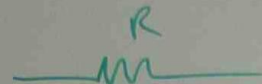
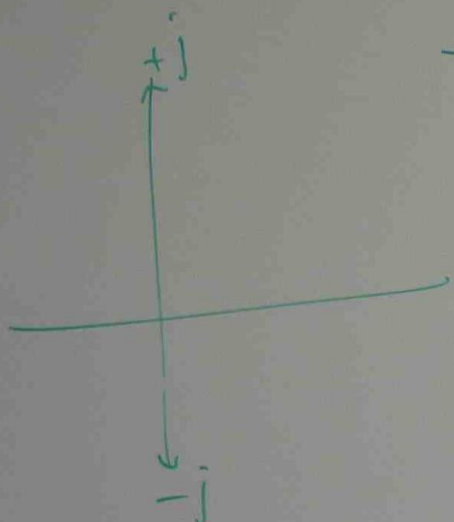
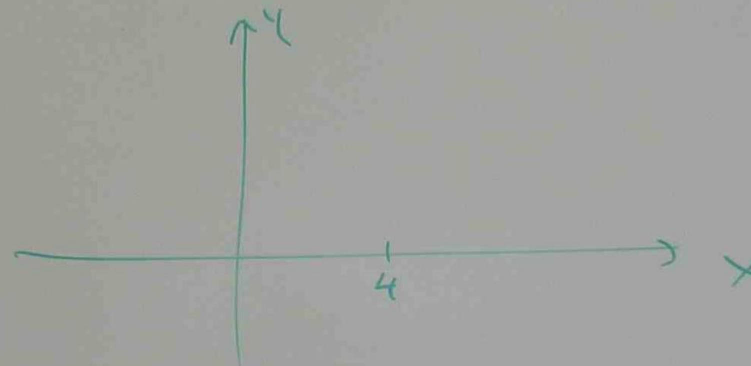
AC NETWORK THEOREM



RESISTOR $\longrightarrow$ REAL NUMBER

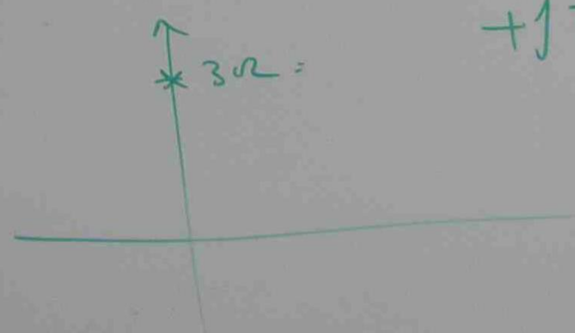


4Ω RESISTOR

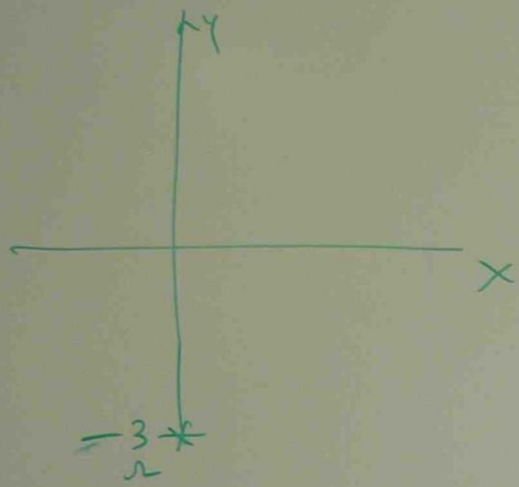


3Ω INDUCTOR $\longrightarrow$ IMAGINARY

$+j3\Omega$



3Ω CAPACITOR = $-j3\Omega$



INDUCTOR L (HENARY)

X_L = INDUCTIVE REACTANCE $= 2\pi f L \ (\Omega)$

f = FREQUENCY (HZ)

