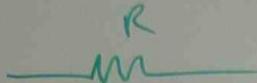
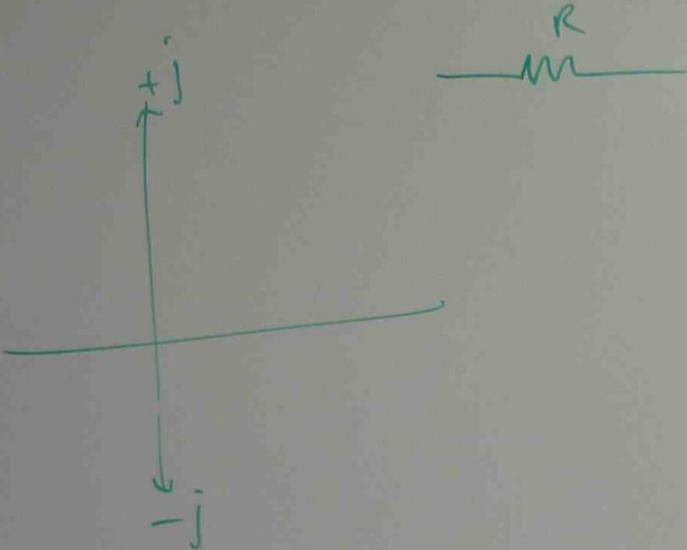
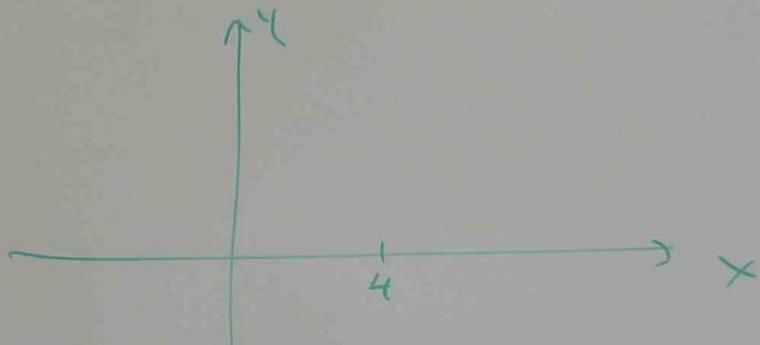


DC NETWORK THEOREM

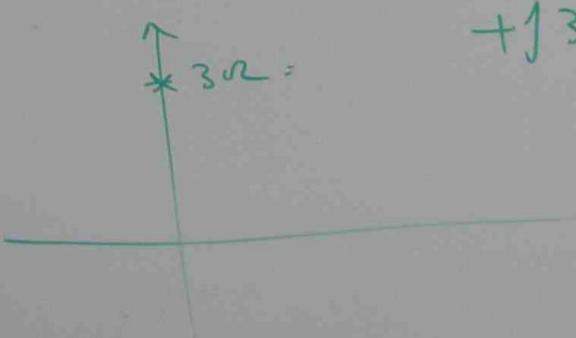


RESISTOR \rightarrow REAL NUMBER

for resistor

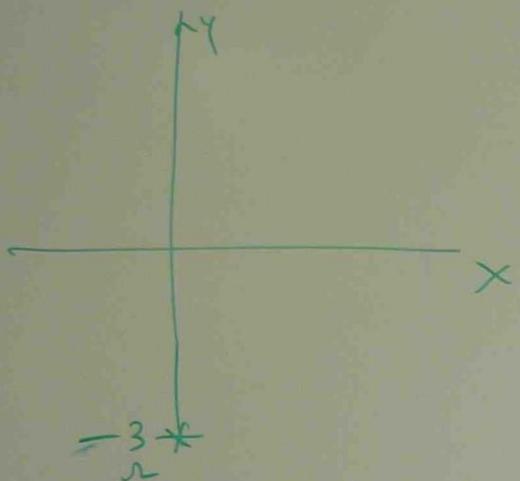


---m 3Ω INDUCTOR \rightarrow IMAGINARY



$$+j3\Omega$$

$$3\Omega \text{ CAPACITOR} = -j3\Omega$$



$$\text{CAPACITIVE REACTANCE } X_c (\Omega) = \frac{1}{2\pi f C}$$

$$C = \text{Farad}$$

$$4\Omega \text{ inductor} = 4$$

$$2\Omega \text{ inductor} = j2\Omega$$

$$3\Omega \text{ capacitor} = -j3\Omega$$

$$4\Omega \text{ inductor} + 2\Omega \text{ inductor} = 4+j2 \Omega$$

$$4\Omega \text{ inductor} - 3\Omega \text{ capacitor} = 4-j3\Omega$$

$$2\Omega \text{ inductor} + 4\Omega \text{ capacitor} = j2 - j4 = -j2$$

INDUCTOR L (HENRY)

$$X_L = \text{INDUCTIVE REACTANCE} = 2\pi f L \quad (\Omega)$$

f = FREQUENCY (Hz)

