Q1

Complex numbers. Given $z_1 = 1 - 2\mathrm{i}$, $z_2 = 3 + 2\mathrm{i}$, find $z_1 z_2$ and z_1 / z_2.

Q2

Roots. Find all the roots of $z^3 = 1$.

Q3

Function of a Complex Variable. Given $u(x, y) = x^2$ and $v(x, y) = y^2$, find $f(z) = u + \mathrm{i}v$, if this exists.

Q4

Derivative (first principles). Find the derivative of z^2 from first principles.

Q5

Line integral I. Evaluate $\int_0^1 \left(t + \mathrm{i}t^2\right)^2 \mathrm{d}t$.

Q6

Cauchy's Integral Formula. Find the value of $\oint_C \frac{z + e^z}{z^2 - 1} \mathrm{d}z$, where C, mapped counter-clockwise, are these circles:

Q7

Power series. Show that $\sum_{n=0}^{\infty} z^n$ converges to $1/(1 - z)$ for $|z| < 1$.

Q8

Evaluate $\int_0^{2\pi} \frac{d\theta}{1+k\sin\theta}$ where $0<|k|<1$ (and k is real). (The condition on k is necessary if the integral is to exist.)

Q9

Example 2.1 *Let Y, X_1, X_2 be independent positive random variables, all with a continuous distribution, and let Y be exponentially distributed. It is well-known that for all $s, t \in \mathbb{R}_+$,*

(1) $P\{Y > s+t\} = P\{Y > s\} \cdot P\{Y > t\}$,

(the property of forgetfulness).

1) Prove by using (1) that we have the following generalization

Q10

Example 3.1 *Given a random variable X, which is normally distributed of mean 2.12. Given that*

$$P\{X \geq 3\} = 0.324,$$

find the variance of X.

Q11

A random variable X has the frequency

$$f(x) = \begin{cases} \frac{1}{B(u,v)} x^{u-1}(1-x)^{v-1}, & 0 < x < 1, \\ 0 & \text{otherwise}, \end{cases}$$

where u and v denote positive constants, and where $B(u,v)$ denotes the Beta function. Find mean and variance of the random variable X.

Q12

Example 8.1 *Given a sequence of random variables $(U_n)_{n=1}^{\infty}$, which converges in distribution towards a random variable U of distribution function $F_U(u)$ and frequency $f_U(u)$.*

Q13

Let $X \in F(n_1, n_2)$. *Prove that* $E\{X\} = \dfrac{n_2}{n_2 - 2}$ *for* $n_2 > 2$.

Q14

Example 2.1 *Prove the formulæ*

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}, \qquad k, n \in \mathbb{N},$$

$$k\binom{n}{k} = n\binom{n-1}{k-1}, \qquad k, n \in \mathbb{N},$$

either by a direct computation, or by a combinatorial argument.

Q15

Example 3.1 *Let X and Y be independent Poisson distributed random variables of the parameters a and b, resp.. Let $Z = X + Y$. Find*

$$P\{X = k \mid Z = n\}, \qquad k = 0, 1, 2, \dots, n.$$

Q16

Example 3.2 *Let X and Y be random variables for which*

X is Poisson distributed, $\quad X \in P(a)$,

and

$$P\{Y = k \mid X = n\} = \binom{n}{k} p^k (1-p)^{n-k}, \qquad k = 0, 1, \dots, n,$$

where $n \in \mathbb{N}_0$ and $p \in]0, 1[$.
Prove that Y is Poisson distributed, $Y \in P(ap)$.

Q17

Example 2.4.2 *Find the distance between the points in* $\mathbb{R}^4$,

$$\mathbf{a} = (1, 2, -4, 6)$$

and

$$\mathbf{b} = (2, 3, -1, 0)$$

Q18

Describe the points which are at the same distance between $(1,2,3)$ *and* $(0,1,2)$.

Q19

Exercise 2.5.1 *Here is a picture of two vectors,* $\mathbf{u}$ *and* $\mathbf{v}$.

Sketch a picture of $\mathbf{u}+\mathbf{v}$, $\mathbf{u}-\mathbf{v}$, *and* $\mathbf{u}+2\mathbf{v}$.

Q20+Q21

20 **Example 3.1.3** *Find* $(1,2,0,-1)\cdot(0,1,2,3)$.

This equals $0+2+0+-3=-1$.

21 **Example 3.1.4** *Find the magnitude of* $\mathbf{a}=(2,1,4,2)$. *That is, find* $|\mathbf{a}|$.

This is $\sqrt{(2,1,4,2)\cdot(2,1,4,2)}=5$.
The dot product satisfies a fundamental inequality known as the **Cauchy Schwarz inequality.**

Q22

Find $(\mathbf{i}-\mathbf{j}+2\mathbf{k})\times(3\mathbf{i}-2\mathbf{j}+\mathbf{k})$.

Q23

Find the solution to the system $x+y=3,\ y-x=5.$

Q26

Solve

$$\begin{pmatrix}1 & 2 & 3\\ 1 & 0 & 4\end{pmatrix}+\begin{pmatrix}5 & 2 & 3\\ -6 & 2 & 1\end{pmatrix}$$

Q27

Compute

$$\begin{pmatrix}1 & 2 & 1 & 3\\ 0 & 2 & 1 & -2\\ 2 & 1 & 4 & 1\end{pmatrix}\begin{pmatrix}1\\ 2\\ 0\\ 1\end{pmatrix}.$$

Q28

Multiply the following.

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 1 \\ -2 & 1 & 1 \end{pmatrix}$$

Q29

Multiply the following

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 1 \\ -2 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \end{pmatrix}$$

Q30

Find $\det \begin{pmatrix} 2 & 4 \\ -1 & 6 \end{pmatrix}.$

Q31

Find co factor of

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 3 & 2 \\ 3 & 2 & 1 \end{pmatrix}.$$

Q32

Compare $\det(AB)$ *and* $\det(A)\det(B)$ *for*

$$A = \begin{pmatrix} 1 & 2 \\ -3 & 2 \end{pmatrix}, B = \begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}.$$

Q33

Find the determinant of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 1 & 2 & 3 \\ 4 & 5 & 4 & 3 \\ 2 & 2 & -4 & 5 \end{pmatrix}$$

Q34

Find the inverse of the matrix

$$A = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ -\frac{1}{6} & \frac{1}{3} & -\frac{1}{2} \\ -\frac{5}{6} & \frac{2}{3} & -\frac{1}{2} \end{pmatrix}$$

Q35

Example 2

Find the value of $\int_0^1 e^{3x} dx$.

Q36

Write down primitives associated with the integrals of each of these functions:

(a) $(2x+1)^5$; (b) $(3-4x)^{3/2}$; (c) $(3x+4)^{-1}$; (d) $2\sin(17x-6)$; (e) $\tan(1-2x)$;
(f) $\frac{3}{x^2+16}$; (g) $\frac{5}{\sqrt{x^2-2x-8}}$; (h) $x^2\sin(x^3)$; (i) $\frac{1+2x}{x^2+x-3}$.

Q37

Evaluate these definite integrals:

(a) $\int_1^2 \left[x^2 + (3x-1)^{-2}\right] dx$; (b) $\int_0^{1/4} 3\sec^2\left(\frac{\pi}{4} - x\pi\right) dx$; (c) $\int_1^2 \frac{dx}{\sqrt{9-x^2}}$

Q38

Use integration by parts to find an equation involving $\int (1+\ln x)^n \, dx$ $(x > 0,\ n \neq -1)$; hence find the value of $\int_1^2 (1+\ln x)^3 \, dx$

Q39

Find $\int \frac{x^3 - x^2 + 2x + 1}{x+1} dx$.

Q40

Example 13

Find $\int \frac{x^6 - 3x^5 + 2x^4 + x^3 - x^2 + 4x - 7}{(x-2)^3} dx$.

Q41

Find the value of $\int_1^2 \frac{x^4 - 2x^2 + 3x - 2}{x^2 - 6x + 9} dx$.

Q42

Find $\int \frac{x^6 - x^3 + 1}{x^3 + x^2 + x + 1} dx$.

Q43

Find the general solution of the equation $y' = 2x(1 + y^2)$

Q44

Find the general solution of the equation $(1 + e^{-x})yy' = 1$, and then that solution which satisfies $y(0) = 1$.

Q45

Find the general solution of the equation $yy' = 2y - x$.

Q46

Find all the solutions of the equation $xy' = y + \sqrt{y^2 - x^2}$.

Q47

Find the general solution of the equation $(1 + x^2)y' + 4xy = 2x$ and then that solution which satisfies $y(0) = 1$.

Q48

Example 2.1 *A motorist shall pass 4 traffic lights. We assume that at each of the traffic lights there is the probability p that he must stop. There is furthermore such a long distance between the traffic signals that there is no synchronization between them. Let X be the random variable, which indicates the number of stops. Find the distribution of X. Sketch in the case $p = \frac{1}{2}$ the corresponding diagram. Let Y have the value k, if the first stop is at signal number k, $k = 1, 2, 3, 4$. Is Y a random variable?*

Q49

Example 2.2 *A random variable X can have the possible values 1, 2, ... of the probabilities*

$$P\{X = k\} = A\frac{q^k}{k}, \qquad k \in \mathbb{N} \qquad (\textit{where } q \in]0, 1[).$$

Find the constant A.

Q50

Example 3.1 *Check if the function*

$$f(x) = \begin{cases} \frac{1}{2} - kx, & x \in [0, 6], \\ 0 & \textit{otherwise}, \end{cases}$$

is a frequency for some k.

Q51

Example 3.2 *Find k, such that*

$$f(x) = \begin{cases} kx^2\left(1 - x^3\right), & x \in [0, 1], \\ 0, & \textit{otherwise}, \end{cases}$$

is a frequency of a random variable, and sketch the function.
Find the median of X.

Q54

. Find the value of $\displaystyle\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$.

Q55

Find the Fourier Transform of

$$f(x)=\begin{cases}1, & |x|\le 1,\\ 0, & |x|>1.\end{cases}$$

Q56

Evaluate the integral of $f(z)=2z-\mathrm{i}z^2$ along the curve $z=\gamma(t)=t^2+\mathrm{i}t$, from $t=0$ to $t=1$.

Q57

The contour C is a circle of radius 2, mapped counter-clockwise, together with the circle of radius 1, mapped clockwise, both centred at the origin; the region R is the annulus between them. Evaluate $\oint_C f(z)\,\mathrm{d}z$ where $f(z)=1/z(z+3)$.

Q58

Example 6

Evaluate $\oint_C \frac{z^2-e^{z^2}}{z(z^2-1)(z+3)}\,\mathrm{d}z$ where C is the Jordan curve $|z|=2$, mapped counter-clockwise.

Q59

Find x, y if

$$\begin{pmatrix}1 & 2 & 1\\ 3 & 2 & 1\\ 2 & -3 & 2\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}1\\ 2\\ 3\end{pmatrix}.$$

Q60

Example 8.3.7 *Find the rank of the following matrix and describe the column and row spaces efficiently.*

$$\begin{pmatrix} 1 & 2 & 1 & 3 & 2 \\ 1 & 3 & 6 & 0 & 2 \\ 1 & 2 & 1 & 3 & 2 \\ 1 & 3 & 2 & 4 & 0 \end{pmatrix} \tag{8.2}$$

The row reduced echelon form is

Q61

Determine whether the vectors $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 2 \\ -1 \end{pmatrix} \right\}$ *are linearly*

Q62

Example 8.4.24 *Let*

$$A = \begin{pmatrix} 1 & 2 & 1 & 0 & 1 \\ 2 & -1 & 1 & 3 & 0 \\ 3 & 1 & 2 & 3 & 1 \\ 4 & -2 & 2 & 6 & 0 \end{pmatrix}$$

Find the null space of A.

Q63

Example 9.2.14 *Let*

$$A = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 2 & 1 & 1 & 2 \\ 4 & 5 & 7 & 2 \end{pmatrix}$$

Find ker (A). *Equivalently, find the solution space to the system of equations* $A\mathbf{x} = \mathbf{0}$.

Q65

Example 11.2.3 *Consider* $z = x_1 - x_2$ *subject to the constraints,* $x_1 + 2x_2 \leq 10, x_1 + 2x_2 \geq 2,$ *and* $2x_1 + x_2 \leq 6, x_i \geq 0$. *Find a simplex tableau.*

Q66

Let $X \in F(n_1, n_2)$. Prove that $E\{X\} = \frac{n_2}{n_2 - 2}$ for $n_2 > 2$.

Q68

Example 11.1 *Let $X \in t(n)$. Prove that $X^2 \in F(1, n)$. Prove that the mean $E\{X\}$ exists, if and only if $n > 1$, and find $E\{X\}$ for $n > 1$.*
Prove that the variance $V\{X\}$ exists, if and only if $n > 2$, and find $V\{X\}$ for $n > 2$.

Q67

Example 12.2 *1) Let $a > 0$ be a positive constant. Prove that the function*

$$f(x) = \begin{cases} \frac{x}{a} \exp\left(-\frac{x^2}{2a}\right), & x > 0, \\ 0, & x \leq 0, \end{cases}$$

can be considered as a frequency of a random variable X.

2) Find mean and variance of the random variable X.

Q69

Example 4.5 *A box contains h white balls, r red balls and s black balls ($h > 0$, $r > 0$, $s > 0$). We draw at random a ball from the box and then return it to the box. This experiment if repeated, so the experiments are identical and independent.*
Denote by X the random variable which gives the number of draws, until a white ball occurs for the first time, and let Y denote the number of draws which are needed before a red ball occurs for the first time.