CAPACITIVE REACTANCE $X_C(\Omega) = \dfrac{1}{2\pi f C}$

C = FARAD

4Ω = 4

2Ω = $j2\Omega$

3Ω = $-j3\Omega$

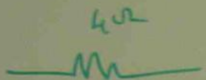
4Ω 2Ω = $4+j2\ \Omega$

4Ω 3Ω = $4-j3\Omega$

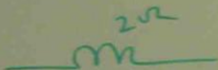
2Ω 4Ω = $j2 - j4 = -j2$

CAPACITIVE REACTANCE $X_c(\Omega) = \dfrac{1}{2\pi f c}$

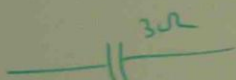
C = FARAD



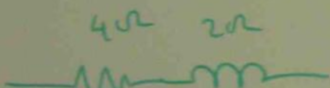
$= 4$



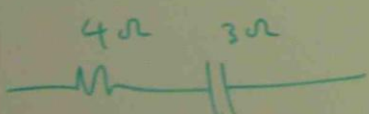
$= j2\Omega$



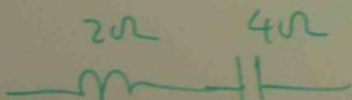
$= -j3\Omega$



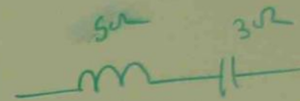
$= 4 + j2\ \Omega$



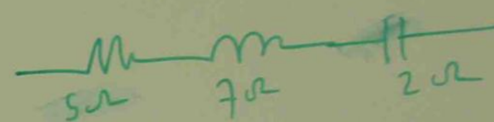
$= 4 - j3\Omega$



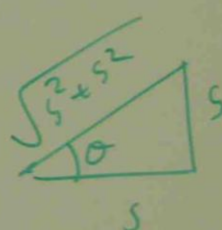
$= j2 - j4 = -j2$



$j5 - j3 = j2\ \Omega$



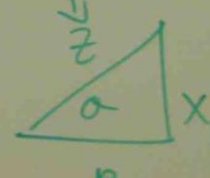
$5 + j7 - j2 = 5 + j5$



$= \sqrt{5^2+5^2}\ \angle \tan^{-1}\frac{5}{5}$

$= 7.07\ \angle 45$

IMPEDANCE



$Z = R + jX$ ← RECTANGULAR FORM

$= \sqrt{R^2+X^2}\ \angle \tan^{-1}\frac{X}{R}$

↑ POLAR FORM

$Z \angle \theta$ = POLAR FORM $\longrightarrow$ RECTANGULAR FORM

$Z\cos\theta + jZ\sin\theta$

$X + jY$ = RECTANGULAR FORM $\longrightarrow$ POLAR FORM

$\sqrt{X^2+Y^2} \angle \tan^{-1}\frac{Y}{X}$

MULTIPLY / DIVIDE $\longrightarrow$ USE POLAR FORM

$$Z_1\angle\theta_1 \times Z_2\angle\theta_2 = Z_1 Z_2 \angle\theta_1+\theta_2$$

$$\frac{Z_1\angle\theta_1}{Z_2\angle\theta_2} = \frac{Z_1}{Z_2}\angle\theta_1-\theta_2$$

ADDITION / SUBSTRACTION

USE RECTANGULAR FORM

$5\angle 53.2 - 5\angle 36.8 = ?$

$(5\cos 53.2 + j\,5\sin 53.2) - (5\cos 36.8 + j\,5\sin 36.8)$

$(5\times 0.6 + j\,5\times 0.8) - (5\times 0.8 + j\,5\times 0.6)$

$(3 + j4) - (4 + j3)$

$3 + j4 - 4 - j3$

$-1 + j1$

ADDITION / SUBSTRACTION

USE RECTANGULAR FORM

$5\angle 53.2 - 5\angle 36.8 = ?$

$(5\cos 53.2 + j\,5\sin 53.2) - (5\cos 36.8 + j\,5\sin 36.8)$

$(5\times 0.6 + j\,5\times 0.8) - (5\times 0.8 + j\,5\times 0.6)$

$(3 + j4) - (4 + j3)$

$3 + j4 - 4 - j3$

$-1 + j1$

EX

$(3+j4)(4+j3)$

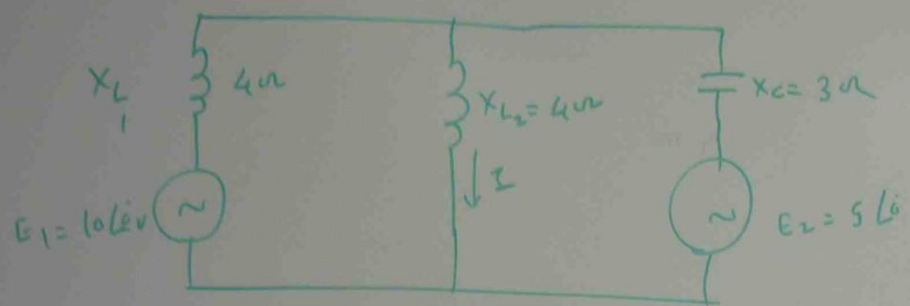
$\sqrt{3^2+4^2}\angle \tan^{-1}\frac{4}{3} \times \sqrt{4^2+3^2}\angle \tan^{-1}\frac{3}{4}$

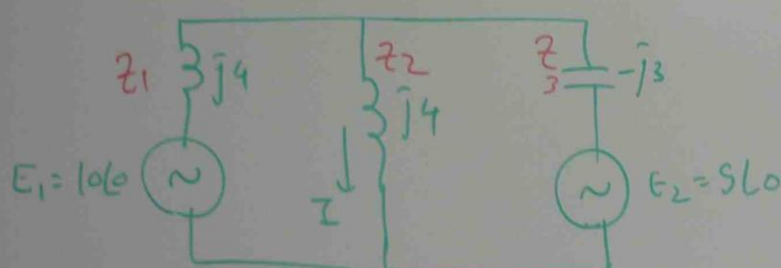
$5\angle 53.2 \times 5\angle 36.8$

$25\angle 53.2 + 36.8$

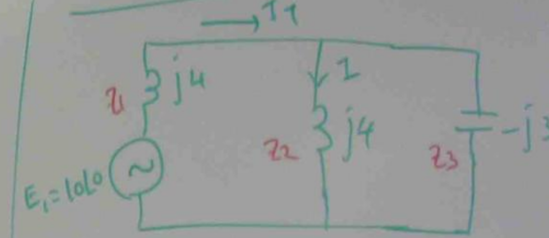
$25\angle 90$

EX USING THE SUPER POSITION THEOREM, FIND THE CURRENT I THROUGH THE 4 Ω REACTANCE.