CAPACITIVE REACTANCE $X_c (\Omega) = \frac{1}{2\pi f C}$

$C = F \text{ farad}$

$$\frac{4\Omega}{m} = 4$$

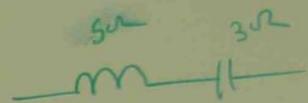
$$\frac{2\Omega}{m} = j2\Omega$$

$$\frac{3\Omega}{H} = -j3\Omega$$

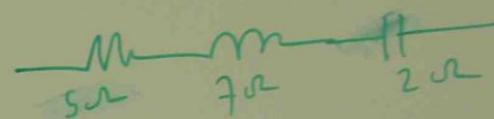
$$\frac{4\Omega}{m} \frac{2\Omega}{m} = 4 + j2 \Omega$$

$$\frac{4\Omega}{m} \frac{3\Omega}{H} = 4 - j3\Omega$$

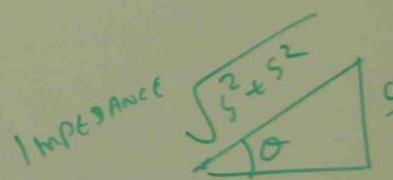
$$\frac{2\Omega}{m} \frac{4\Omega}{H} = j2 - j4 = -j2$$



$$j_5 - j_3 = j_2 \Omega$$



$$s + j7 - j2 = s + j5$$

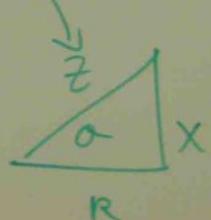


$$= \sqrt{s^2 + s^2} \left(\tan^{-1} \frac{s}{s} \right)$$

$$= 7.07 \angle 45^\circ$$

RECTANGULAR FORM

$$z = R + jX = \sqrt{R^2 + X^2} \left(\tan^{-1} \frac{X}{R} \right)$$



POLAR FORM

$$z \angle \theta = \text{Polar Form} \longrightarrow \text{RECTANGULAR FORM}^{-4/}$$

$$z \cos \theta + j z \sin \theta$$

$$x + j y = \text{RECTANGULAR FORM} \longrightarrow \text{Polar Form}$$

$$\sqrt{x^2 + y^2} \quad \tan^{-1} \frac{y}{x}$$

MULTIPLY (DIVIDE → USE POLAR FORM)

$$z_1 \angle \theta_1 \times z_2 \angle \theta_2 = z_1 z_2 \angle \underline{\theta_1 + \theta_2}$$

$$\frac{z_1 \angle \theta_1}{z_2 \angle \theta_2} = \frac{z_1}{z_2} \angle \underline{\theta_1 - \theta_2}$$

ADDITION / SUBTRACTION

USE RECTANGULAR FORM

$$5 \angle 53.2^\circ - 5 \angle 36.8^\circ = ?$$

$$(5 \cos 53.2 + j 5 \sin 53.2) - (5 \cos 36.8 + j 5 \sin 36.8)$$

$$(5 \times 0.6 + j 5 \times 0.8) - (5 \times 0.8 + j 5 \times 0.6)$$

$$(3 + j4) - (4 + j3)$$

$$3 + j4 - 4 - j3$$

$$-1 + j1$$

ADDITION / SUBTRACTION

USE RECTANGULAR FORM

$$5 \angle 53.2^\circ - 5 \angle 36.8^\circ = ?$$

$$(5 \cos 53.2 + j 5 \sin 53.2) - (5 \cos 36.8 + j 5 \sin 36.8)$$

$$(5 \times 0.6 + j 5 \times 0.8) - (5 \times 0.8 + j 5 \times 0.6)$$

$$(3 + j4) - (4 + j3)$$

$$3 + j4 - 4 - j3$$

$$-1 + j1$$

EF
 $(3+j4)(4+j3)$

$$\sqrt{3^2+4^2} \sqrt{\tan^{-1} \frac{4}{3}} \times \sqrt{4^2+3^2} \sqrt{\tan^{-1} \frac{3}{4}}$$

$$5 \angle 53.2^\circ \times 5 \angle 36.8^\circ$$

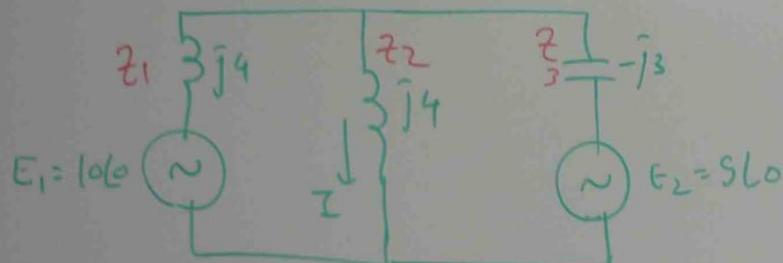
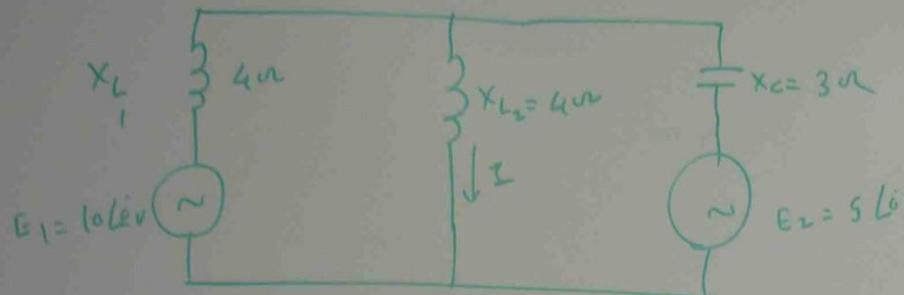
$$25 \angle 53.2^\circ + 36.8^\circ$$

$$25 \angle 90^\circ$$

Ex

USING THE SUPER POSITION THEOREM, FIND THE CURRENT I

THROUGH THE 4Ω REACTANCE.