1) Find the distributions of the random variables X and Y.

2) Find the means $E\{X\}$ and $E\{Y\}$.

3) Find for $n \in \mathbb{N}$ the probability that the first white ball is drawn in experiment number n, and that no red ball has occurred in the previous $n-1$ experiment, thus $P\{X = n \wedge Y > n\}$.

4) Find $P\{X < Y\}$.

Q71

Example 5.1 *Let X and Y be random variables, both having values in $\mathbb{N}_0$. We say that the random variable X is stochastically larger than Y, if*

$$P\{X > k\} \geq P\{Y > k\} \qquad \text{for every } k \in \mathbb{N}_0.$$

1) Prove that if X is stochastically larger than Y, and if $E\{X\}$ exists, then

$$E\{Y\} \leq E\{X\}.$$

2) Let $X \in \text{Pas}(r, p_1)$ and $Y \in \text{Pas}(r, p_2)$, where $r \in \mathbb{N}$ and $0 < p_1 < p_2 < 1$. Prove that X is stochastically larger than Y.

Q72

Example 7.1 BANACH'S MATCH STICK PROBLEM. *A person B has the habit of using two boxes of matches at the same time. We assume that a matchbox contains 50 matches. When B shall apply a match, he chooses at random one of the two matchboxes without noticing afterwards if it is empty. Let X denote the number of matches in one of the boxes, when we discover that the other one is empty, and let Y denote the number of matches in the first box, when the second one is emptied.*
It can be proved that

$$a_r = P\{X = r\} = \binom{100-r}{50} \left(\frac{1}{2}\right)^{100-r}, \qquad r = 0, 1, \dots, 50.$$

Find analogously

$$b_r = P\{Y = r\}, \qquad r = 1, 2, \dots, 50.$$

Q73

Find the inverse of

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$$

Q74

Find eigenvalues and eigenvectors of

$$\begin{bmatrix} 1 & 3 & 0 \\ 3 & -2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

Q75

Example 3.2.2 1. Let

$$f(h) = \frac{(x+h)^2 - x^2}{h}.$$

Then

$$f(h) = \frac{x^2 + 2hx + h^2 - x^2}{h} = \frac{h^2 + 2hx}{h},$$

By the sum rule,

Q77

Differentiate

$$(y + x)^5 = \sin(x^7)e^{3y}.$$

Q78

Evaluate, from first principles,

$$\int_0^b x^2 dx$$

Q79

Solve

$$\int \sin x \, e^{\cos x} dx$$

Q80

Example 5.5.1 1. Find real numbers A and B such that for all x we have

$$\frac{x+1}{(x-1)(x-7)} = \frac{A}{x-1} + \frac{B}{x-7}$$

Q81

Find

$$\int \frac{2-x}{(x+1)(x-3)} dx$$

Q82

Calculate

$$\int e^{x} \sin x \, dx$$

Q84

Calculate

$$\int_{1}^{\infty} \frac{1}{x^2} \, dx$$

Q87

Find the value of

Example 6.5.1 Let $\underline{v}(x, y) = (y, x)$ and the curve C be parameterized by $\underline{r}(\lambda) = (\cos\lambda, \sin\lambda)$ for $0 \leqslant \lambda \leqslant \frac{\pi}{4}$. Then $\underline{r}'(\lambda) = (-\sin\lambda, \cos\lambda)$ and

Q88

Solve

$$\frac{dy}{dx} + y = e^{x}.$$

Q89

Find the characteristics equation of

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$$

Q90

Solve

$$\oint_C \frac{1}{z-a}\,dz$$

Q91

Example 3.5 *Let $n > 3$. We randomly choose from the numbers $1, 2, 3, \dots, n$, in a sequence (without replacing the numbers), until they have all been taken.*

1) Find the probability that the numbers 1 and 2 are chosen successively in the given order.

2) Find the probability that the numbers 1, 2 and 3 are chosen successively in the given order.

We have several possibilities of solutions, of which we only give one.

First notice that the n numbers can be chosen in

$n!$ different orders.

Q92

Example 4.4 *By a dealing the 52 cards are taken one by one. Find the probability that the four aces are succeeding each other.*

Q92A

Example 4.4 *By a dealing the 52 cards are taken one by one. Find the probability that the four aces are succeeding each other.*

Q92B

Example 4.5 *Find the probability that each of the 4 bridge players get precisely one ace.*
Find the probability in 7 games that at least one of these 7 games have this uniform distribution of the aces, and find also the probability that precisely one of the 7 games has this uniform distribution of the aces.

Q93

Example 6.2 *Some airplane companies allow for that some of their passengers who have booked a ticket do not show up at take off. Therefore, the airplane companies possibly use to overbook.*
We assume in the following that each passenger has the probability of $\frac{1}{10}$ *for not showing up and that this happens independently of the other passengers.*
Company A always sells 10 tickets to its small airplane of 9 seats.
Company B always sells 20 tickets to its larger airplane of 18 seats.
Find the probability that company A at take off must reject one passenger.
Find the probability that company B at take off must reject 1 or 2 passengers.
Which one is the larger probability?

Q94

Example 10.1 *A family has 3 children, of which it is given that at least one is a boy. Find the probability that all 3 children are boys.*
(We assume that boys and girls have the same probability of being born).

There are in total $2^3 = 8$ possibilities. In 7 of these there is at least 1 boy, and there is only one possibility that they are all three boys. Since the 7 events have the same probability, the searched probability is $\frac{1}{7}$.

Q95

Multiply the following.

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 1 \\ -2 & 1 & 1 \end{pmatrix}$$

Q96

Multiply if possible $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 & 0 \end{pmatrix}$.

Q97

7 *Let* $A = \begin{pmatrix} 1 & 2 & 2 \\ 1 & 0 & 2 \\ 3 & 1 & -1 \end{pmatrix}$. *Find* A^{-1}.

Q98

Example 4.2.6 *Find the rank of the following matrix and identify columns whose linear combinations yield all the other columns.*

$$\begin{pmatrix} 1 & 2 & 1 & 3 & 2 \\ 1 & 2 & 6 & 0 & 2 \\ 3 & 6 & 8 & 6 & 6 \end{pmatrix} \tag{4.2}$$

Q110

Example 2.2 *A random variable* X *can have the possible values* $1, 2, \ldots$ *of the probabilities*

$$P\{X = k\} = A\frac{q^k}{k}, \qquad k \in \mathbb{N} \qquad (\text{where } q \in]0, 1[).$$

Find the constant A.

Q111

Example 3.2 *Find k, such that*

$$f(x) = \begin{cases} kx^2\left(1-x^3\right), & x \in [0,1], \\ 0, & \textit{otherwise}, \end{cases}$$

is a frequency of a random variable, and sketch the function.
Find the median of X.

Q112

Example 3.4 *Prove for some choice of the constant k that the function*

$$f(x) = k \cdot e^{-|x-2|}, \qquad x \in \mathbb{R},$$

is the frequency of a random variable X.
Find the distribution function of X, and compute $P\{-1 \le X \le 3\}$ and $P\{X \ge 0\}$.
Find the median of X.

Q113

Example 3.6 *Prove that the function*

$$f(x) = \begin{cases} \dfrac{b}{\theta}\left(\dfrac{x}{\theta}\right)^{b-1} \exp\left(-\left\{\dfrac{x}{\theta}\right\}^{b}\right), & x > 0, \\ 0, & x \le 0, \end{cases}$$

Q114

Example 4.3 *Prove that the function*

$$f(x,y) = \begin{cases} x e^{-x(y+1)}, & x > 0,\ y > 0, \\ 0, & \textit{otherwise}, \end{cases}$$

is a frequency of a 2-dimensional random variable (X,Y).
Find the frequencies and the distribution functions of the random variables X and Y, and find the medians of these two distributions.

Q115

Example 5.2 *A discrete random variable* (X, Y) *has its distribution given by the following table*

$Y \setminus X$	1	2	3
1	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$
2	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{12}$
3	$\frac{1}{12}$	$\frac{1}{12}$	0

Find the marginal distributions of X *and* Y.
Compute $P\{X \cdot Y \text{ is even}\}$.
Compute $P\{X \geq Y\}$.
Are X *and* Y *independent?*

Q116

Example 4.1 *Find the exact value of*

$$(83) \quad \int_0^{\pi/2} f_\alpha(x)\,dx = \int_0^{\pi/2} \frac{dx}{1+\tan^\alpha x} \qquad \text{for every } \alpha \in \mathbb{R}.$$

Q117

Find

$$\int_{-1}^{1} \sqrt{1-x^2}\,dx$$

Q118

Solve

$$f(x) = 1 \qquad \text{and} \qquad G(x) = \operatorname{Arcsin} x, \qquad x \in]-1,1[,$$

Q119

Solve

$$F(x) = \int_0^x \exp\left(-t^2\right) dt, \qquad \text{for } x \in [0,1].$$

Q120

Solve

$$\int \frac{1}{x^3} \exp\left(\frac{1}{x}\right) dx.$$

Q121

Find the decomposition of the fraction $\dfrac{1}{x^4 - 1}$.

Q122

Solve

$$\frac{df}{dx} - \tan x \cdot f(x) = 2 \sin x, \qquad x \in \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[,$$

Q123

Decompose

$$\frac{1}{x^4 - 1}.$$

Q128

Example 2.1

A *Let A be a closed and bounded (i.e. compact) subset of the plane where the boundary ∂A is a closed curve of the parametric representation*

$$\mathbf{r}(t) = \left(4t^{\frac{1}{3}}(1-t)^{\frac{2}{3}}, 4t^{\frac{2}{3}}(1-t)^{\frac{1}{3}}\right), \qquad t \in [0,1].$$

Find the maximum and minimum in A for the C^∞-function

$$f(x,y) = x^3 + y^3 - 3xy, \qquad (x,t) \in A.$$

Q129

Example 3.1

A. *Calculate* $\int_B xy\,dS$, *where* B *is given on the figure.*

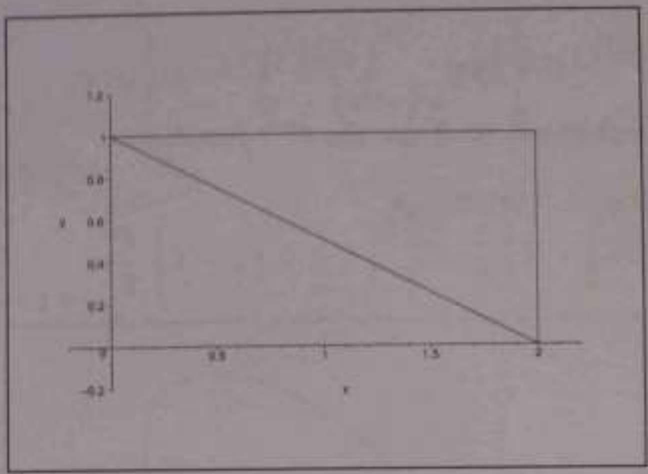
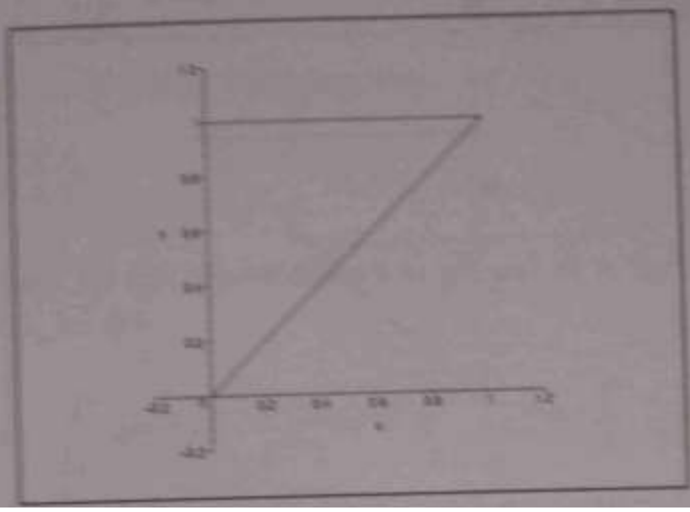


Figure 19: The domain B is the upper triangle.

Q130

Example 3.2

A. *Calculate* $\int_B x\exp\left(y^3\right)\,dS$, *where* B *is given on the figure for* $a = 1$.



Q131

Find the value of

$$E = Lz\int_B \frac{y}{(x^2+y^2+z^2)^2}\,dS, \quad \text{where } B = [0,a]\times[0,b] \text{ and } z > 0.$$

Q132

Calculate

$$I = \int_b (x^2 + y^2)\, dS,$$

where B is described in polar *coordinates by*

$$A = \left\{ (\varrho, \varphi) \;\middle|\; a \le \varrho \le 2a,\ \frac{\varrho}{2a} \le \varphi \le \frac{\varphi}{a} \right\}.$$

Q133

Example 3.6

A. *Calculate* $I = \int_B x\, dS$, *where* $B = K\left(\left(\frac{a}{2}, 0\right); \frac{a}{2}\right)$, $a > 0$.

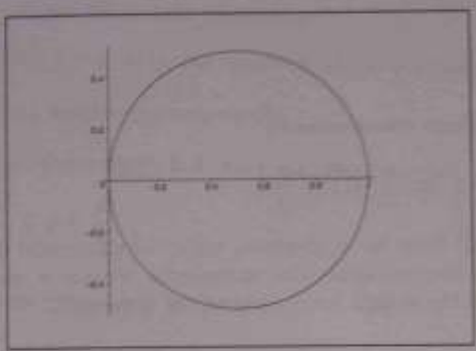


Figure 29: The domain B for $a = 1$, i.e. $-\sqrt{x - x^2} \le y \le \sqrt{x - x^2}$ for $0 \le x \le 1$.