KILL (E_2)



$$Z_T = Z_1 + Z_2 \parallel Z_3$$

$$Z_T = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$$

$$= j4 + \frac{j4 \times (-j3)}{j4 + (-j3)}$$

$$= j4 + \frac{4\angle 90 \times 3\angle -90}{j1}$$

$$= j4 + \frac{12\angle 90-90}{1\angle 90}$$

$$= j4 + \frac{12\angle 0}{1\angle 90}$$

$Z_T = j4 + 12\angle{-90}$

$= j4 - j12$

$= -j8$

$I_T = \dfrac{E1}{Z_T}$

$= \dfrac{10\angle 0}{-j8}$

$= \dfrac{10\angle 0}{8\angle{-90}}$

$= 1.25\angle 90$

$I_{(Z_2)} = I_T \times \dfrac{Z_3}{Z_2+Z_3}$

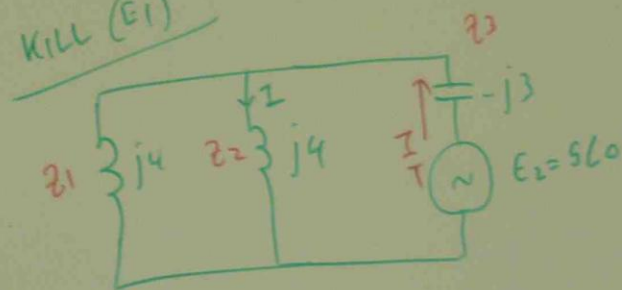
$= 1.25\angle 90 \times \dfrac{-j3}{j4+(-j3)}$

$I'_{(Z_2)} = 1.25\angle 90 \times \dfrac{3\angle{-90}}{j1}$

$= 1.25\angle 90 \times \dfrac{3\angle{-90}}{1\angle 90}$

$= 3.75\angle{-90}$ Amp

KILL (E1)



$Z_T = Z_3 + \dfrac{Z_1 Z_2}{Z_1+Z_2}$

$= (-j3) + \dfrac{j4 \times j4}{j4+j4}$

$= -j3 + \dfrac{4\angle 90 \times 4\angle 90}{j8}$

$= -j3 + \dfrac{16\angle 180}{8\angle 90}$

$= -j3 + 2\angle{180-90}$

$= -j3 + 2\angle 90$

$= -j3 + j2$

$= -j1 = 1\angle{-90}$

$I_T = \dfrac{E2}{Z_T} = \dfrac{5\angle 0}{1\angle{-90}}$

$= 5\angle 90$ Amp

$I''_{Z2} = I_T \times \dfrac{Z_1}{Z_1+Z_2}$

$= 5\angle 90 \times \dfrac{j4}{j4+j4}$

$= 5\angle 90 \times \dfrac{4\angle 90}{j8}$

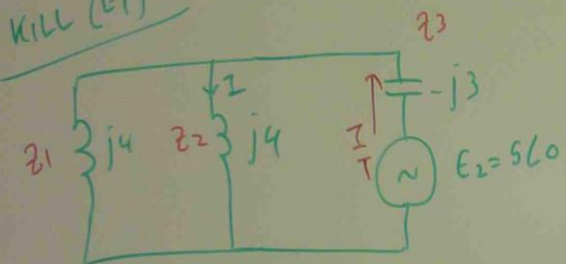
$= 5\angle 90 \times \dfrac{4\angle 90}{8\angle 90} =$

$$I'_{(Z2)} = 1.25\angle 90 \times \frac{3\angle -90}{j1}$$

$$= 1.25\angle 90 \times \frac{3\angle -90}{1\angle 90}$$

$$= 3.75\angle -90 \text{ Amp} \downarrow$$

KILL (E1)



$$Z_T = Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= (-j3) + \frac{j4 \times j4}{j4 + j4}$$