KILL (E_2) $\rightarrow z_1$

$E_1 = 10\angle 0^\circ \text{ V}$

$$z_T = z_1 + z_2 // z_3$$

$$z_T = z_1 + \frac{z_2 z_3}{z_2 + z_3}$$

$$= j4 + \frac{j4 \times (-j3)}{j4 + (-j3)}$$

$$= j4 + \frac{4 \angle 90^\circ \times 3 \angle -90^\circ}{j1}$$

$$= j4 + \frac{12 \angle 90^\circ - 90^\circ}{1 \angle 90^\circ}$$

$$= j4 + \frac{12 \angle 0^\circ}{1 \angle 90^\circ}$$

$$Z_T = j4 + 12 \angle -90^\circ$$

$$= j4 - j12$$

$$= -j8$$

$$I_T = \frac{E_1}{Z_T}$$

$$= \frac{10 \angle 0^\circ}{-j8}$$

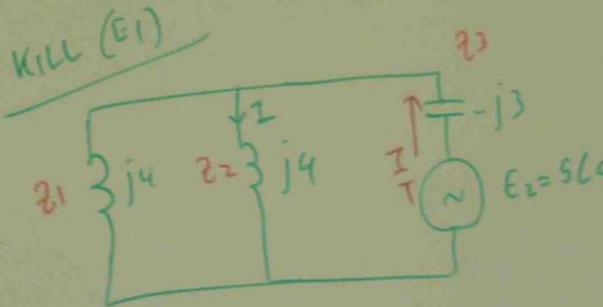
$$= \frac{10 \angle 0^\circ}{8 \angle -90^\circ}$$

$$= 1.25 \angle 90^\circ$$

$$(Z_2) \quad I = I_T \times \frac{Z_3}{Z_2 + Z_3}$$

$$= 1.25 \left(\frac{40 \times -j3}{j4 + (-j3)} \right)$$

$$\begin{aligned} I_{(Z_2)} &= 1.25 \angle 90^\circ \times \frac{3 \angle -90^\circ}{j1} \\ &= 1.25 \angle 90^\circ \times \frac{3 \angle -90^\circ}{1 \angle 90^\circ} \\ &= 3.75 \angle -90^\circ \text{ Amp} \end{aligned}$$



$$Z_T = Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2}$$

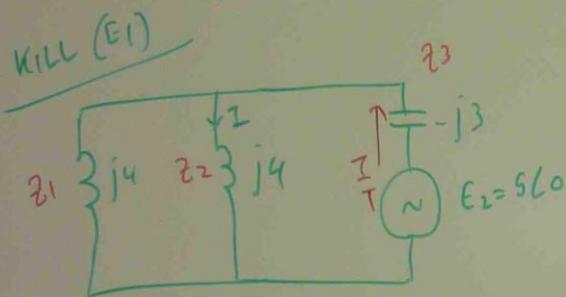
$$= (-j3) + \frac{j4 \times j4}{j4 + j4}$$

$$= -j3 + \frac{4 \angle 90^\circ \times 4 \angle 90^\circ}{j8}$$

$$\begin{aligned} &= -j3 + \frac{16 \angle 180^\circ}{8 \angle 90^\circ} \\ &= -j3 + 2 \angle 180 - 90^\circ \\ &= -j3 + 2 \angle 90^\circ \\ &= -j3 + j2 \\ &= -j1 = 1 \angle -90^\circ \\ I_T &= \frac{E_2}{Z_T} = \frac{560}{1 \angle -90^\circ} \\ &= 560 \text{ Amp} \end{aligned}$$

$$\begin{aligned} I_{Z_2} &= I_T \times \frac{Z_1}{Z_1 + Z_2} \\ &= 5 \angle 90^\circ \times \frac{j4}{j4 + j4} \\ &= 5 \angle 90^\circ \times \frac{4 \angle 90^\circ}{j8} \\ &= 5 \angle 90^\circ \times \frac{4 \angle 90^\circ}{8 \angle 90^\circ} = \end{aligned}$$

$$\begin{aligned} I_{(22)} &= 1.25 \angle 90^\circ \times \frac{3 \angle -90^\circ}{j1} \\ &= 1.25 \angle 90^\circ \times \frac{3 \angle -90^\circ}{1 \angle 90^\circ} \\ &= 3.75 \angle -90^\circ \text{ Amp} \end{aligned}$$



$$\begin{aligned} Z_T &= Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2} \\ &= (-j3) + \frac{j4 \times j4}{j4 + j4} \\ &= -j3 + \frac{4 \angle 90^\circ \times 4 \angle 90^\circ}{j8} \end{aligned}$$

$$\begin{aligned} &= -j3 + \frac{16 \angle 180^\circ}{8 \angle 90^\circ} \\ &= -j3 + 2 \angle 180^\circ - 90^\circ \\ &= -j3 + 2 \angle 90^\circ \\ &= -j3 + j2 \\ &= -j1 = 1 \angle -90^\circ \\ I_T &= \frac{E_2}{Z_T} = \frac{5 \angle 0^\circ}{1 \angle -90^\circ} \\ &= 5 \angle 90^\circ \text{ Amp.} \end{aligned}$$

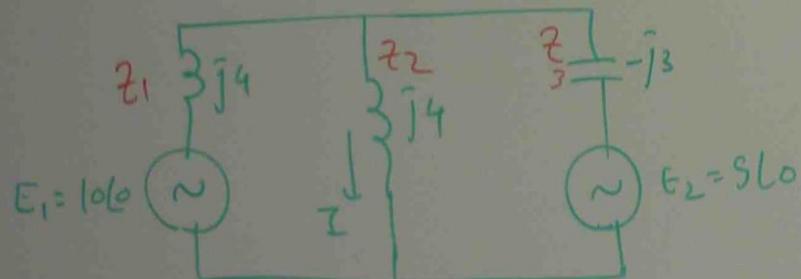
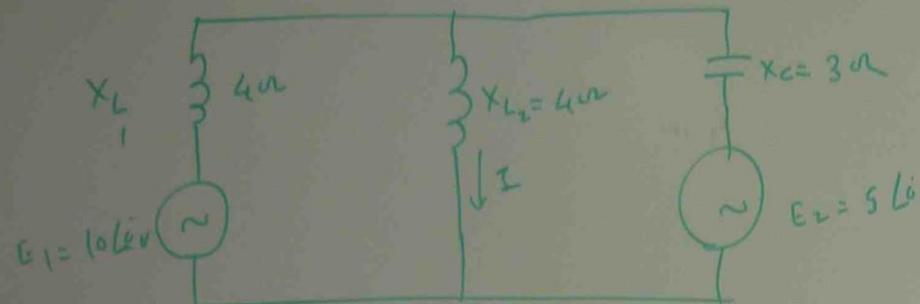
$$\begin{aligned} I_{(22)} &= I_T \times \frac{z1}{z1 + z2} \\ &= 5 \angle 90^\circ \times \frac{j4}{j4 + j4} \\ &= 5 \angle 90^\circ \times \frac{4 \angle 90^\circ}{j8} \end{aligned}$$