Q134

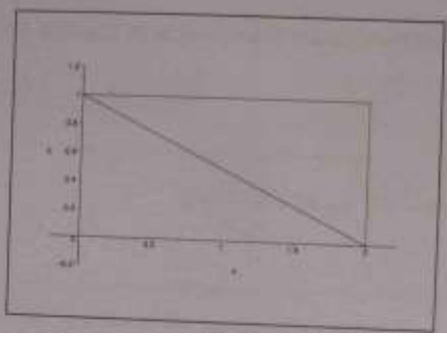
A. *Calculate the space integral*

$$I = \int_A (3 + y - z)\, x\, d\Omega,$$

where

$$A = \{(x, y, z) \in \mathbb{R}^3 \mid (x, y) \in B,\ 0 \le z \le 2y\},$$

and B is the upper triangle shown on the figure.



Q135

A. *Calculate the space integral*

$$\int_A (x+2y+z)\exp\left(z^4\right)\,d\Omega,$$

where

$$A = \{(x,y,z) \mid z \in [2,0],\ (x,y) \in B(z)\}$$

with a cut at the height z,

$$B(z) = [0,z] \times \left[0,\frac{z}{2}\right], \qquad z \in]0,2].$$

D. Apply the second reduction theorem in 3 dimensions.

Q136

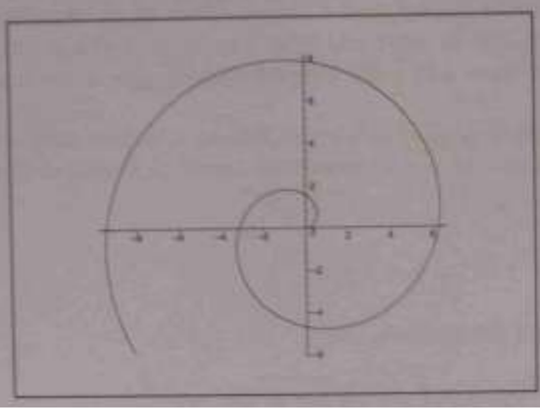
Example 5.1

A. *Find the curve length from* $(0,0)$ *of any finite piece* $(0 \le \varphi \le \alpha)$ *of* the Archimedes's spiral, *given in polar coordinates by*

$$\varrho = a\varphi, \qquad 0 \le \varphi < +\infty, \qquad \text{hvor } a > 0,$$

i.e. calculate the line integral

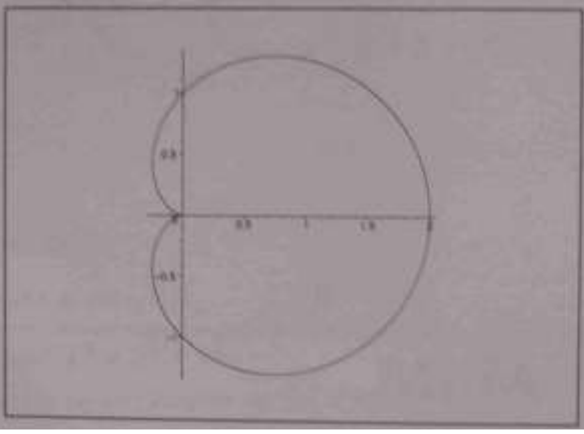
$$\ell = \int_{\varphi=0}^{\alpha} ds.$$



Q137

. *Find the value of the line integral* $I = \int_{\mathcal{K}} |y|\, ds$, *where* $\mathcal{K}$ *is the cardioid given in polar coordinates by*

$$\varrho = P(\varphi) = a(1 + \cos\varphi), \qquad -\pi \le \varphi \le \pi.$$



Q138

A. *Let* $a,\ h > 0$. *Consider the* helix

$$\mathbf{r}(t) = (x, y, z) = (a\,\cos t, a\,\sin t, h\,t), \qquad t \in \mathbb{R}.$$

This is lying on the cylinder $x^2 + y^2 = a^2$.

Find the natural parametric representaion *of the curve from* $(a, 0, 0)$, *corresponding to* $t = 0$.

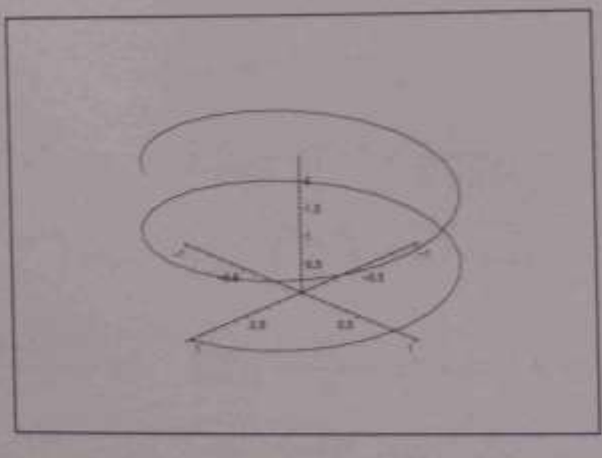


Figure 45: The helix for $a = 1$ and $h = \dfrac{1}{5}$.

Q141

Example 5.3 Let $f(x) = \sum_{n=1}^{\infty} nx^n$, $|x| < 1$. Then $f(0) = 0$, and we get for $x \neq 0$

$$f(x) = \sum_{n=1}^{\infty} nx^n = x \sum_{n=1}^{\infty} nx^{n-1} = x \cdot g(x), \quad \text{where } g(x) = \sum_{n=1}^{\infty} nx^{n-1}.$$

It follows that $g(0) = 1$, and by an *integration*,

Q142

Estimate the possible values.

$$x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + xy = 0, \qquad x \in \mathbb{R},$$

Q143

Example 1.1 *Check if the sequence*

$$a_n = \frac{n}{n+1} - \frac{n+1}{n}$$

is convergent or divergent. Find its limit, if it is convergent.

Q144

Example 1.2 *Check if the sequence*

$$a_n = \frac{n^2}{n+1} - \frac{n^2+1}{n}$$

is convergent or divergent. In case of convergence, find its limit.

Q145

Example 1.3 *Check if the sequence*

$$a_n = \cos\frac{n\pi}{2}$$

is convergent or divergent. Find the limit in case of convergence.

Q146

Example 1.4 . *Check if the sequence*

$$a_n = n^{(-1)^n}$$

is convergent or divergent. Find the limit in case of convergence.

Q147

Example 1.5 . *Check if the sequence*

$$a_n = \frac{a^n}{n}, \qquad a \in \mathbb{R},$$

is convergent or divergent. Find the limit in case of convergence.

Q148

Example 1.7 . *Check if the sequence*

$$a_n = \frac{(-1)^n}{n} + \frac{1+(-1)^n}{2}$$

is convergent or divergent. Find the limit in case of convergence.

Q149

Example 1.8 . *Check if the sequence*

$$a_n = \frac{1}{n}\sin^5 n$$

is convergent or divergent. Find the limit if the sequence is convergent.

Q150

Example 1.10 . *Check if the sequence*

$$a_n = \left(\frac{2n-3}{3n+7}\right)^4$$

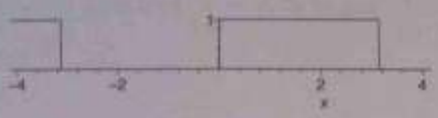
is convergent or divergent. Find its limit if it is convergent.

Q151

Example 1.2 *Find the Fourier series for the function* $f \in K_{2\pi}$, *which is given in the interval* $]-\pi, \pi]$ *by*

$$f(t) = \begin{cases} 0 & \text{for } -\pi < t \leq 0, \\ 1 & \text{for } 0 < t \leq \pi, \end{cases}$$

and find the sum of the series for $t = 0$.

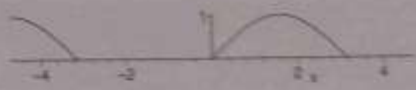


Q152

Example 1.3 *Find the Fourier series for the function $f \in K_{2\pi}$, given in the interval $]-\pi, \pi]$ by*

$$f(t) = \begin{cases} 0 & \text{for } -\pi < t \le 0, \\ \sin t & \text{for } 0 < t \le \pi, \end{cases}$$

and find the sum of the series for $t = p\pi$, $p \in \mathbb{Z}$.

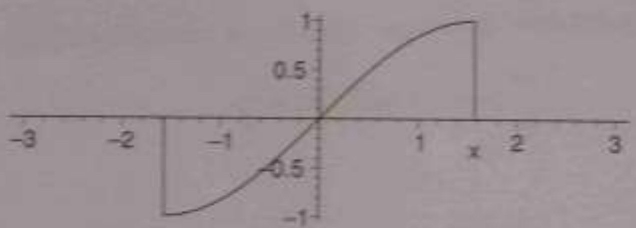


Q153

Example 1.4 *Let the periodic function $f : \mathbb{R} \mapsto \mathbb{R}$, of period 2π, be given in the interval $]-\pi, \pi]$ by*

$$f(t) = \begin{cases} 0, & \text{for } t \in \,]-\pi, -\pi/2[, \\ \sin t, & \text{for } t \in [-\pi/2, \pi/2], \\ 0 & \text{for } t \in \,]\pi/2, \pi]. \end{cases}$$

Find the Fourier series of the function and its sum function.



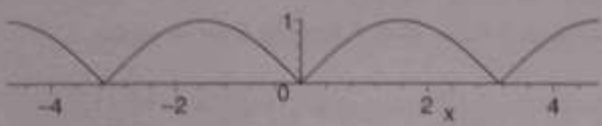
Q154

Example 1.5 *Find the Fourier series for the periodic function $f \in K_{2\pi}$, given in the interval $]\pi, \pi]$ by*

$$f(t) = |\sin t|.$$

Then find the sum of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4n^2 - 1}.$$



Q155

Example 1.6 *Let the periodic function $f : \mathbb{R} \mapsto \mathbb{R}$ of period 2π, be given by*

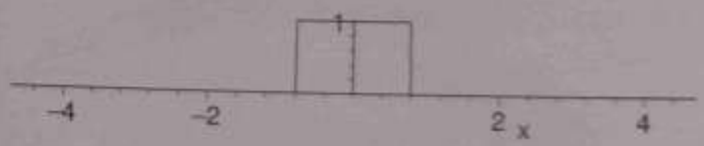
$$f(t) = \begin{cases} 0, & \text{for } t \in \left]-\pi, -\pi/4\right[, \\ 1, & \text{for } t \in [-\pi/4, \pi/4], \\ 0 & \text{for } t \in \left]\pi/4, \pi\right]. \end{cases}$$

1) Prove that f has the Fourier series

$$\frac{1}{4} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{4}\right)\cos nt.$$

2) Find the sum of the Fourier series for $t = \frac{\pi}{4}$, and then find the sum of the series

$$\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{2}\right)$$



Q157

Example 2

Find the value of $\int_0^1 e^{3x}dx$.

Q158

Write down primitives associated with the integrals of each of these functions:

(a) $(2x+1)^5$; (b) $(3-4x)^{3/2}$; (c) $(3x+4)^{-1}$; (d) $2\sin(17x-6)$; (e) $\tan(1-2x)$;
(f) $\frac{3}{x^2+16}$; (g) $\frac{5}{\sqrt{x^2-2x-8}}$; (h) $x^2\sin(x^3)$; (i) $\frac{1+2x}{x^2+x-3}$.

Q159

Evaluate these definite integrals:

(a) $\int_1^2 \left[x^2 + (3x-1)^{-2}\right] dx$; (b) $\int_0^{1/4} 3\sec^2\left(\frac{\pi}{4} - x\pi\right) dx$; (c) $\int_1^2 \frac{dx}{\sqrt{9-x^2}}$

Q160

Use the method of integration by parts to find these definite and indefinite integrals:

(a) $\int x^n \ln x \, dx$ $(x>0,\ n \neq -1)$; (b) $\int \arctan x \, dx$; (c) $\int_0^1 x^2 e^{-x} dx$; (d) $\int_0^2 \sqrt{4-x^2} \, dx$.

Q161

Use integration by parts to find an equation involving $\int (1+\ln x)^n dx$ $(x>0,\ n \neq -1)$; hence find the value of $\int_1^2 (1+\ln x)^3 dx$.

Q162

Decide whether these integrals exist; when an integral exists, find its value.

(a) $\int_0^\infty \frac{dx}{1+x^2}$; (b) $\int_0^\infty \sin x \, dx$; (c) $\int_0^\infty \frac{dx}{1+x}$.

Q163

Decide whether these integrals exist; when an integral exists, find its value.

$\int_0^1 \frac{dx}{\sqrt{x}}$; (b) $\int_{-1}^2 \frac{dx}{1+x}$; (c) $\int_0^1 \ln x \, dx$.

Q164

Find $\int \frac{x^3 - x^2 + 2x + 1}{x + 1} dx$.

Q165

Find $\int \frac{x^3 - 1}{x^2 + 4x + 3} dx$.

Q166

Find the value of $\int_1^2 \frac{x^4 - 2x^2 + 3x - 2}{x^2 - 6x + 9} dx$.

Q167

Find $\int \frac{x^3 - 3x + 1}{x^2 - 8x + 25} dx$.