$$= -j3 + \frac{4\angle 90 \times 4\angle 90}{j8}$$

$$= -j3 + \frac{16\angle 180}{8\angle 90}$$

$$= -j3 + 2\angle 180-90$$

$$= -j3 + 2\angle 90$$

$$= -j3 + j2$$

$$= -j1 = 1\angle -90$$

$$I_T = \frac{E2}{Z_T} = \frac{5\angle 0}{1\angle -90}$$

$$= 5\angle 90 \text{ Amp.}$$

$$I''_{Z2} = I_T \times \frac{Z1}{Z1 + Z2}$$

$$= 5\angle 90 \times \frac{j4}{j4 + j4}$$

$$= 5\angle 90 \times \frac{4\angle 90}{j8}$$

$$= 5\angle 90 \times \frac{4\angle 90}{8\angle 90} = 2.5\angle 90 \downarrow$$

$$I_{Z2} = I'_{Z_2} + I''_{Z2}$$

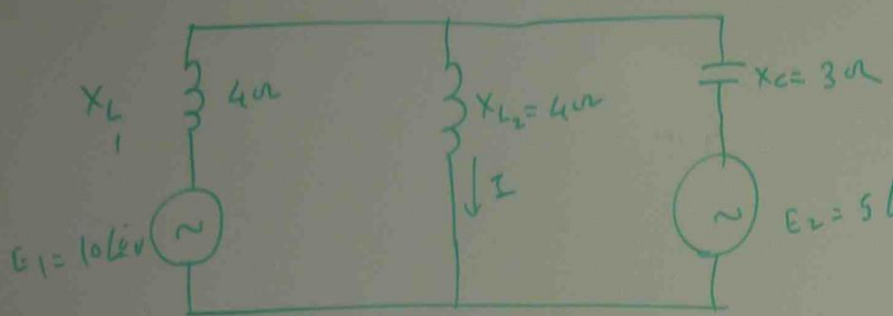
$$= 3.75\angle -90 + 2.5\angle 90$$

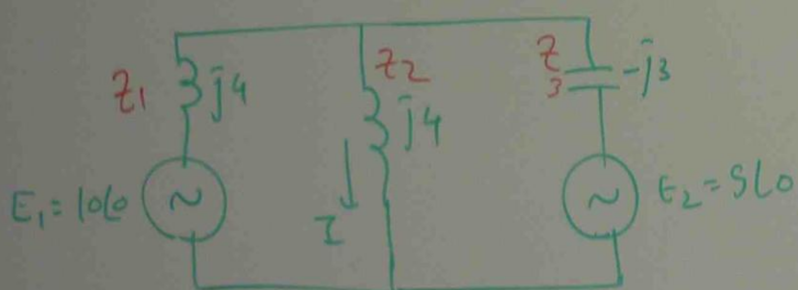
$$= -j3.75 + j2.5$$

$$= -j1.25$$

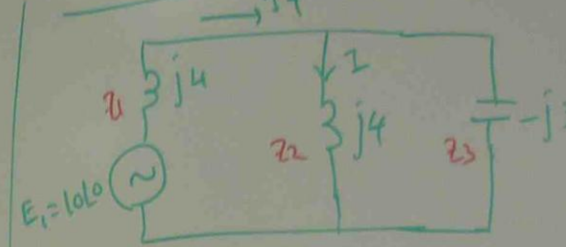
$$= 1.25\angle -90 \downarrow$$

Ex USING THE SUPER POSITION THEOREM, FIND THE CURRENT I THROUGH THE 4Ω REACTANCE.





KILL (E_2)



$Z_T = Z_1 + Z_2 \,||\, Z_3$

$$Z_T = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$$

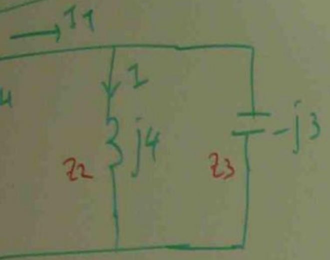
$$= j4 + \frac{j4 \times (-j3)}{j4 + (-j3)}$$

$$= j4 + \frac{4\angle 90 \times 3\angle -90}{j1}$$

$$= j4 + \frac{12\angle 90-90}{1\angle 90}$$

$$= j4 + \frac{12\angle 0}{1\angle 90}$$

(E2)



$Z_T = Z_1 + Z_2 \,||\, Z_3$

$= Z_1 + \dfrac{Z_2 Z_3}{Z_2 + Z_3}$

$= j4 + \dfrac{j4 \times (-j3)}{j4 + (-j3)}$

$= j4 + \dfrac{4\angle 90 \times 3\angle -90}{j1}$

$= j4 + \dfrac{12\angle 90-90}{1\angle 90}$

$= j4 + \dfrac{12\angle 0}{1\angle 90}$

$Z_T = j4 + 12\angle -90$

$= j4 - j12$

$= -j8$

$I_T = \dfrac{E1}{Z_T}$

$= \dfrac{10\angle 0}{-j8}$

$= \dfrac{10\angle 0}{8\angle -90}$

$= 1.25\angle 90$

$I_{(Z_2)} = I_T \times \dfrac{Z_3}{Z_2 + Z_3}$

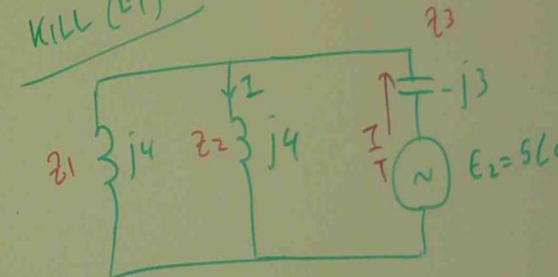
$= 1.25\angle 90 \times \dfrac{-j3}{j4 + (-j3)}$

$I'_{(Z_2)} = 1.25\angle 90 \times \dfrac{3\angle -90}{j1}$

$= 1.25\angle 90 \times \dfrac{3\angle -90}{1\angle 90}$

$= 3.75\angle -90$ Amp

KILL (E1)



$Z_T = Z_3 + \dfrac{Z_1 Z_2}{Z_1 + Z_2}$

$= (-j3) + \dfrac{j4 \times j4}{j4 + j4}$

$= -j3 + \dfrac{4\angle 90 \times 4\angle 90}{j8}$

$= -j3 + \dfrac{16\angle 180}{8\angle 90}$

$= -j3 + 2\angle 18$ [illegible]

$= -j3 + 2\angle 9$ [illegible]

$= -j3 + j2$

$= -j1 =$ [illegible]

$I_T = \dfrac{E2}{Z_T} =$ [illegible]

$I''_{Z_2} = I_T \times$ [illegible]

$= 5\angle 90 \times$ [illegible]

$= 5\angle 90 \times$ [illegible]