$$= 5 \angle 90^\circ \times \frac{4 \angle 90^\circ}{8 \angle 90^\circ} = 2.5 \angle 90^\circ$$

$$\begin{aligned} I_{22} &= I_{(22)} + I_{(22)}'' \\ &= 3.75 \angle 90^\circ + 2.5 \angle 90^\circ \\ &= -j3.75 + j2.5 \\ &= -j1.25 \\ &\Rightarrow 1.25 \angle -90^\circ \end{aligned}$$

Ex

USING THE SUPER POSITION THEOREM, FIND THE CURRENT I
THROUGH THE 4Ω REACTANCE.



KILL (E_2)

$E_1 = 10\angle 0^\circ V$

$Z_1 = j4$

$Z_2 = j4$

$Z_3 = -j3$

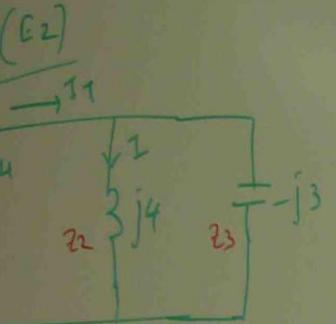
$Z_T = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$

$= j4 + \frac{j4 \times (-j3)}{j4 + (-j3)}$

$= j4 + \frac{4 \angle 90^\circ \times 3 \angle -90^\circ}{j1}$

$= j4 + \frac{12 \angle 90^\circ - 90^\circ}{1 \angle 90^\circ}$

$= j4 + 12 \angle 0^\circ$



$$Z_1 + Z_2 // Z_3$$

$$I_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$$

$$= j4 + \frac{j4 \times (-j3)}{j4 + (-j3)}$$

$$= j4 + \frac{4 \angle 90^\circ \times 3 \angle -90^\circ}{j1}$$

$$= j4 + \frac{12 \angle 90^\circ - 90^\circ}{1 \angle 90^\circ}$$

$$= j4 + \frac{12 \angle 0^\circ}{1 \angle 90^\circ}$$

$$\begin{aligned} Z_T &= j4 + 12 \angle -90^\circ \\ &= j4 - j12 \\ &= -j8 \end{aligned}$$

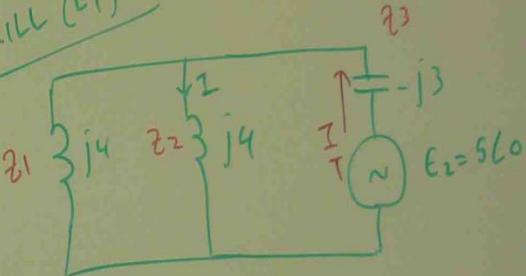
$$\begin{aligned} I_T &= \frac{E1}{Z_T} \\ &= \frac{10 \angle 0^\circ}{-j8} \\ &= \frac{10 \angle 0^\circ}{8 \angle -90^\circ} \\ &= 1.25 \angle 90^\circ \end{aligned}$$

$$(Z_2) \quad I = I_T \times \frac{Z_3}{Z_2 + Z_3}$$

$$= 1.25 \angle 90^\circ \times \frac{-j3}{j4 + (-j3)}$$

$$\begin{aligned} I_{(Z_2)} &= 1.25 \angle 90^\circ \times \frac{3 \angle -90^\circ}{j1} \\ &= 1.25 \angle 90^\circ \times \frac{3 \angle -90^\circ}{1 \angle 90^\circ} \\ &= 3.75 \angle -90^\circ \text{ Amp} \end{aligned}$$

KILL (E1)



$$Z_T = Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$= (-j3) + \frac{j4 \times j4}{j4 + j4}$$

$$= -j3 + \frac{4 \angle 90^\circ \times 4 \angle 90^\circ}{j8}$$

$$\begin{aligned} &= -j3 + \frac{16 \angle 180^\circ}{8 \angle 90^\circ} \\ &= -j3 + 2 \angle 180^\circ \\ &= -j3 + 2 \angle 90^\circ \\ &= -j3 + j2 \\ &= -j1 = 1 \\ I_T - \frac{E2}{Z_T} &= \end{aligned}$$

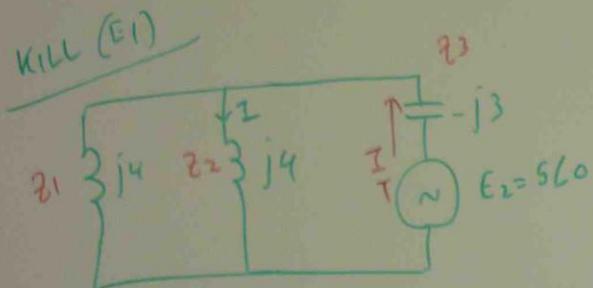
$$I_{T2} = I_T \times \frac{\dots}{\dots}$$

$$= 5 \angle 90^\circ \times \dots$$

$$= 5 \angle 90^\circ \times \dots$$

$$= 5 \angle 90^\circ$$

$$\begin{aligned} I_{(22)} &= 1.15 \angle 40^\circ \times \frac{3 \angle -90^\circ}{j1} \\ &= 1.15 \angle 40^\circ \times \frac{3 \angle -90^\circ}{1 \angle 90^\circ} \\ &= 3.75 \angle -90^\circ \text{ Amp} \end{aligned}$$