Q168

Find $\int \frac{x^6 - x^3 + 1}{x^3 + x^2 + x + 1} dx$.

Q169

Find $\int \frac{x-2}{(x+1)(x^2-1)(x^2+2x+2)(x^2+2x+5)}\,dx$..

Q170

Find $\int \sin 2x \cos 3x\,dx$.

Q171

Find $\int \sin^4 x\,dx$.

Q172

Find $\int \frac{dx}{5+4\cos x}$.

Q173

Find the general solution of the equation $y' = 2x\left(1+y^2\right)$

Q174

Find the general solution of the equation $\left(1+e^{-x}\right)yy' = 1$, and then that solution which satisfies $y(0) = 1$.

Q175

Find the general solution of the equation $\left(1-e^{x}\right)y'\sec^2 y + 2e^{x}\tan y = 0$.

Q176

Find all the solutions of the equation $xy' = y + \sqrt{y^2 - x^2}$.

Q177

Find the general solution of the equation $(1+x^2)y' + 4xy = 2x$ and then that solution which satisfies $y(0) = 1$.

Q178

Find the general solution of the equation $y' - xy = xy^3$.

Q179

Seek a solution of $y'' + 3y' + 2y = 3 + 2x$ in the form $y(x) = \sum_{n=0}^{\infty} a_n x^n$.

Q180

Classify the point at infinity for the equation $x^2y'' + 3xy' + 2y = 0$

Q181

Use the method of Frobenius to find the general solution of the equation

$$4xy'' + 2y' - y = 0,$$

as a power series about $x = 0$.

Q182

Use the method of Frobenius to find the general solution of the equation

$$xy'' + y' - xy = 0$$

as a suitable power series about $x = 0$.

Q183

Use the method of Frobenius to find the general solution of the equation

$$xy'' + y = 0,$$

as a suitable power series about $x = 0$.

Q184

Write down the general solution of the equation $xy'' + y' + a^2xy = 0$, where $a > 0$ is a constant.

Q186

EXAMPLE 1.1

A girl whose mass is 40 kg is using a swing set. The diameter of the wire used for constructing the links of the chain is 5 mm. Determine the average normal stress in the links at the bottom of the swing, assuming that the inertial forces can be neglected.

Q187

EXAMPLE 1.3

All members of the truss shown in Figure 1.12 have a cross-sectional area of 500 mm² and all pins have a diameter of 20 mm. Determine: (a) The axial stresses in members *BC* and *DE*, (b) The shear stress in the pin at *A*, assuming the pin is in double shear.

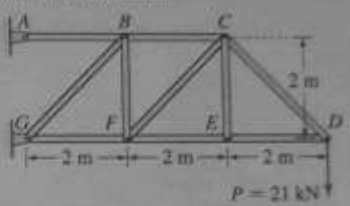


Figure 1.12 Truss.

Q189

A thin ruler, 12 in. long, is deformed into a circular arc with a radius of 30 in. that subtends an angle of 23° at the center. Determine the average normal strain in the ruler.

Q190

EXAMPLE 2.4

A belt and a pulley system in a VCR has the dimensions shown in Figure 2.10. To ensure adequate but not excessive tension in the belts, the average normal strain in the belt must be a minimum of 0.019 mm/mm and a maximum of 0.034 mm/mm. What should be the minimum and maximum undeformed lengths of the belt to the nearest 1/10th of a millimeter?

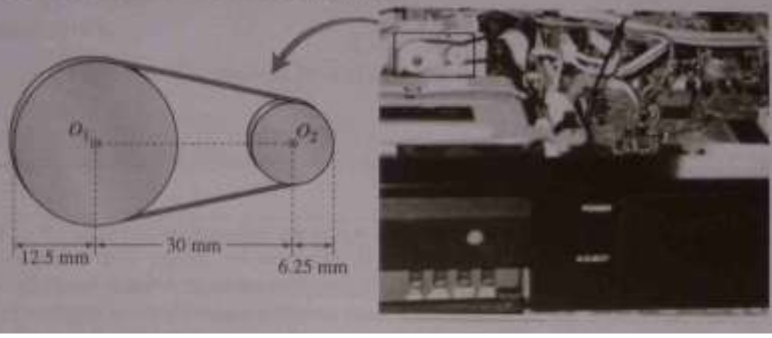


Figure 2.10 Belt and pulley in a VCR.

Q191

EXAMPLE 2.6

A gap of 0.18 mm exists between the rigid plate and bar *B* before the load *P* is applied on the system shown in Figure 2.15. After load *P* is applied, the axial strain in rod *B* is −2500 μm/m. Determine the axial strain in rods *A*.

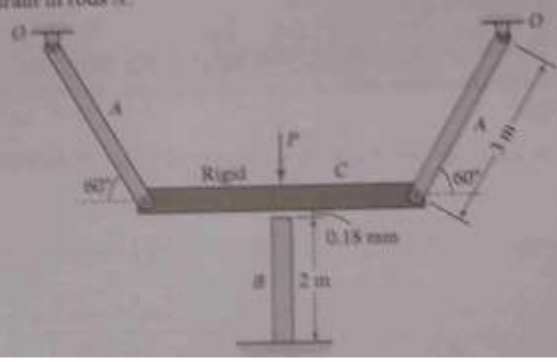


Figure 2.15 Undeformed geometry in Example 2.6.

Q192

Two bars of hard rubber are attached to a rigid disk of radius 20 mm as shown in Figure 2.17. The rotation of the rigid disk by an angle $\Delta\phi$ causes a shear strain at point *A* of 2000 μrad. Determine the rotation $\Delta\phi$ and the shear strain at point *C*.

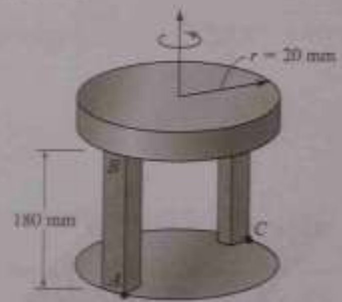


Figure 2.17 Geometry in Example 2.7.

Q193

The displacements of pins of the truss shown Figure 2.19 were computed by the finite-element method (see Section 4.8) and are given below. u and v are the pin displacement x and y directions, respectively. Determine the axial strains in members BC, HB, HC, and HG.

$u_B = 2.700$ mm $v_B = -9.025$ mm

$u_C = 5.400$ mm $v_C = -14.000$ mm

$u_G = 8.000$ mm $v_G = -14.000$ mm

$u_H = 9.200$ mm $v_H = -9.025$ mm

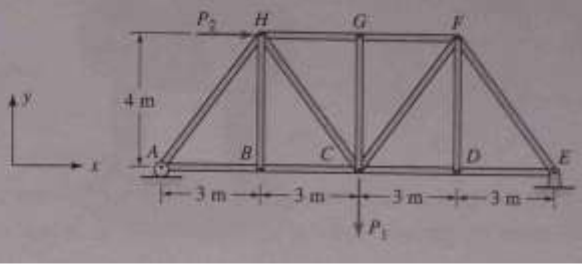


Figure 2.19 Truss in Example 2.8.

Q194

EXAMPLE 2.9

Displacements u and v in x and y directions, respectively, were measured at many points on a body by the geometric Moiré method (See Section 2.7). The displacements of four points on the body of Figure 2.23 are as given. Determine strains ε_{xx}, ε_{yy}, and γ_{xy} at point A.

$u_A = -0.0100$ mm $v_A = 0.0100$ mm

$u_B = -0.0050$ mm $v_B = -0.0112$ mm

$u_C = 0.0050$ mm $v_C = 0.0068$ mm

$u_D = 0.0100$ mm $v_D = 0.0080$ mm

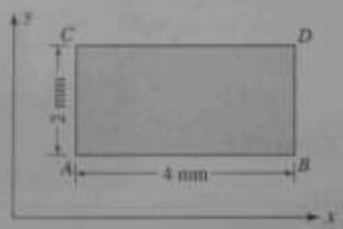


Figure 2.23 Undeformed geometry in Example 2.9.

Q197

EXAMPLE 3.1

A tension test was conducted on a circular specimen of titanium alloy. The gage length of the specimen was 2 in. and the diameter in the test region before loading was 0.5 in. Some of the data from the tension test are given in Table 3.2, where P is the applied load and δ is the corresponding deformation. Calculate the following quantities: (a) Stress at proportional limit. (b) Ultimate stress. (c) Yield stress at offset strain of 0.4%. (d) Modulus of elasticity. (e) Tangent and secant moduli of elasticity at a stress of 136 ksi. (f) Plastic strain at a stress of 136 ksi.

TABLE 3.2 Tension test data in Example 3.1

#	P (kips)	δ (10^{-3} in.)
1	0.0	0.0
2	5.0	3.2
3	15.0	9.5
4	20.0	12.7
5	24.0	15.3
6	24.5	15.6
7	25.0	15.9
8	25.2	16.9
9	25.4	19.7
10	26.0	28.5
11	26.5	36.9
12	27.0	46.5
13	27.5	58.3
14	28.0	75.2
15	28.2	87.1
16	28.3	100.0
17	28.2	112.9
18	28.0	124.8

TABLE 3.3 Stress and strain in Example 3.1

#	σ (ksi)	ε (10^{-3})
1	0.0	0.0
2	25.5	1.6
3	76.4	4.8
4	101.9	6.4
5	122.2	7.6
6	124.8	7.8
7	127.3	8.0
8	128.3	8.5
9	129.9	10.5
10	132.4	14.3
11	135.0	18.4
12	137.5	23.3
13	140.1	29.1
14	142.6	37.6
15	143.6	43.5
16	144.0	50.0
17	143.6	56.5
18	142.6	62.4

Q198

EXAMPLE 3.2

For the titanium alloy in Example 3.1, determine: (a) The modulus of resilience. Use proportional limit as an approximation for yield point. (b) Strain energy density at a stress level of 136 ksi. (c) Complementary strain energy density at a stress level of 136 ksi. (d) Modulus of toughness.

Q199

A rigid plate is attached to two 10 mm × 10 mm square bars (Figure 3.16). The bars are made of hard rubber with a shear modulus G = 1.0 MPa. The rigid plate is constrained to move horizontally due to action of the force F. If the horizontal movement of the plate is 0.5 mm, determine the force F assuming uniform shear strain in each bar.

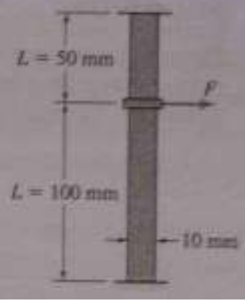


Figure 3.16 Geometry in Example 3.3.

Q200

In the leaf spring design in Figure 3.22 the formulas for the maximum stress σ and deflection δ given in Equation (3.11) are derived from theory of bending of beams (see Example 7.4):

$$\sigma = \frac{3PL}{nbt^2} \qquad \delta = \frac{3PL^3}{4Enbt^3} \tag{3.11}$$

where P is the load supported by the spring, L is the length of the spring, n is the number of leaves b is the width of each leaf, t is the thickness of each leaf, and E is the modulus of elasticity. A spring has the following data: L = 20 in., b = 2 in., t = 0.25 in., and E = 30,000 ksi. The failure stress is $\sigma_{failure}$ = 120 ksi, and the failure deflection is $\delta_{failure}$ = 0.5 in. The spring is estimated to carry a maximum force P = 250 lb and is to have a factor of safety of K_{safety} = 4. (a) Determine the minimum number of leaves. (b) For the answer in part (a) what is the real factor of safety?

Figure 3.22 Leaf spring in Example 3.6.

Q201

The strains at a point on aluminum (E = 70 GPa, G = 28 GPa, and ν = 0.25) were found to be ε_{xx} = 650 μ, ε_{yy} = 300 μ, and γ_{xy} = 750 μ. Determine the stresses σ_{xx}, σ_{yy}, and τ_{xy} and the strain ε_{zz} assuming the point is in plane stress.

Q202

Finite-element analysis (see Section 4.8) shows that a long structural component in Figure 3.31 carries a uniform axial stress of $\sigma_{nominal} = 35$ MPa (T). A hole in the center needs to be drilled for passing cables through the structural component. The yield stress of the material is $\sigma_{yield} = 200$ MPa. If failure due to yielding is to be avoided, determine the maximum diameter of the hole that can be drilled using a factor of safety of $K_{safety} = 1.6$.

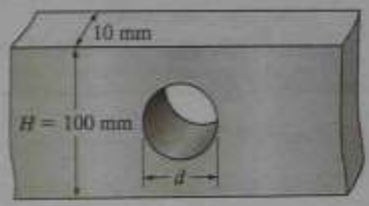


Figure 3.31 Component geometry in Example 3.9.

Q203

Aluminum has a yield stress $\sigma_{yield} = 40$ ksi in tension, a yield strain $\varepsilon_{yield} = 0.004$, an ultimate stress $\sigma_{ult} = 45$ ksi, and the corresponding ultimate strain $\varepsilon_{ult} = 0.17$. Determine the material constants and plot the corresponding stress–strain curves for the following models: (a) the elastic–perfectly plastic model. (b) the linear strain-hardening model. (c) the nonlinear power-law model.

Q204

At a cross section of a beam shown in Figure 3.46, the normal strain due to bending about the z axis was found to vary as $\varepsilon_{xx} = -0.0125y$, with y measured in meters. Write the expressions for normal stress σ_{xx} as a function of y and plot the σ_{xx} distribution across the cross section. Assume the beam is made from elastic–perfectly plastic material that has a yield stress $\sigma_{yield} = 250$ MPa and a modulus of elasticity $E = 200$ GPa.