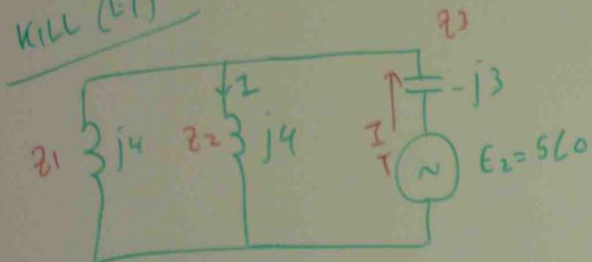
$= 5\angle 90$ [illegible]

$$I'_{(Z2)} = 1.25\angle 90 \times \frac{3\angle -90}{j1}$$

$$= 1.25\angle 90 \times \frac{3\angle -90}{1\angle 90}$$

$$= 3.75\angle -90 \text{ Amp} \downarrow$$

KILL (E1)



$$Z_T = Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= (-j3) + \frac{j4 \times j4}{j4 + j4}$$

$$= -j3 + \frac{4\angle 90 \times 4\angle 90}{j8}$$

$$= -j3 + \frac{16\angle 180}{8\angle 90}$$

$$= -j3 + 2\angle 180-90$$

$$= -j3 + 2\angle 90$$

$$= -j3 + j2$$

$$= -j1 = 1\angle -90$$

$$I_T = \frac{E_2}{Z_T} = \frac{5\angle 0}{1\angle -90}$$

$$= 5\angle 90 \text{ Amp.}$$

$$I''_{Z2} = I_T \times \frac{Z_1}{Z_1 + Z_2}$$

$$= 5\angle 90 \times \frac{j4}{j4 + j4}$$

$$= 5\angle 90 \times \frac{4\angle 90}{j8}$$

$$= 5\angle 90 \times \frac{4\angle 90}{8\angle 90} = 2.5\angle 90 \downarrow$$

$$I_{Z2} = I'_{Z2} + I''_{Z2}$$

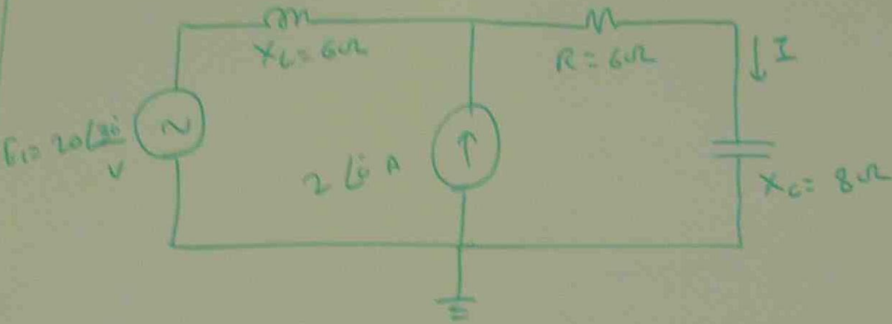
$$= 3.75\angle -90 + 2.5\angle 90$$

$$= -j3.75 + j2.5$$

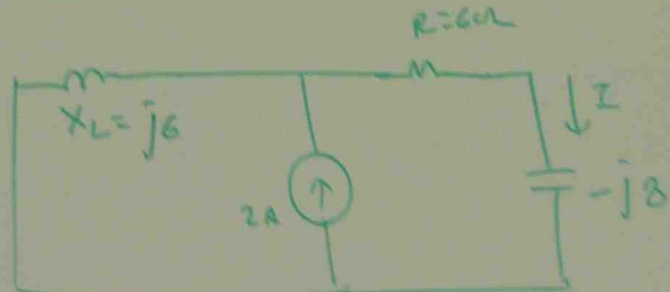
$$= -j1.25$$

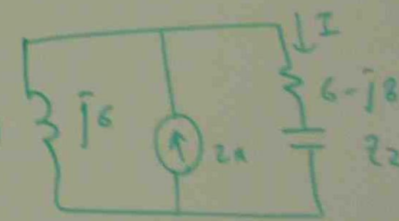
$$= 1.25\angle -90 \downarrow$$

EX USING SUPERPOSITION, FIND THE CURRENT I THROUGH THE 6Ω RESISTOR.



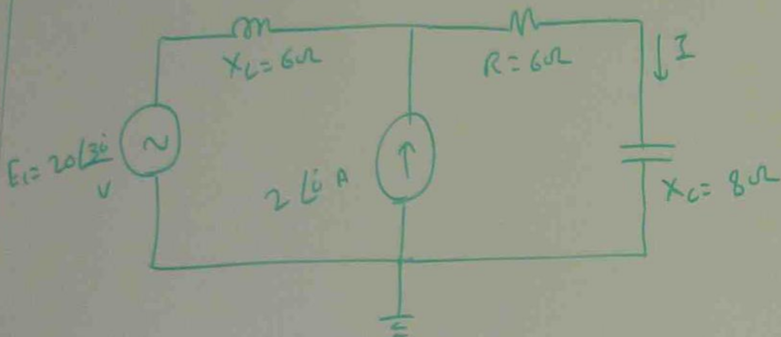
KILL $E_1 = 20\angle 30°$ V



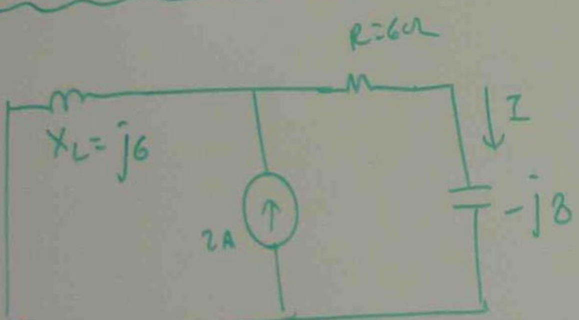


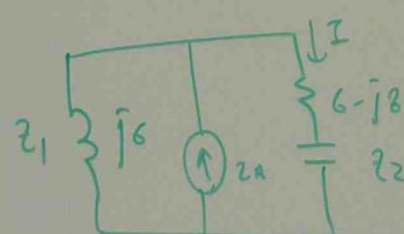
$$I'_{Z_2}\text{ (OR) CAPACITOR CURRENT} = I_T \times \frac{Z_1}{Z_1+Z_2} = 2 \times \frac{j6}{j6+6-j8} \downarrow$$

EX USING SUPER POSITION, FIND THE CURRENT I THROUGH THE 6Ω RESISTOR.