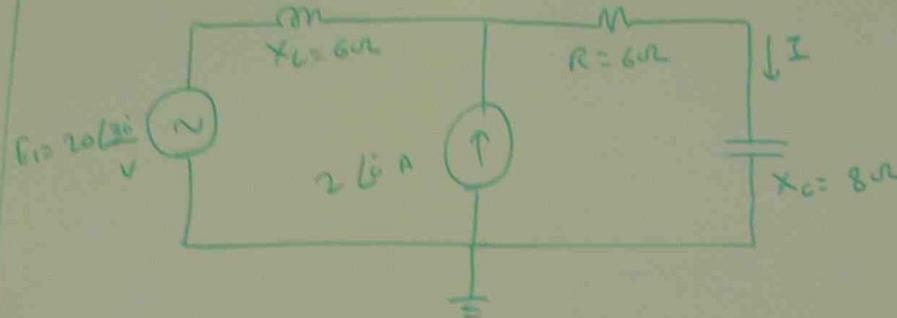


$$\begin{aligned} z_T &= z_3 + \frac{z_1 z_2}{z_1 + z_2} \\ &= (-j3) + \frac{j4 \times j4}{j4 + j4} \\ &= -j3 + \frac{4 \angle 90^\circ \times 4 \angle 90^\circ}{j8} \end{aligned}$$

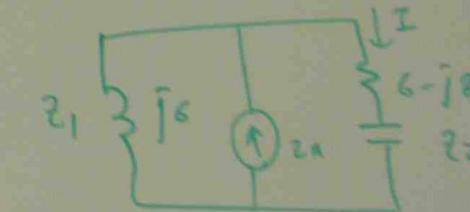
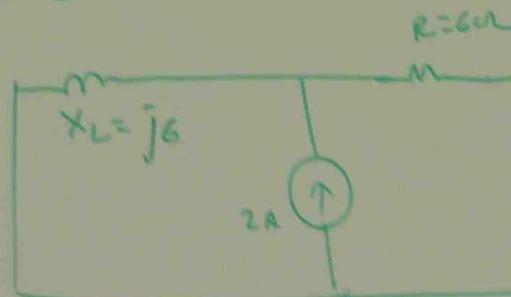
$$\begin{aligned} &= -j3 + \frac{16 \angle 180^\circ}{8 \angle 90^\circ} \\ &= -j3 + 2 \angle 180^\circ - 90^\circ \\ &= -j3 + 2 \angle 90^\circ \\ &= -j3 + j2 \\ &= -j1 = 1 \angle -90^\circ \\ I_T &= \frac{E_2}{z_T} = \frac{5 \angle 0^\circ}{1 \angle -90^\circ} \\ &= 5 \angle 90^\circ \text{ Amp.} \\ I_{T2} &= I_T \times \frac{z_1}{z_1 + z_2} \\ &= 5 \angle 90^\circ \times \frac{j4}{j4 + j4} \\ &= 5 \angle 90^\circ \times \frac{4 \angle 90^\circ}{j8} \\ &= 5 \angle 90^\circ \times \frac{4 \angle 90^\circ}{8 \angle 90^\circ} = 2.5 \angle 90^\circ \end{aligned}$$

$$\begin{aligned} I_{22} &= I_{T2} + I_{22}'' \\ &= 3.75 \angle -90^\circ + 2.5 \angle 90^\circ \\ &= -j3.75 + j2.5 \\ &= -j1.25 \\ &\rightarrow 1.25 \angle -90^\circ \end{aligned}$$

BY USING SUPERPOSITION, FIND THE CURRENT I THROUGH THE 6Ω RESISTOR.

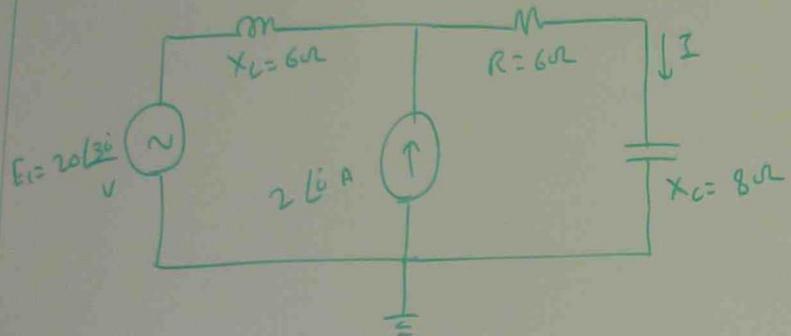


KILL $E_1 = 20 \angle 30^\circ \text{ V}$

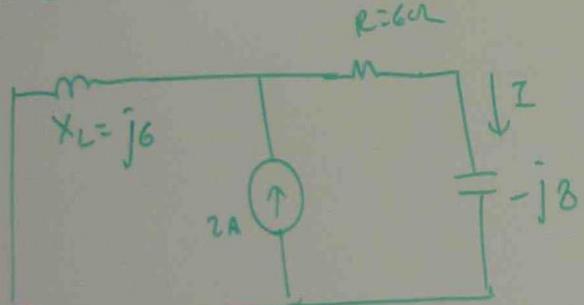


$$I'_{Z_2} (\text{OR CAPACITOR CURRENT}) = I_T \times \frac{Z_1}{Z_1 + Z_2} = 2 \times \frac{j6}{j6 + 6 - j8}$$

