{j0.75} = -j1.2103$$ per unit

$$I_{ma1} = I_{fa1} \times \frac{j0.5}{j0.75} = -j2.4206$$ per unit

Similarly from Fig. 8.12, the negative sequence currents are given as

$$I_{ga2} = I_{fa2} \times \frac{j0.25}{j0.75} = j0.7893$$ per unit

$$I_{ma2} = I_{fa2} \times \frac{j0.5}{j0.75} = j1.5786$$ per unit

Finally notice from Fig. 8.13 that the zero sequence current flowing from the generator to the fault is 0. Then we have

$$I_{ga0} = 0$$

$$I_{ma0} = j1.2631$$ per unit

Therefore the fault currents flowing from the generator side are

$$\begin{bmatrix} I_{ga} \\ I_{gb} \\ I_{gc} \end{bmatrix} = C^{-1} \begin{bmatrix} I_{ga0} \\ I_{ga1} \\ I_{ga2} \end{bmatrix} = \begin{bmatrix} 0.4210\angle -90° \\ 1.7445\angle 173.07° \\ 1.7445\angle 6.93° \end{bmatrix}$$

and those flowing from the motor are

$$\begin{bmatrix} I_{ma} \\ I_{mb} \\ I_{mc} \end{bmatrix} = C^{-1} \begin{bmatrix} I_{ma0} \\ I_{ma1} \\ I_{ma2} \end{bmatrix} = \begin{bmatrix} 0.4210\angle 90° \\ 3.8512\angle 154.07° \\ 3.8512\angle 25.93° \end{bmatrix}$$

It can be easily verified that adding I_g and I_m we get I_f given above.

In the above two examples we have neglected the phase shifts of the Y/ Δ transformers. However according to the American standard, the positive sequence components of the high tension side lead those of the low tension side by 30° , while the negative sequence behavior is reverse of the positive sequence behavior. Usually the high tension side of a Y/ Δ transformer is Y-connected. Therefore as we have seen in Fig. 7.16, the positive sequence component of Y side leads the positive sequence component of the Δ side by 30° while the negative sequence component of Y side lags that of the Δ side by 30° . We shall now use this principle to compute the fault current for an unsymmetrical fault.

Let us do some more examples

Example 8.6

Example 8.7