KILL $E_1 = 20\angle 30°$ V





I'_{Z_2} (OR) CAPACITOR CURRENT $= I_T \times \dfrac{Z_1}{Z_1+Z_2} = 2 \times \dfrac{j6}{j6+6-j8} \downarrow$

$= 2 * \dfrac{6\angle 90}{6-j2}$

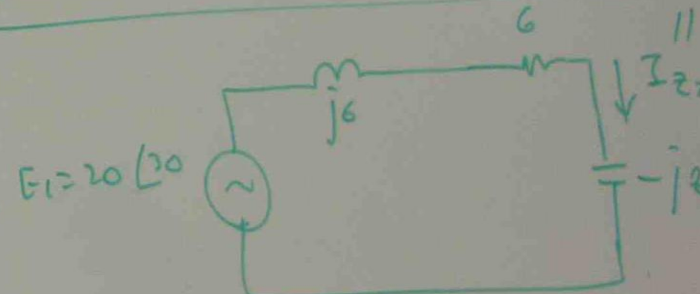
$= 2 * \dfrac{6\angle 90}{\sqrt{6^2+2^2}\ \angle -\tan^{-1}\frac{2}{6}}$

$= \dfrac{12\angle 90}{6.32\angle -18.43}$

$= 1.9\angle 90-(-18.43)$

$I'_{Z_2} = 1.9\angle 108.43$ A $\downarrow$

KILL $2\angle 0°$ A CURRENT SOURCE



$$Z_T = j6 + 6 + (-j8) = j6 + 6 - j8 = 6 - j2\ \Omega = \sqrt{6^2+2^2}\ \angle -\tan^{-1}\frac{2}{6}$$

$$= 6.32\ \angle -18.43$$

$$I_{Z_2}'' = \frac{E1}{Z_T} = \frac{20\angle 30}{6.32\angle -18.43} = 3.16\ \angle 30-(-18.43)$$

$$I_{Z_2}'' = 3.16\ \angle 48.43\ \text{A} \downarrow$$

$$I_{Z_2} = I_{Z_2}' + I_{Z_2}''$$

$$= 1.9\angle 108.43 + 3.16\angle 48.43$$

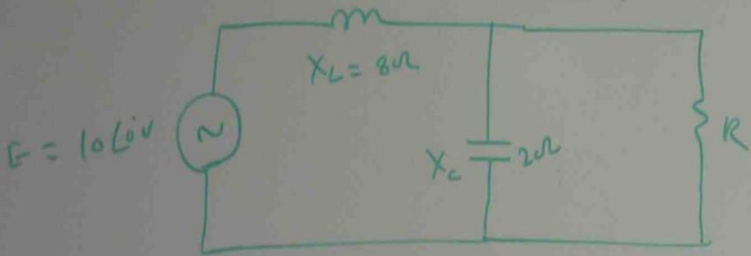
$$= 1.9\cos 108.43 + j1.9\sin 108.43 + 3.16\cos 48.43 + j3.16\sin 48.43$$

$$= -0.6 + j1.8 + 2.1 + j2.36$$

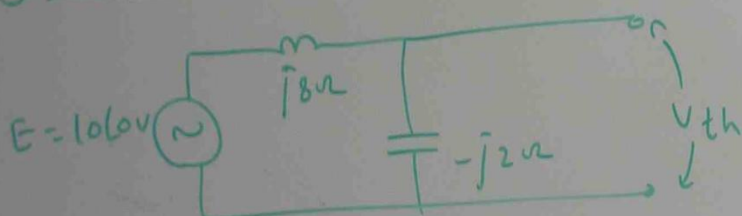
$$= 1.5 + j4.16$$

$$= \sqrt{1.5^2+4.16^2}\ \angle \tan^{-1}\frac{4.16}{1.5} = 4.42\ \angle 70.2\ \text{A} \downarrow$$

EX FIND THE THEVENIN'S EQUIVALENT CIRCUIT FOR THE NETWORK EXTERNAL TO RESISTOR "R" IN GIVEN FIGURE



1) REMOVE RESISTOR "R"



$$V_{R_2} = E \times \frac{R_2}{R_1 + R_2}$$

2) FIND V_{th}

$$V_{th} = 10\angle 0 \times \frac{(-j2)}{j8 + (-j2)}$$

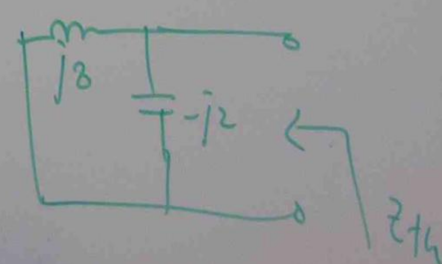
$$= 10\angle 0 \times \frac{2\angle -90}{j6}$$