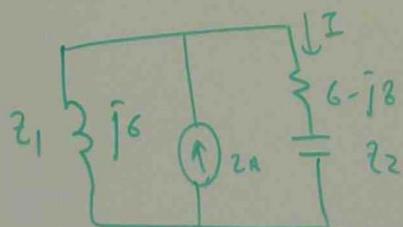
EY USING SUPERPOSITION, FIND THE CURRENT I THROUGH THE 6Ω RESISTOR.



$$\text{KILL } E_1 = 20 \angle 30^\circ \text{ V}$$



$$I'_{z_2} (\text{OR CAPACITOR CURRENT}) = I_T \times \frac{z_1}{z_1 + z_2} = 2 \times \frac{j6}{j6 + 6 - j8} \downarrow$$



$$= 2 \times \frac{6 \angle 90^\circ}{6 - j2}$$

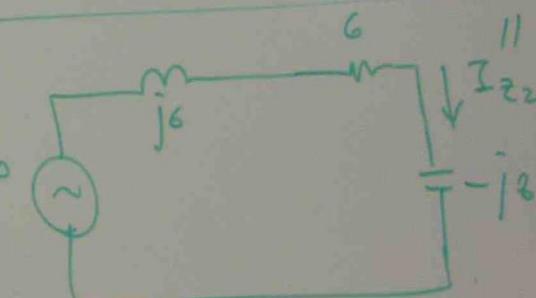
$$= 2 + \frac{6 \angle 90^\circ}{\sqrt{6^2 + 2^2}} \angle -\tan^{-1} \frac{2}{6}$$

$$= \frac{12 \angle 90^\circ}{6 \cdot 3.2 \angle -18.43^\circ}$$

$$I' = 1.9 \angle 90^\circ - (-18.43^\circ)$$

$$I'_{z_2} = 1.9 \angle 108.43^\circ \downarrow$$

KILL $2 \angle 0^\circ$ CURRENT SOURCE



$$Z_T = j6 + 6 + (-j8) = \sqrt{6^2 + 6^2} \angle -\tan^{-1} \frac{8}{6} = \sqrt{6^2 + 6^2} \angle -18.43^\circ$$

$$I_{22}'' = \frac{EI}{Z_T} = \frac{20 \angle 30^\circ}{6.32 \angle -18.43^\circ} = 3.16 \angle 30^\circ - (-18.43^\circ)$$

$$I_{22}'' = 3.16 \angle 48.43^\circ \text{ A}$$

$$I_{22} = I_{22}' + I_{22}''$$

$$= 1.9 \angle 108.43^\circ + 3.16 \angle 48.43^\circ$$

$$= 1.9 \cos 108.43^\circ + j 1.9 \sin 108.43^\circ + 3.16 \cos 48.43^\circ + j 3.16 \sin 48.43^\circ$$

$$= -0.6 + j 1.8 + 2.1 + j 2.36$$

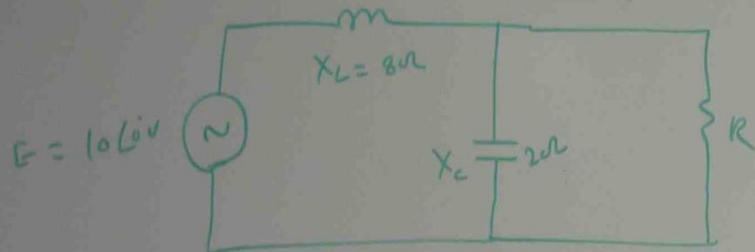
$$= 1.5 + j 4.16$$

$$= \sqrt{1.5^2 + 4.16^2} \angle \tan^{-1} \frac{4.16}{1.5} = 4.42 \angle 70.2^\circ \text{ A}$$

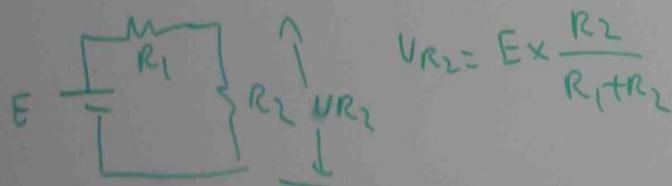
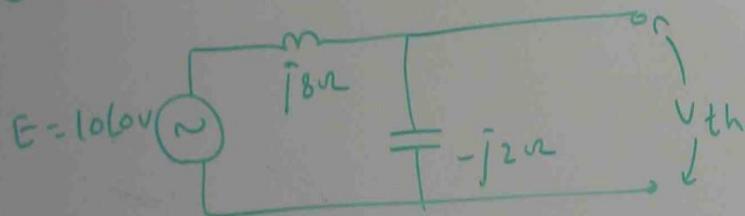
E_t

FIND THE THEVENIN'S EQUIVALENT CIRCUIT FOR THE NETWORK EXTERNAL

TO RESISTOR "R" IN GIVEN FIGURE



① REMOVE RESISTOR "R"