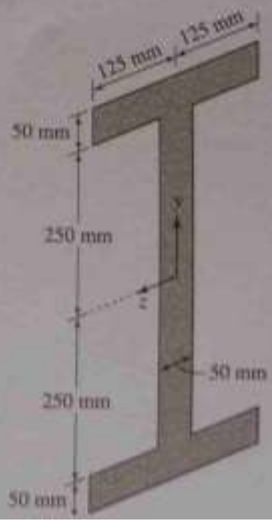


Figure 3.46 Beam cross section in Example 3.16.

Q206

Two thin bars are securely attached to a rigid plate, as shown in Figure 4.2. The cross-sectional area of each bar is 20 mm². The force F is to be placed such that the rigid plate moves only horizontally by 0.05 mm without rotating. Determine the force F and its location h for the following two cases: (a) Both bars are made from steel with a modulus of elasticity $E = 200$ GPa. (b) Bar 1 is made of steel ($E = 200$ GPa) and bar 2 is made of aluminum ($E = 70$ GPa).

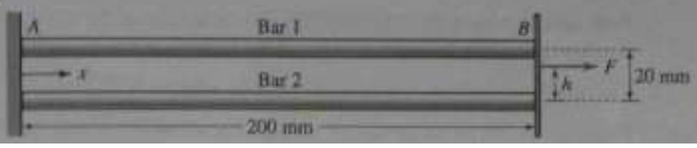


Figure 4.2 Axial bars in Example 4.1.

Q207

Figure 4.5 shows a homogeneous wooden cross section and a cross section in which the wood is reinforced with steel. The normal strain for both cross sections is uniform, $\varepsilon_{xx} = -200\ \mu$. The moduli of elasticity for steel and wood are $E_{steel} = 30{,}000$ ksi and $E_{wood} = 8000$ ksi. (a) Plot the σ_{xx} distribution for each of the two cross sections shown. (b) Calculate the equivalent internal axial force N for each cross section using Equation (4.1).

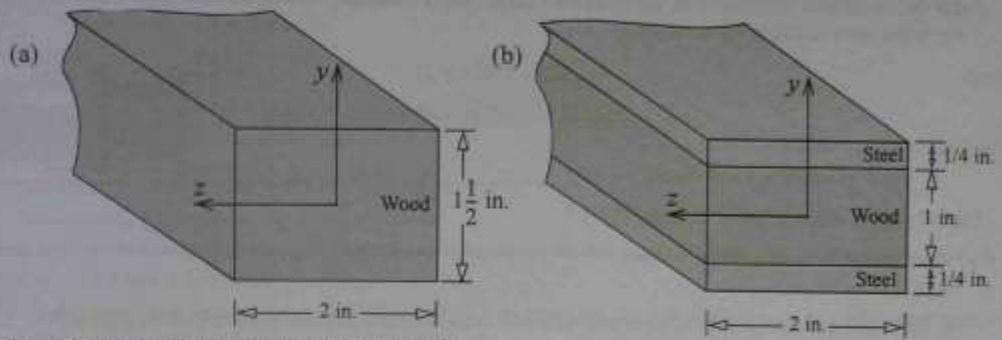


Figure 4.5 Cross sections in Example 4.2. (*a*) Homogeneous. (*b*) Laminated.

Q208

Solid circular bars of brass ($E_{br} = 100$ GPa, $\nu_{br} = 0.34$) and aluminum ($E_{al} = 70$ GPa, $\nu_{al} = 0.33$) having 200 mm diameter are attached to a steel tube ($E_{st} = 210$ GPa, $\nu_{st} = 0.3$) of the same outer diameter, as shown in Figure 4.12. For the loading shown determine: (a) The movement of the plate at C with respect to the plate at A. (b) The change in diameter of the brass cylinder. (c) The maximum inner diameter to the nearest millimeter in the steel tube if the factor of safety with respect to failure due to yielding is to be at least 1.2. The yield stress for steel is 250 MPa in tension.

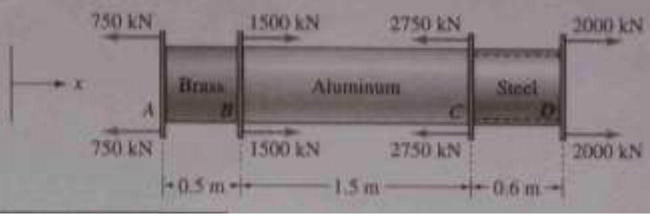


Figure 4.12 Axial member in Example 4.3.

Q209

The three bars in Figure 4.25 are made of steel ($E = 30{,}000$ ksi) and have cross-sectional areas of 1 in^2. Determine the displacement of point D.

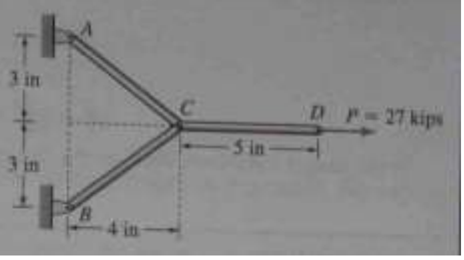


Figure 4.25 Geometry in Example 4.9

Q210

The lid is bolted to the tank in Figure 4.48 along the flanges using 1-in.-diameter bolts. The tank is made from sheet metal that is $\frac{1}{2}$ in. thick and can sustain a maximum hoop stress of 24 ksi in tension. The normal stress in the bolts is to be limited to 60 ksi in tension. A manufacturer can make tanks of diameters from 2 ft to 8 ft in steps of 1 ft. Develop a table that the manufacturer can use to advise customers of the size of tank and the number of bolts per lid needed to hold a desired gas pressure.

Figure 4.48 Cylindrical tank in Example 4.14

Q212

The two thin bars of hard rubber shown in Figure 5.2 have shear modulus $G = 280$ MPa and cross-sectional area of 20 mm². The bars are attached to a rigid disc of 20-mm radius. The rigid disc is observed to rotate about its axis by an angle of 0.04 rad due to the applied torque T_{ext}. Determine the applied torque T_{ext}.

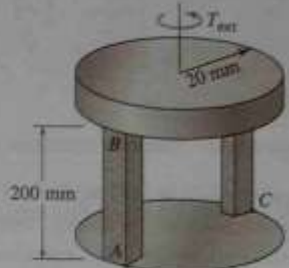


Figure 5.2 Geometry in Example 5.1.

Q214

The two shafts shown in Figure 5.24 are of the same material and have the same amount of material cross-sectional areas A. Show that the hollow shaft has a larger polar moment of inertia than the solid shaft.

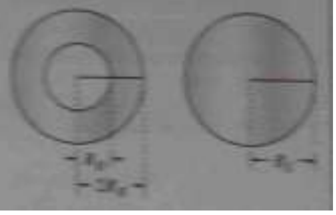


Figure 5.24 Hollow and solid shafts of Example 5.5.

Q215

A 1-m-long hollow shaft in Figure 5.32 is to transmit a torque of 400 N·m. The shaft can be made of either titanium alloy or aluminum. The shear modulus of rigidity G, the allowable shear stress τ_{allow}, and the density γ are given in Table 5.1. The outer diameter of the shaft must be 25 mm to fit existing attachments. The relative rotation of the two ends of the shaft is limited to 0.375 rad. Determine the inner radius to the nearest millimeter of the lightest shaft that can be used for transmitting the torque.

TABLE 5.1 **Material properties in Example 5.8**

Material	G (GPa)	τ_{allow} (MPa)	γ (Mg/m³)
Titanium alloy	36	450	4.4
Aluminum	28	150	2.8

Figure 5.32 Shaft in Example 5.8.

Q216

EXAMPLE 5.12

A solid circular steel shaft ($G_s = 12{,}000$ ksi, $E_s = 30{,}000$ ksi) of 4-in. diameter is loaded as shown in Figure 5.62. Determine the maximum shear stress in the shaft.

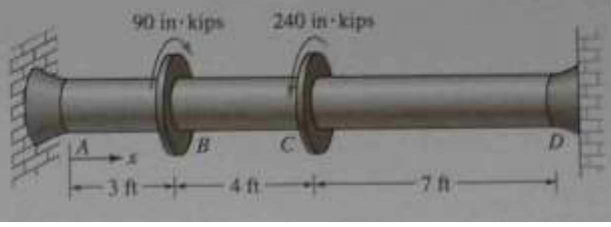
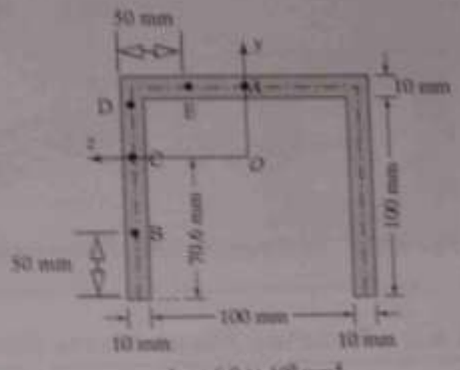


Figure P5.62 Shaft in Example 5.12.

Q223

A positive shear force $V_y = 30$ kN acts on the thin cross sections shown in Figure 6.60 (not drawn to scale). Determine the shear stress at points B, C, D, and E. Report the answers as τ_{xy} or τ_{xz}.



$I_{zz} = 6.8 \times 10^6$ mm^4

Figure 6.60 Cross sections in Example 6.15.

Q222

Assuming a positive shear force V_y, sketch the direction of the shear flow along the center line on the thin cross sections shown in Figure 6.51.

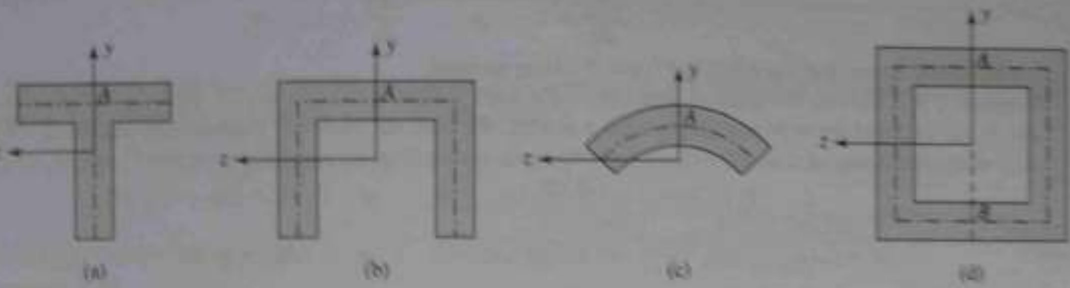


Figure 6.51 Cross sections in Example 6.13.

Q221

(a) Write the equations for the internal shear force V_y and the internal bending moments M_z as a function of x for the entire beam shown in Figure 6.38. (b) Determine the values of V_y and M_z just before and after point B.

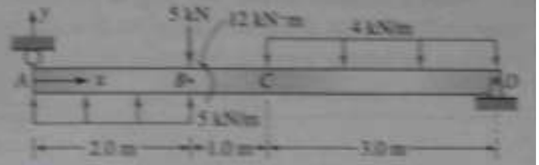


Figure 6.38 Beam in Example 6.10.

Q220

Figure 6.24 shows a beam and loading in three different coordinate systems. Determine for the three cases the internal shear force and bending moment at a section 36 in. from the free end using the sign conventions described in Figures 6.21 and 6.23.

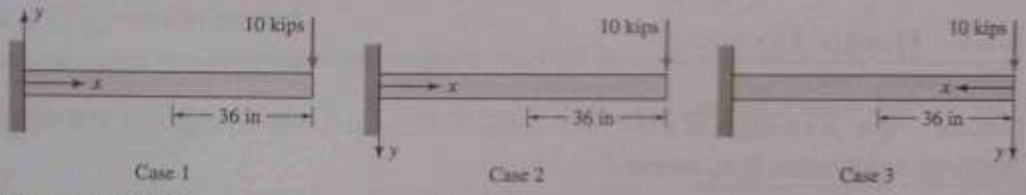


Figure 6.24 Example 6.6 on sign convention.

Q219

The maximum bending normal strain on a homogeneous steel ($E = 30{,}000$ ksi) cross section shown in Figure 6.19 was found to be $\varepsilon_{xx} = +1000\ \mu$. Determine the bending normal stress at point A.

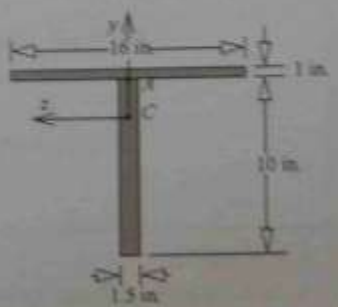


Figure 6.19 T cross section in Example 6.5.

Q218

EXAMPLE 6.1

The left ends of three bars are built into a rigid wall, and the right ends are welded to a rigid plate, as shown in Figure 6.2. The undeformed bars are straight and perpendicular to the wall and the rigid plate. The rigid plate is observed to rotate due to the applied moment by an angle of 3.5° from the vertical plane. If the normal strain in bar 2 is zero, determine the normal strains in bars 1 and 3.

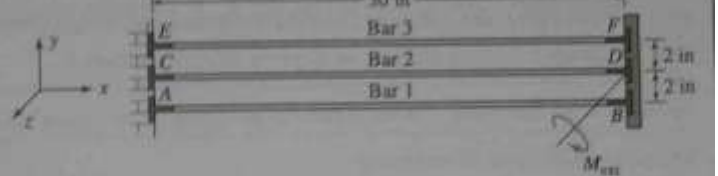


Figure 6.2 Geometry in Example 6.1.