$$= 10\angle 0 \times \frac{2\angle -90}{6\angle 90}$$

$$= 3.33\angle -90 - 90$$

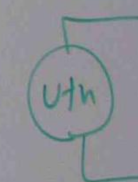
$$= 3.33\angle -180\ V$$

3) KILL THE SOURCE, FIND Z_{th}



$Z_{th} =$

$=$



$3.33\angle -1$

FOR THE NETWORK EXTERNAL

② FIND Vth

$$V_{th} = 10\angle 0 \times \frac{(-j2)}{j8+(-j2)}$$

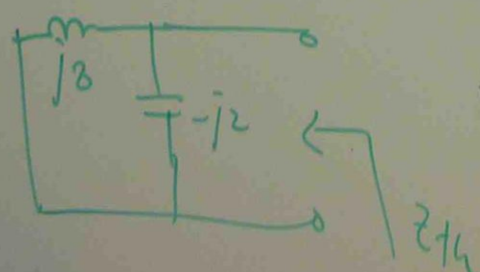
$$= 10\angle 0 \times \frac{2\angle -90}{j6}$$

$$= 10\angle 0 \times \frac{2\angle -90}{6\angle 90}$$

$$= 3.33\angle -90-90$$

$$= 3.33\angle -180\ V$$

③ KILL THE SOURCE, FIND Zth

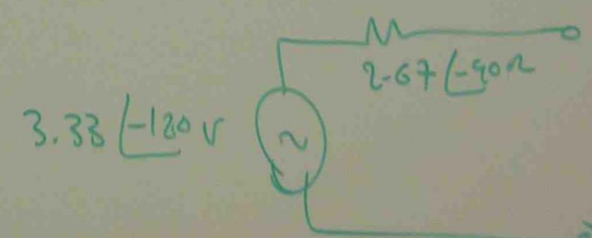


$$Z_{th} = \frac{j8 \times (-j2)}{j8+(-j2)}$$

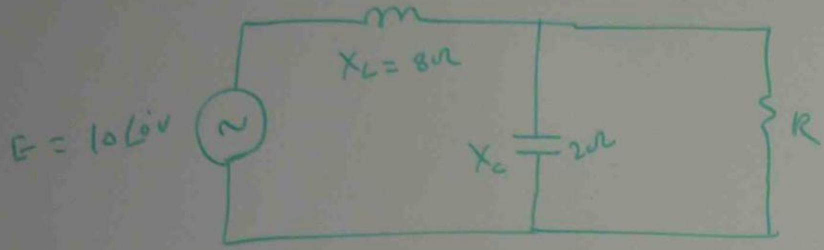
$$= \frac{8\angle 90 \times 2\angle -90}{j6}$$

$$= \frac{8\angle 90 \times 2\angle -90}{6\angle 90}$$

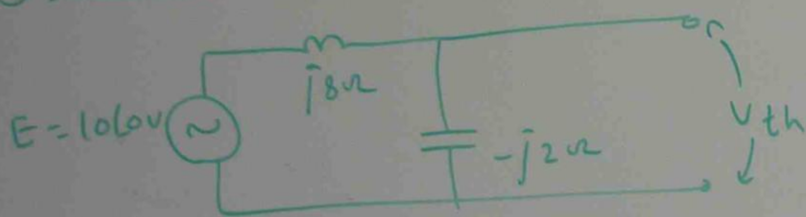
$$= 2.67\angle -90$$

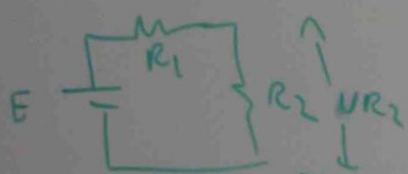


EX FIND THE THEVENIN'S EQUIVALENT CIRCUIT FOR THE NETWORK EXTERNAL TO RESISTOR "R" IN GIVEN FIGURE



1. REMOVE RESISTOR "R"





$$V_{R_2} = E \times \frac{R_2}{R_1 + R_2}$$

2. FIND Vth

$$V_{th} = 10\angle 0 \times \frac{(-j2)}{j8 + (-j2)}$$

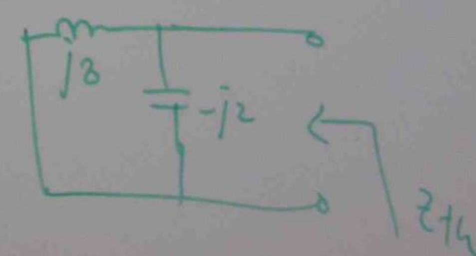
$$= 10\angle 0 \times \frac{2\angle -90}{j6}$$

$$= 10\angle 0 \times \frac{2\angle -90}{6\angle 90}$$

$$= 3.33\angle -90 - 90$$

$$= 3.33\angle -180\ V$$

3. KILL THE SOURCE, FIND Z_{th}

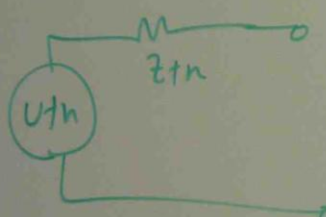


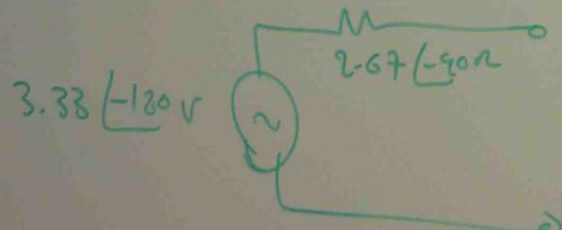
$$Z_{th} = \frac{j8 \times (-j2)}{j8 + (-j2)}$$