② FIND V_{th}

$$V_{th} = 10\angle 0^\circ \times \frac{(-j2)}{j8 + (-j2)}$$

$$= 10\angle 0^\circ \times \frac{2 \angle -90^\circ}{j6}$$

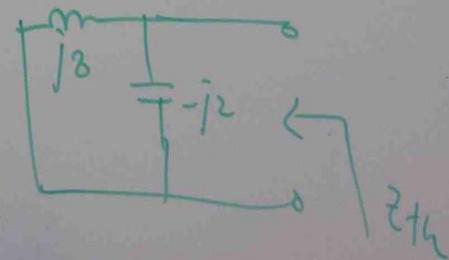
$$= 10\angle 0^\circ \times \frac{2 \angle -90^\circ}{6 \angle 90^\circ}$$

$$= 3.33 \angle -90^\circ - 90^\circ$$

$$= 3.33 \angle -180^\circ V$$

③

KILL THE SOURCE, FIND Z_{th}



$Z_{th} =$

=

V_{th}

3.33∠

FOR THE NETWORK EXTERNAL

② FIND U_{th}

$$U_{th} = 10 \angle 0^\circ \times \frac{(-j2)}{j8 + (-j2)}$$

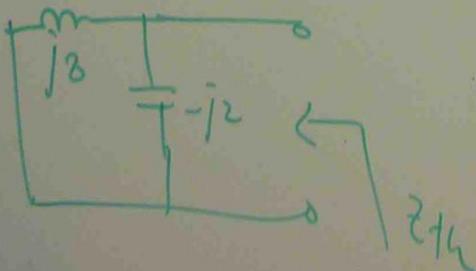
$$= 10 \angle 0^\circ \times \frac{2 \angle -90^\circ}{j6}$$

$$= 10 \angle 0^\circ \times \frac{2 \angle -90^\circ}{6 \angle 90^\circ}$$

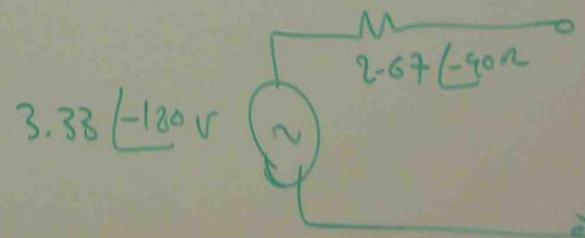
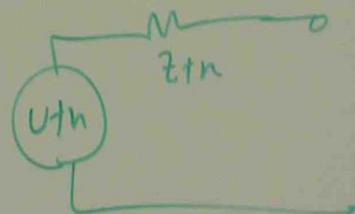
$$= 3.33 \angle -90^\circ - 90^\circ$$

$$= 3.33 \angle -180^\circ$$

③ KILL THE SOURCE, FIND Z_{th}

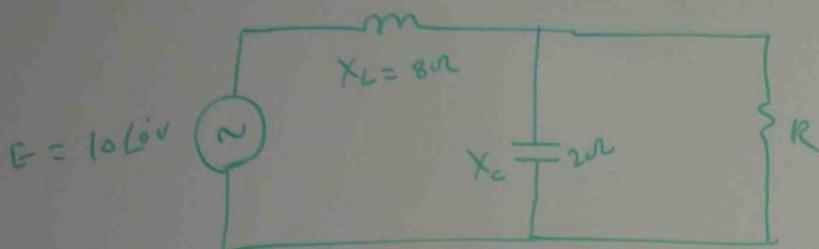


$$\begin{aligned} Z_{th} &= \frac{j8 \times (-j2)}{j8 + (-j2)} \\ &= \frac{8 \angle 90^\circ \times 2 \angle -90^\circ}{j6} \\ &= \frac{8 \angle 90^\circ \times 2 \angle -90^\circ}{6 \angle 90^\circ} \\ &= 2.67 \angle -90^\circ \end{aligned}$$

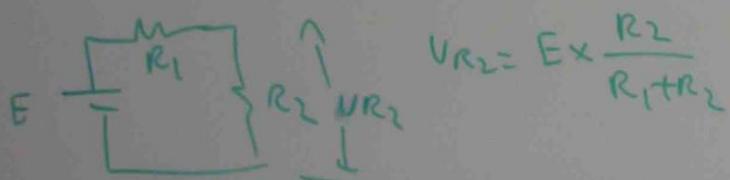
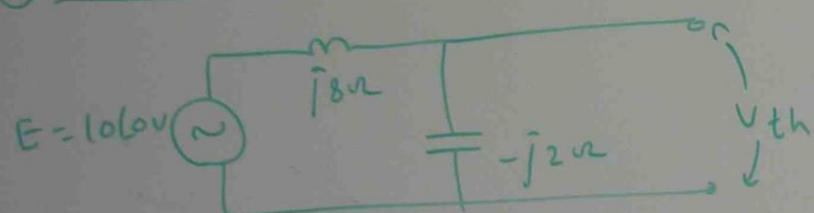


Ex

FIND THE THEVENIN'S EQUIVALENT CIRCUIT FOR THE NETWORK EXTERNAL
TO RESISTOR " R " IN GIVEN FIGURE



① REMOVE RESISTOR " R "



$$U_{R2} = E \times \frac{R_2}{R_1 + R_2}$$

② FIND V_{th}

$$V_{th} = 10\angle 0^\circ \times \frac{(-j2)}{|8 + (-j2)|}$$

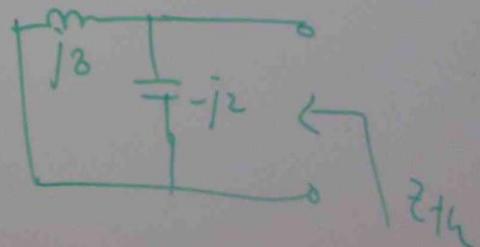
$$= 10\angle 0^\circ \times \frac{2 \angle -90^\circ}{\sqrt{64 + 4}}$$

$$= 10\angle 0^\circ \times \frac{2 \angle -90^\circ}{6 \angle 60^\circ}$$

$$= 3.33 \angle -90^\circ - 90^\circ$$

$$= 3.33 \angle -180^\circ \text{ V}$$

③ KILL THE SOURCE, FIND Z_{th}

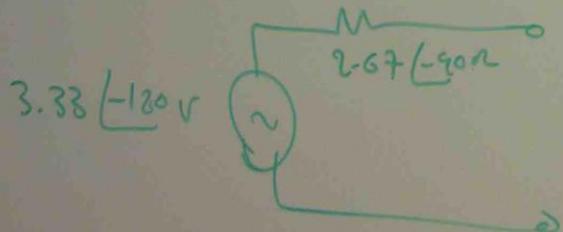
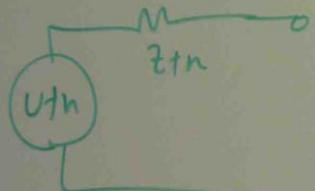