Q219

The maximum bending normal strain on a homogeneous steel ($E = 30{,}000$ ksi) cross section shown in Figure 6.19 was found to be $\varepsilon_{xx} = +1000\ \mu$. Determine the bending normal stress at point A.

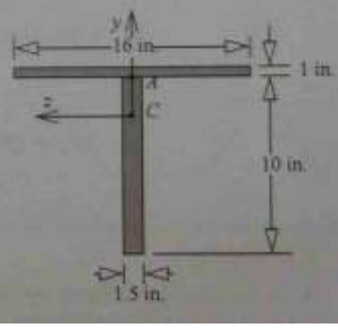


Figure 6.19 T cross section in Example 6.5.

Q225

A beam has a linearly varying distributed load, as shown in Figure 7.5. Determine: (a) The equation of the elastic curve in terms of E, I, w, L, and x. (b) The maximum intensity of the distributed load if the maximum deflection is to be limited to 20 mm. Use $E = 200$ GPa, $I = 600\ (10^6)\ \text{mm}^4$, and $L = 8$ m.

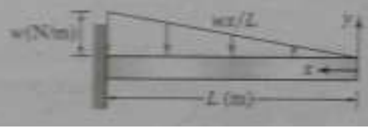


Figure 7.5 Beam and loading in Example 7.1.

Q226

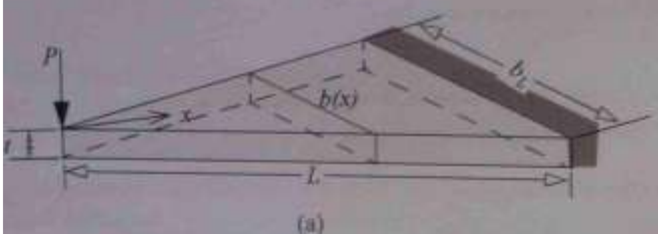
For the beam and loading shown in Figure 7.7, determine: (a) the equation of the elastic curve in terms of E, I, L, P, and x; (b) the maximum deflection in the beam.

Figure 7.7 Beam and loading in Example 7.2.

Q227

EXAMPLE 7.4

A cantilever beam with variable width $b(x)$ is shown in Figure 7.11. Determine the maximum deflection in terms of P, b_L, t, L, and E.



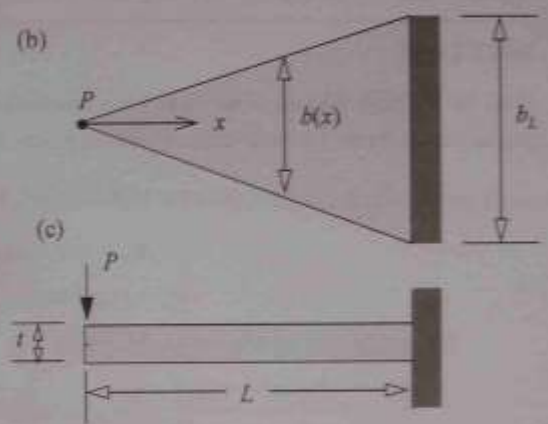


Figure 7.11 (a) Geometry of variable-width beam in Example 7.4. (b) Top view. (c) Front view.

Q228

EXAMPLE 7.7

A light pole is subjected to a wind pressure that varies as a quadratic function, as shown in Figure 7.25. In terms of E, I, w, L, and x determine (a) the deflection at the top of the pole; (b) the ground reactions.

Figure 7.25 Beam and loading in Example 7.7.

Q229

For the beam shown in Figure 7.28, using the principle of superposition and Table C.3, determine (a) the reactions at *A*; (b) the maximum deflection.

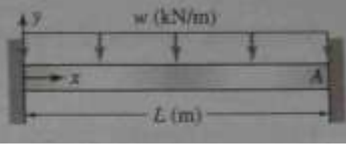


Figure 7.28 Beam in Example 7.8.

Q232

A steel beam in a bridge was repaired by welding along a line that is 35° to the axis of the beam. The normal stress near the bottom of the beam is estimated using beam theory and is shown on the stress cube. Determine the normal and shear stress on the plane containing the weld line.

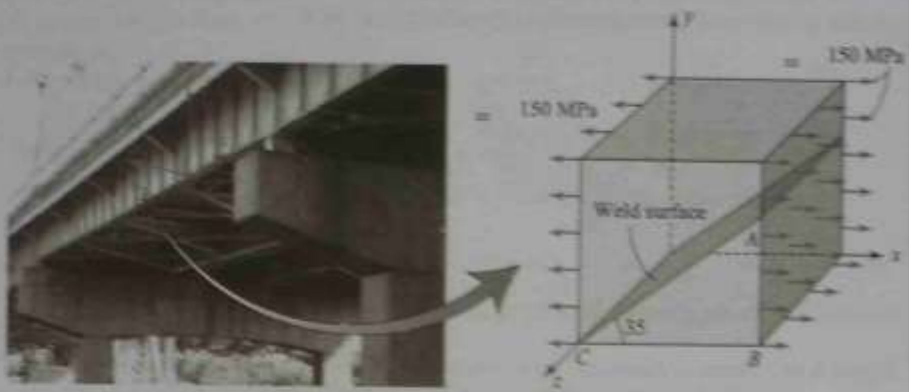


Figure 8.3 Stress cube at a point on a bridge.

Q233

Solve Example 8.2 using Equations (8.1) and (8.2). Also determine the principal stresses, principal angle 1, and the maximum shear stress at the point.

Q232

A steel beam in a bridge was repaired by welding along a line that is 35° to the axis of the beam. The normal stress near the bottom of the beam is estimated using beam theory and is shown on the stress cube. Determine the normal and shear stress on the plane containing the weld line.

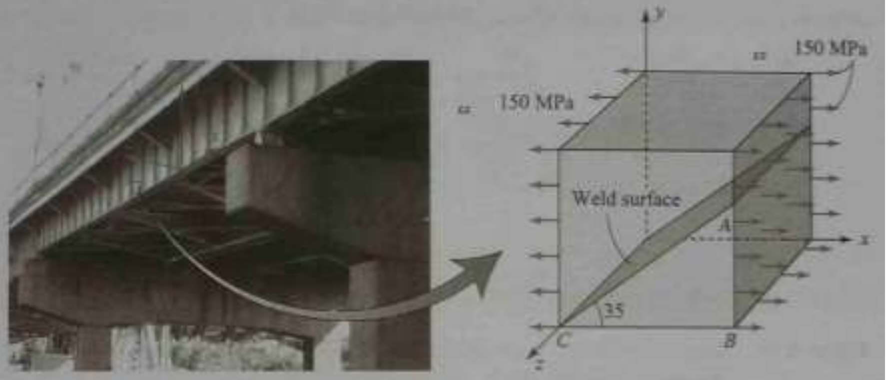


Figure 8.3 Stress cube at a point on a bridge.

Q234

A 30-in.-long thin cylindrical tube is to transmit a torque of 25π in.·kips. The tube is to be fabricated by butt welding a $\frac{1}{16}$-in.-thick steel plate ($G = 12{,}000$ ksi) along a spiral seam, as shown in Figure 8.28. Buckling considerations limit the allowable stress in steel to 10 ksi in compression. The allowable shear stress in the weld is 12 ksi, and the allowable tensile stress in the weld is 20 ksi. Stiffness considerations limit the relative rotation of the two ends to 3°. Determine the minimum outer radius of the tube to the nearest $\frac{1}{16}$ in.

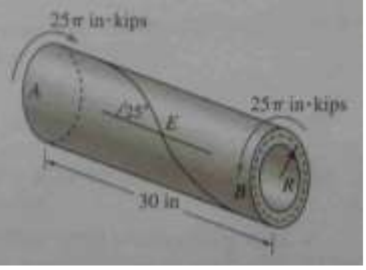


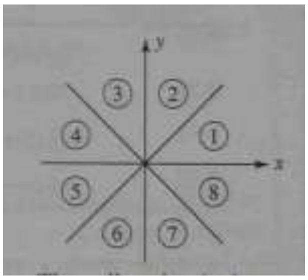
Figure 8.28 Geometry of shaft and loading in Example 8.6.

Q236

At a point, the only nonzero strain component is $\varepsilon_{xx} = 200\ \mu$. Determine the strain components in the n, t coordinate system that is rotated 25° counterclockwise to the x axis.

Q237

At a point in plane strain, the strain components are $\varepsilon_{xx} = 200\ \mu$, $\varepsilon_{yy} = 500\ \mu$, and $\gamma_{xy} = 600\ \mu$. Estimate the orientation of the principal directions and report your results using the sectors shown in Figure 9.7.



Q240

A hollow shaft that has an outside diameter of 100 mm, and an inside diameter of 50 mm is loaded as shown in Figure 10.8. For the three cases shown, determine the principal stresses and the maximum shear stress at point A. Point A is on the surface of the shaft.

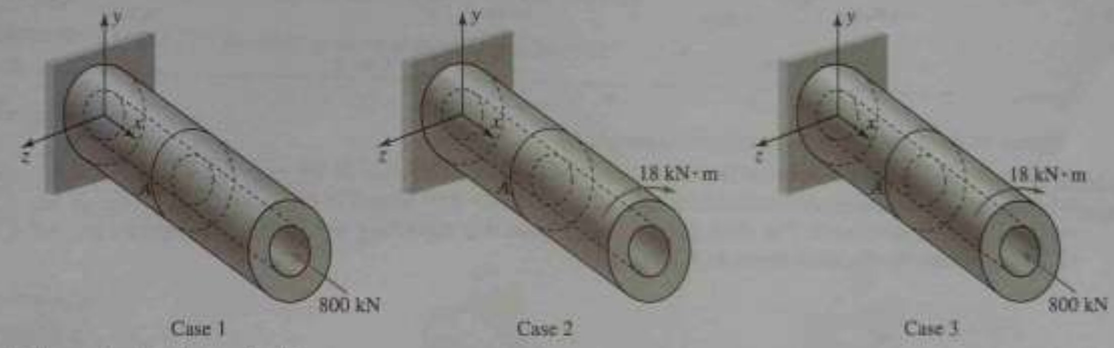


Figure 10.8 Hollow cylinder in Example 10.1.

Q241

A hollow shaft has an outside diameter of 100 mm and an inside diameter of 50 mm, is shown in Figure 10.13. Strain gages are mounted on the surface of the shaft at 30° to the axis. For each case determine the applied axial load P and the applied torque T_{ext} if the strain gage readings are $\varepsilon_a = -500\ \mu$ and $\varepsilon_b = 400\ \mu$. Use $E = 200$ GPa, $G = 80$ GPa, and $\nu = 0.25$.

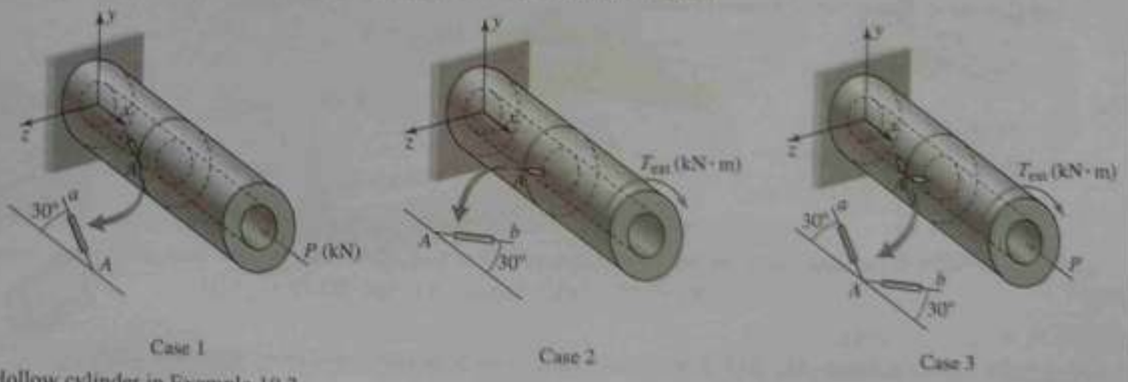


Figure 10.13 Hollow cylinder in Example 10.2.

Q242

EXAMPLE 10.4

A thin cylinder with an outer diameter of 100 mm and a thickness of 10 mm is loaded as shown in Figure 10.25. At point A, which is on the surface of the cylinder, determine the normal and shear stresses in the x, y, z coordinate system. Show your results on a stress cube.

Figure 10.25 Geometry and loading in Example 10.4.

Q243

A rectangular wooden beam of 60 mm × 180 mm cross section is supported at the right end by an aluminum circular rod of 8-mm diameter, as shown in Figure 10.37. The allowable normal stress in the wood is 14 MPa and the allowable shear stress in aluminum is 60 MPa. The moduli of elasticity for wood and aluminum are $E_w = 12.6$ GPa and $E_{al} = 70$ GPa. Determine the maximum intensity w of the distributed load that the structure can support.

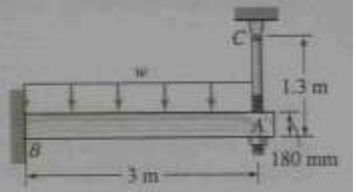


Figure 10.37 Beam in Example 10.7.