$$= \frac{8\angle 90 \times 2\angle -90}{j6}$$

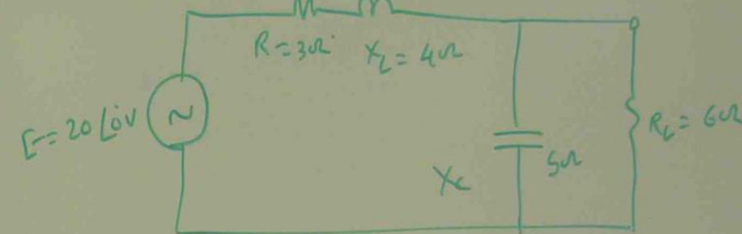
$$= \frac{8\angle 90 \times 2\angle -90}{6\angle 90}$$

$$= 2.67\angle -90$$

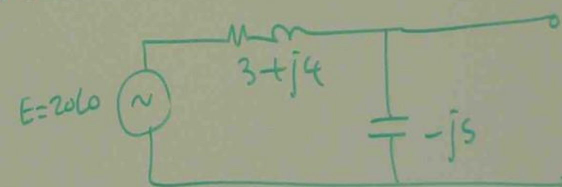




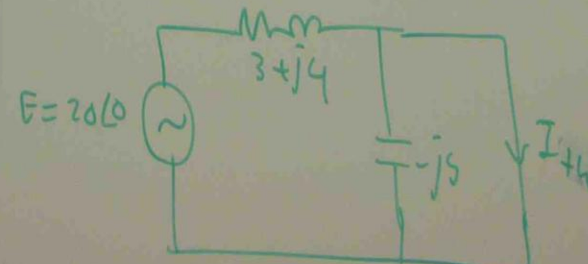
EX DETERMINE THE NORTON EQUIVALENT CIRCUIT FOR THE NETWORK EXTERNAL TO THE 6Ω RESISTOR.

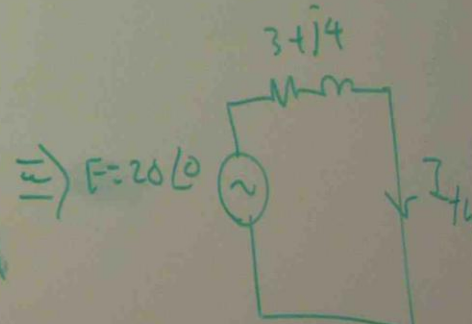


① REMOVE RESISTOR



② FIND I_{th} — SHORT CIRCUIT

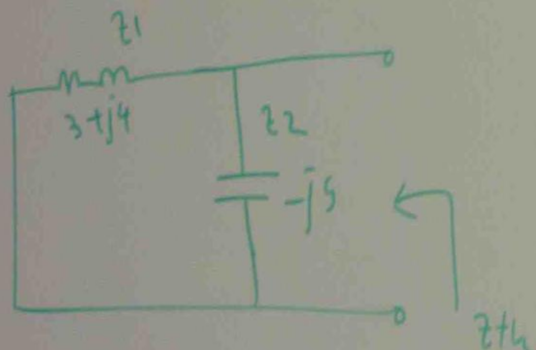




$$I_{th} = \frac{20\angle 0}{3+j4} = \frac{20\angle 0}{\sqrt{3^2+4^2}\ \angle \tan^{-1} 4/3}$$

$$= \frac{20\angle 0}{5\angle 53.2} = 4\angle -53.2 \text{ Amp.}$$

(3) KILL THE SOURCE, FIND Z_{th}



$$Z_{th} = \frac{Z_1 Z_2}{Z_1+Z_2} = \frac{(3+j4)(-j5)}{3+j4+(-j5)}$$

$$Z_{th} = \frac{\sqrt{3^2+4^2}\ \angle \tan^{-1} 4/3 \times 5\angle -90}{3-j1}$$

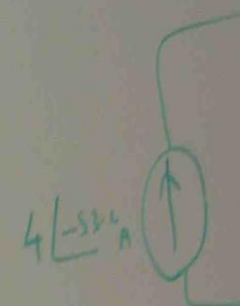
$$= \frac{5\angle 53.2 \times 5\angle -90}{\sqrt{3^2+1^2}\ \angle -\tan^{-1} 1/3}$$

$$= \frac{25\angle -36.8}{3.16\angle -18.43}$$

$$= 7.91\angle -18.44$$

$$= 7.91(\cos 18.44 - j\sin 18.44)$$

$$= 7.5 - j2.5\ \Omega$$



$$Z_{th} = \frac{\sqrt{3^2+4^2}\,\angle \tan^{-1} 4/3 \times 5\angle -90}{3 - j1}$$

$$= \frac{5\angle 53.2 \times 5\angle -90}{\sqrt{3^2+1^2}\,\angle -\tan^{-1} 1/3}$$

$$= \frac{25\angle -36.8}{3.16\angle -18.43}$$

$$= 7.91\angle -18.44$$

$$= 7.91(\cos 18.44 - j\sin 18.44)$$

$$= 7.5 - j2.5\ \Omega$$