$$Z_{th} = \frac{j8 \times (-j2)}{j8 + (-j2)}$$

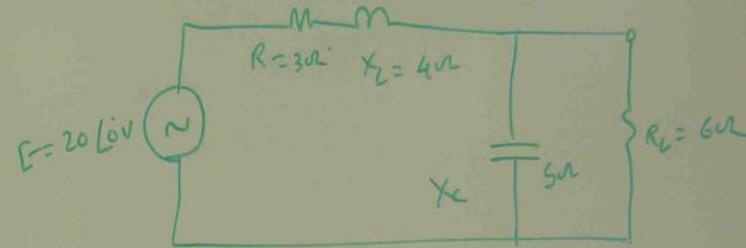
$$= \frac{8 \angle 90^\circ \times 2 \angle -90^\circ}{j^6}$$

$$= \frac{8 \angle 90^\circ \times 2 \angle -90^\circ}{6 \angle 90^\circ}$$

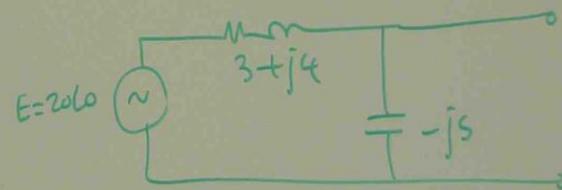
$$= 2.67 \angle -90^\circ$$



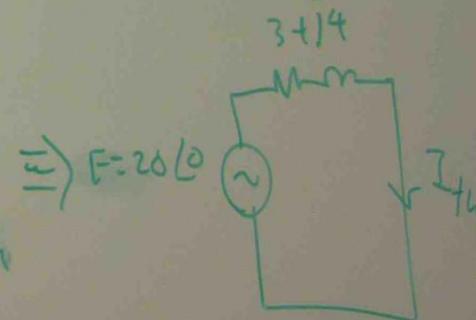
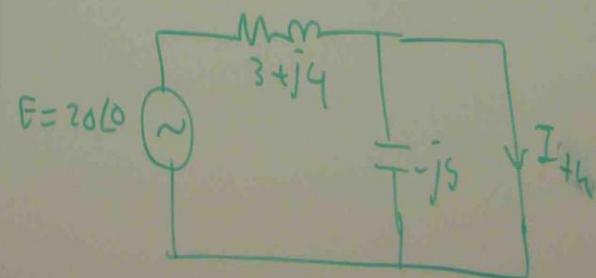
Ex DETERMINE THE NORTON EQUIVALENT CIRCUIT FOR THE NETWORK EXTERNAL TO THE 6Ω RESISTOR.



① REMOVE RESISTOR



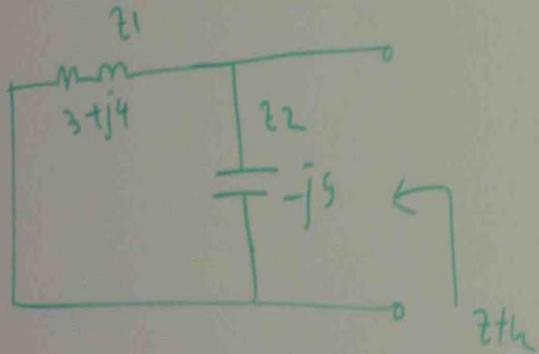
② FIND I_th - SHORT CIRCUIT



$$I_{th} = \frac{20\text{Lo}}{3+j4} = \frac{20\text{Lo}}{\sqrt{3^2+4^2}} \angle \tan^{-1} 4/3$$

$$= \frac{20\text{Lo}}{5 \angle 53.2^\circ} = 4 \angle -53.2^\circ \text{ Amp.}$$

③ KILL THE SOURCE, FIND Z_{th}



$$Z_{th} = \frac{z_1 z_2}{z_1 + z_2} = \frac{(3+j4)(-j5)}{3+j4+(-j5)}$$

$$Z_{th} = \frac{\sqrt{3^2+4^2} \angle \tan^{-1} 4/3}{3-j1} \times 5 \angle -90^\circ$$

$$= \frac{5 \angle 53.2^\circ \times 5 \angle -90^\circ}{\sqrt{3^2+1^2} \angle -\tan^{-1} 1/3}$$

$$= \frac{25 \angle -36.8^\circ}{3.66 \angle -18.43^\circ}$$

$$= 7.91 \angle -18.44^\circ$$

$$= 7.91 (\cos 18.44^\circ - j \sin 18.44^\circ)$$

$$\approx 7.5 - j2.5 \Omega$$

$4 \angle -53.2^\circ$

$$\begin{aligned}
 Z_{fh} &= \frac{\sqrt{3^2 + 4^2} \left[\tan^{-1} 4/3 \right] + 5 \left[-90^\circ \right]}{3 - j1} \\
 &= \frac{5 \left[53.1^\circ \right] + 5 \left[-90^\circ \right]}{\sqrt{3^2 + 1^2} \left[-\tan^{-1} 1/3 \right]} \\
 &= \frac{25 \left[-36.8^\circ \right]}{3.16 \left[-18.43^\circ \right]} \\
 &= 7.91 \left[-18.44^\circ \right] \\
 &= 7.91 (\cos 18.44^\circ - j \sin 18.44^\circ) \\
 &\approx 7.5 - j2.5 \quad \Omega
 \end{aligned}$$