Q244

EXAMPLE 10.8

A circular member was repaired by welding a crack at point A that was 30° to the axis of the shaft, as shown in Figure 10.40. The allowable shear stress at point A is 24 ksi and the maximum normal stress the weld material can support is 9 ksi (T). Calculations show that the stresses at point A are $\sigma_{xx} = 9.55P_2$ ksi (T) and $\tau_{xy} = -6.79P_1$ ksi. (a) Draw the failure envelope for the applied loads P_1 and P_2. (b) Determine the values of loads P_1 and P_2. (c) If $P_1 = 2$ kips and $P_2 = 1.5$ kips, determine the factor of safety.

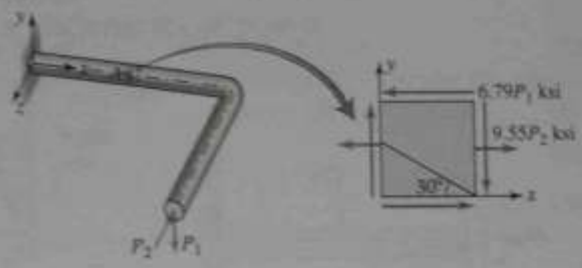


Figure 10.40 Problem geometry in Example 10.8.

Q245

At a critical point on a machine part made of steel, the stress components were found to be $\sigma_{xx} = 100$ MPa (T), $\sigma_{yy} = 50$ MPa (C), and $\tau_{xy} = 30$ MPa. Assuming that the point is in plane stress and the yield stress in tension is 220 MPa, determine the factor of safety using (a) the maximum shear stress theory; (b) the maximum octahedral shear stress theory.

Q247

A hollow circular steel column ($E = 30{,}000$ ksi) is simply supported over a length of 20 ft. The inner and outer diameters of the cross section are 3 in. and 4 in., respectively. Determine (a) the slenderness ratio; (b) the critical buckling load; (c) the axial stress at the critical buckling load. (d) If roller supports are added at the midpoint, what would be the new critical buckling load?

Q248

Determine the maximum deflection of the column shown in Figure 11.15 in terms of the modulus of elasticity E, the length of the column L, the area moment of inertia I, the axial force P, and the intensity of the distributed force w.

Figure 11.15 Buckling of beam with distributed load in Example 11.4.

Q249

A wooden box column ($E = 1800$ ksi) is constructed by joining four pieces of lumber together, as shown in Figure 11.19. The load $P = 80$ kips is applied at a distance of $e = 0.667$ in. from the centroid of the cross section. (a) If the length is $L = 10$ ft, what are the maximum stress and the maximum deflection? (b) If the allowable stress is 3 ksi, what is the maximum permissible length L to the nearest inch?

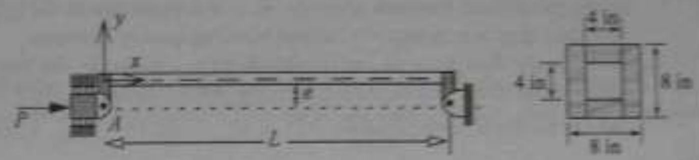


Figure 11.19 Eccentrically loaded box column.

Q250

EXAMPLE 11.6

A wooden box column ($E = 1800$ ksi) is constructed by joining four pieces of lumber together, as shown in Figure 11.19. The ultimate stress is 5 ksi. Determine the maximum load P that can be applied.

Q252

A stone is thrown vertically upwards at a speed of 14 m/s. What is the maximum height achieved?

Q253

Example 2.1

A ball, travelling horizontally at 7 m/s, rolls over the edge of a cliff. Where will it be 2 seconds later?

Q254

Example 2.2

In the same situation as *Example 2.1*, what is the velocity of the ball after 2 seconds?

Q255

Find the maximum range for the arrow of *Example 2.3*. Find also the angle of projection for which the range of the arrow is exactly 50% of the maximum achievable.

Q256

A lift of mass 300 kg is pulled upwards by a cable in which the tension is 3300 newtons. What is the acceleration produced?

Q257

A bucket of mass 2 kg is suspended from a hook by a chain of mass 0.5 kg. What is the force exerted by the chain on (a) the bucket and (b) the hook?

Q258

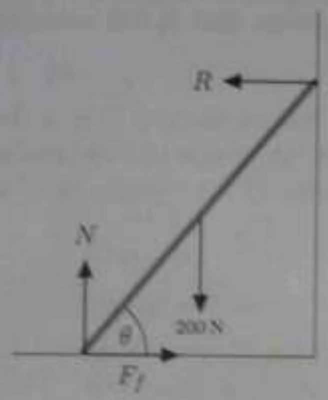
Mr B is in his garden and places a pile of flowerpots against the shed door to stop it blowing shut. The coefficient of friction between the pots and the ground is 0.7. How many pots, mass 0.5 kg each, are needed to withstand a force from the door of 20 newtons?

Q259

A shelf 2 metres long, supported by brackets at its two ends L and R, carries a set of encyclopædias of total weight 100 newtons. The volumes occupy a one metre length of the shelf at the left hand end. What are the loads F_L and F_R on the two brackets?

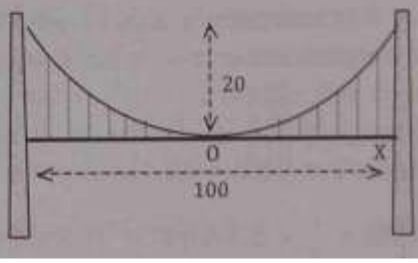
Q260

A ladder of length 6.5 m and weight 200 N rests against a wall with its base on rough ground 2.5 m away from the bottom of the wall. The wall is perfectly "smooth" so the reaction R from the wall on the ladder is exactly perpendicular to the wall. What is the minimum value of μ, the coefficient of friction between the ground and the ladder, for the ladder to remain in equilibrium in this position?



Q261

A suspension bridge has a span of 100 metres and the deck, which weighs 200 tonnes, hangs from cables fixed to piers on either side. The cable fixing points are 20 metres above the level of the deck. Assuming that the weight of the cables is small compared with the weight of the deck, calculate the tension in the cables **(a)** at the centre of the span and **(b)** at the fixing points.



Q262

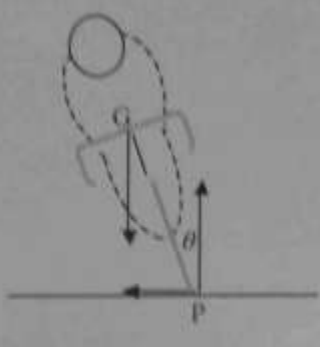
A railway truck A, travelling at speed 6 m/s, collides with a similar truck B. After the collision the two trucks couple together and move off before hitting a third truck C. Finally, the three trucks move off together down the track. What is their common speed?

Q263

Snooker ball C travelling at speed 0.5 m/s, collides head-on with an identical stationary ball D, the coefficient of restitution being $e = 0.96$. What are the velocities after impact?

Q264

Mr C, mass 60 kg, turns a corner on his bicycle, mass 5 kg, in an arc of radius 20 m. If he is travelling at 5 m/s, what is the angle at which he will need to 'lean in' to the curve?



Q265

Calculate the heat conducted through a 0.2 m thick industrial furnace wall made of fireclay brick. Measurements made during steady-state operation showed that the wall temperatures inside and outside the furnace are 1500 and 1100 K respectively. The length of the wall is 1.2m and the height is 1m.

Q266

A refrigerator stands in a room where the air temperature is 20oC. The surface temperature on the outside of the refrigerator is 16oC. The sides are 30 mm thick and have an equivalent thermal conductivity of 0.1 W/m K. The heat transfer coefficient on the outside 9 is 10 W/m2K. Assuming one dimensional conduction through the sides, calculate the net heat flow and the surface temperature on the inside.

Q267

The walls of the houses in a new estate are to be constructed using a 'cavity wall' design. This comprises an inner layer of brick (k = 0.5 W/m K and 120 mm thick), an air gap and an outer layer of brick (k = 0.3 W/m K and 120 mm thick). At the design condition the inside room temperature is 20°C, the outside air temperature is -10°C; the heat transfer coefficient on the inside is 10 W/m2 K, that on the outside 40 W/m2 K, and that in the air gap 6 W/m2 K. What is the heat flux through the wall?

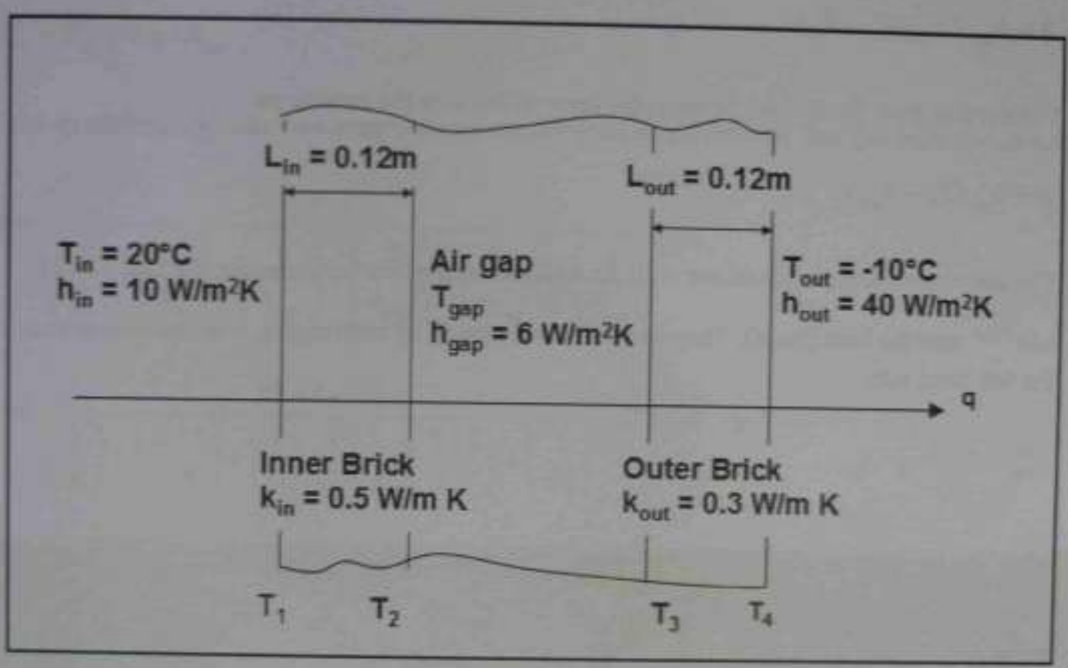


Figure 2-2: Conduction through a plane wall

Q268

The design of a single 'pin fin' which is to be used in an array of identical pin fins on an electronics heat sink is shown in Figure 2.6. The fin is made from cast aluminium, k = 180 W / m K, the diameter is 3 mm and the length 15 mm. There is a heat transfer coefficient of 30 W / m2 K between the surface of the fin and surrounding air which is at 25°C.

1. Use the expression for a fin with an adiabatic tip to calculate the heat flow through a single pin fin when the base has a temperature of 55°C.

2. Calculate also the efficiency and the effectiveness of this fin design.

3. How long would this fin have to be to be considered "infinite" ?

Q269

Air at temperature 527oC and 1 bar pressure flows with a velocity of 10m/s over a flat plate 0.5m long. Estimate the cooling rate per unit width of the plate needed to maintain it at a surface temperature of 27oC assuming the contribution of radiation contribution is negligible.

Q270

A square flat plate of 2m each side is maintained at a uniform temperature of 230 oC by means of an embedded electric wire heater. If the atmospheric air is flowing at 25 oC over the plate with a velocity of 60m/s, what is the electrical power input required?

Q272

Example 4.1

The following examples illustrate a number of different techniques to calculate view factors.

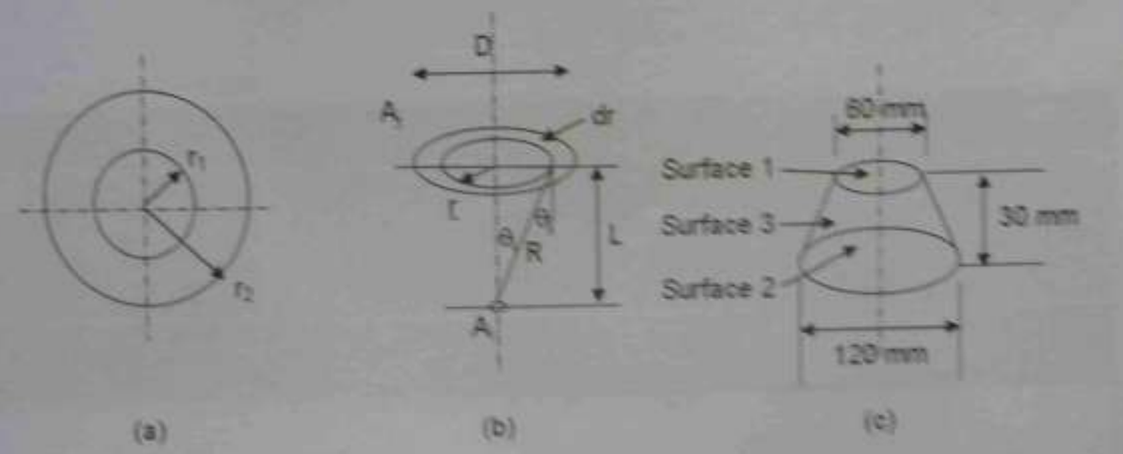


Figure 4-3: View factor geometries for Example 4.1

(a) This demonstrates the use of simple view factor algebra applied to two concentric cylinders.

F11 = 0 (convex surface)

so

F12 = 1

F21 = F12 (A1 / A2) = A1 / A2 = r1 / r2

How do we use this relationship ?

Q273

High temperature gas flows through the inside of a pipe of outer radius r2 = 30 mm. To reduce the thermal radiation from the pipe to an electrical control panel mounted nearby, a semi-circular radiation shield of radius r1 = 100 mm is placed concentrically around the pipe. Thermal radiation from the pipe is radiated to bothe the shield and the surroundings which are at 310 K. The radiation view factor from the shield to itself is F11 = 0.3345.

a) Obtain values for the remaining view factors

b) Consider a radiation balance on the system. The pipe temperature is 900 K; the shield temperature is T1; the emissivity of the inner surface of the shield is ε1 = 0.8 and the emissivity of the outer surface of the pipe is ε2 = 0.5. Assume the surroundings can be approximated by a black body at 310 K and neglecting any heat loss by convection, estimate the surface temperature of the shield when the emissivity of the outer surface of the radiation shield is ε0 = 0.1

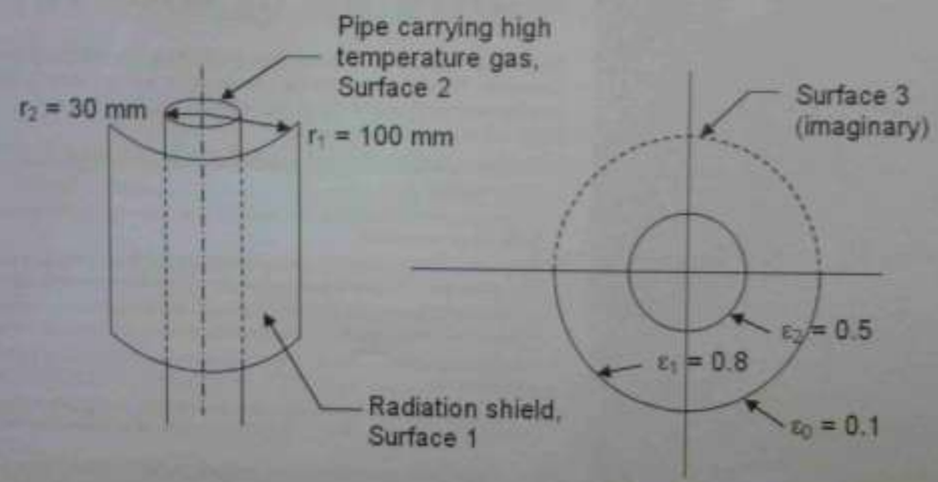


Figure 4.4 Radiation shield - Example 4.2

Q275

A concentric tube heat exchanger is used to cool lubricating oil for a large diesel engine. The inner tube is constructed of 2 mm wall thickness stainless steel, having k = 16 W/m K. The flow rate of cooling water through the inner tube (ri = 30mm) is 0.3 kg/s. The flow rate of oil through the tube (ro = 50mm) is 0.15 kg/s. Assume fully developed flow, if the oil cooler is to be used to cool oil from 90oC to 50oC using water available at 10oC, calculate:

The length of the tube required for parallel flow;

The length of the tube required for counterflow;

The area required for a single pass cross-flow heat exchanger with both streams umixed, operating at the same temperatures and flow rates and with the same value of U as in a and b above.

Q276

The double pipe heat exchanger of Example 5.2 is to be used to cool 0.15kg/s of oil at 90oC using 0.3kg/s of seawater at 10oC. The area of the heat exchanger is 11.5m2 and the overall heat transfer coefficient is 21.9W/m2K. What are the exit states of oil and water from the heat exchanger?

Q296

Self ignition would occur in the engine using certain brand of petrol if the temperature due to compression reached 350°C.

Calculate the highest ratio of compression that may be used to avoid pre-ignition if the law of compression is

$PV^{1.3} = c$

$PV^{1.4} = c$

Calculate the final pressure in each case. Assume inlet condition of 27°C and 1 bar.

Q297

Find the specific volume for H_2O at 10 MPa and 500°C using:

a) Compressibility chart;

b) Steam tables (below)

T	p = 10.0 MPa (311.06 deg-C)			
	v	u	H	s
Sat.	0.018026	2544.4	2724.7	5.6141
325	0.019861	2610.4	2809.1	5.7568
350	0.02242	2699.2	2923.4	5.9443
400	0.02641	2832.4	3096.5	6.2120
450	0.02975	2943.4	3240.9	6.4190
500	0.03279	3045.8	3373.7	6.5966
550	0.03564	3144.6	3500.9	6.7561
600	0.03837	3241.7	3625.3	6.9029

Q298

A closed rigid container has a volume of 1 m^3 and holds air at 345 kPa and 20°C. Heat is added until the temperature is 327°C. Determine the change in Internal Energy:-

a) Using an average value of the specific heat.
b) Taking into account the variation of specific heat with temperature.

Q299

An adiabatic steam turbine expands steam from a pressure of 6 MPa and a temperature of 500°C to a pressure of 10 KPa The isentropic efficiency of the turbine is 0.82 and changes in kinetic and potential energy may be neglected. Determine the state of the steam at exit from the turbine and the specific work transfer.

T	p = 6.0 MPa (257.64 deg-C)			
	v	*u*	*h*	*s*
Sat.	0.03244	2589.7	2784.3	5.8892
500	0.05665	3082.2	3422.2	6.8803

p (kPa)	t_s (°C)	v_f (m^3/kg)	v_g (m^3/kg)	h_f (kJ/kg)	h_g (kJ/kg)	s_f (kJ/kg.K)	s_g (kJ/kg.K)
10	45.81	0.001	14.674	191.83	2,585	0.6493	8.1502

Q300

Air at 27°C receives heat at constant volume until its temperature reaches 927°C. Determine the heat added per kilogram? Assume for air C_V = 0.718 kJ/kgK.

Q301

The gas expanding in the combustion space of a reciprocating engine has an initial pressure of 50 bar and an initial temperature of 1623 °C. The initial volume is 50000 mm^3 and the gas expands through a volume ratio of 20 according to the law $pV^{1.25}$ = constant. Calculate

a) the work transfer and
b) heat transfer in the expansion process.

Take R = 270 J/Kg K and C_v = 800 J/Kg K.

Q302

A reciprocating compressor delivers 0.1 kg/s of air at a pressure of 12 bar. The air enters the compressor at a pressure of 1 bar and a temperature of 15 °C. Calculate the delivery temperature of the air, the work transfer rate and the heat transfer rate in the compression process for:

i. reversible polytropic compression, $PV^{1.2}$ = constant;
ii. reversible adiabatic compression;
iii. reversible isothermal compression.

Air - R = 0.287 kJ/kgK, C_p = 1.005 kJ/kg K, C_v = 0.718 kJ/kgK and n = 1.4.

Q303

A reciprocating steam motor is supplied with dry saturated steam at a pressure of 1.6MPa (specific volume = 0.1238 m^3/kg). The stroke of the motor is 0.8m and the bore is 0.3m. The clearance volume is negligible. The steam enters the cylinder, expands at constant pressure for ¼ of the stroke and then expands reversibly according to a law PV = constant, till the end of the stroke. Calculate

a) the mass of the steam,
b) the work transfer and the heat transfer in the process?

Q304

A reversible adiabatic air turbine drives a small generator which requires a power of 2kW. The air supply for the turbine is provided by a reservoir and the pressure and temperature at turbine entry may be considered constant at 9 bar, 20°C respectively. The velocity of the air at inlet to the turbine is small and may be neglected but at exit the velocity is 55m/s. The exit pressure is 1.2 bar. Calculate:

a) the air temperature at exit from the turbine, and
b) the mass flow rate of air stating any assumptions made.

Air may be considered a perfect as for which the specific heat capacity at constant pressure C_p = 1.005 kJ/kg K and n = 1.4.

Q305

Steam at a pressure of 2 MPa and a temperature of 240°C enters a nozzle with a velocity of 15m/s. The steam expands reversibly and adiabatically in the nozzle to a pressure of 100 kPa and a dryness fraction of 0.9. Calculate the velocity of the steam at exit from the nozzle,

Saturated table extract for P = 100 KPa

p (kPa)	t_s (°C)	v_f (m^3/kg)	v_g (m^3/kg)	h_f (kJ/kg)	h_g (kJ/kg)	s_f (kJ/kg.K)	s_g (kJ/kg.K)
100	99.63	0.00104	1.694	417.46	2,676	1.3026	7.3594

Superheated table extract at P = 2 MPa

T (C)	V (m^3/kg)	h (kJ/kg)	S (kJ/kgK)
240	0.1084	2876	6.495

Q306

A mass of air at 330°C, contained in a cylinder expanded polytropically to five times its initial volume and 1/8th its initial pressure which is 1 bar. Calculate:

a) the value of the expansion index,
b) the work transfer per unit mass.

Q307

You have a 200 gram cup of coffee at 100 C, too hot to drink.

a) How much will you cool it by adding 50 gm of water at 0 C?
b) How much will you cool it by adding 50 gm of ice at 0 C?
for ice assume h_i = -333.5 and h_f = 417 kJ/kgK

Q310

The table below summarises the data for a wind turbine to be installed. Determine the annual power output for this turbine on this particular site.

Set	Wind Speed (m/sec)	Turbine Output (kW)	No of hours per year at given wind speed
(a)	(b)	(c)	(d)
1	4	2	1100
2	5	4	1100
3	6	6	1000
4	7	8	900
5	8	10	800
6	9	10	600
7	10	10	400
8	11	10	300
9	12	10	200
10	13	10	100

Q311

The following specifications for two HAWT are supplied by the manufacturers.

Item	Turbine A	Turbine B
Rotor diameter	25m	28m
Power coefficient	38	35
Gearbox efficiency	90	88
Generator efficiency	98	95
Capital cost	£99,000	£103,000
Maintenance cost/year	£4,000	£4,000

a. draw up a table for the performance of each turbine for wind speeds 4-12 m/s in intervals of 2 m/s
b. Assume the site wind availability to be 2000 hours per year, and average wind speed of 6 m/s, select the wind turbine which will be most economical. Assume life expectancy for each to be 20 years, and the unit cost of power to be 6 pence per kWh to remain constant.

Q312

If the average wind speed of 10 mph which yields 100 watts per square meter, Determine the power produced by a wind mill when the wind speed is 40 mph.

Q322

Example 4 (dynamic force calculation)

An actuator must reciprocate a load in horizontal direction (fig. 3-27). The load mass is 1000 kg, the friction coefficient is 0.12, the acceleration travel distance is 40 mm and the stationary speed after acceleration is 600 mm/s. The system pressure is 600 kPa (6 bar) calculate the piston diameter

Q323

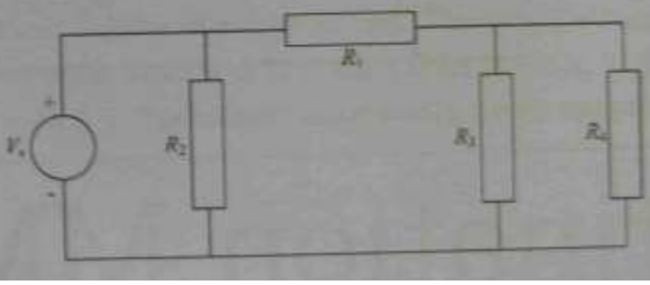
Example 6

Piston diameter	= 40 mm (0.04 m)
Piston rod diameter (d)	= 16 mm (0.016 m)
Stroke length	= 550 mm (1.10 m)
Euler case (fig. 3-26)	= situation 2
Safety factor (S)	= 5 (max. safety)
Mounting type	= rear end swivel

Find safe operating pressure

Q330

Calculate the current supplied by the 30 V source and the current through each resistor in the circuit diagram shown in Figure 3.6 using (1) nodal analysis, (2) mesh analysis and (3) circuit reduction techniques, where $R_1 = R_2 = R_3 = R_4 = 10\ \Omega$.



Q331

Find the voltage V_L using Superposition theorem in the circuit diagram of Figure 3.10.

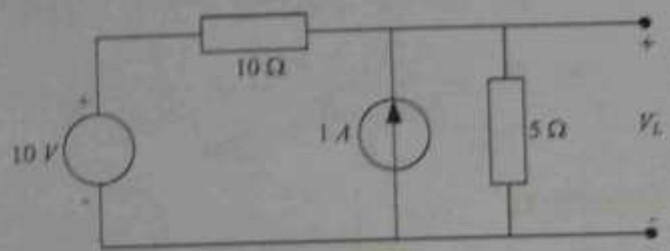


Figure 3.10: An electric circuit containing multiple sources.

Q333

For the circuit diagram depicted in Figure 3.13, use Norton's theorem to determine the current through the 5 Ω resistance.

Q334

Determine the voltage across the 1 A current source in the circuit diagram of Figure 3.20 using nodal analysis.

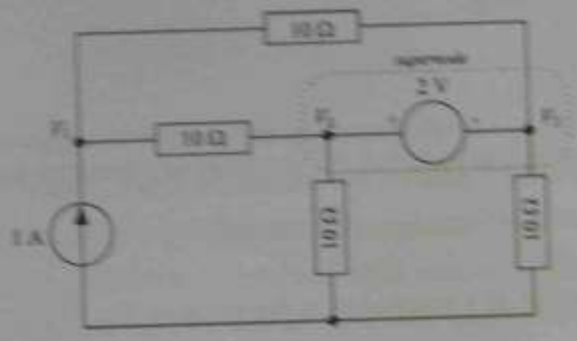


Figure 3.20: Illustration of a supernode.

Q335

Express the following as phasors and write the corresponding polar and cartesian forms.

1. $5\sin(\omega t + 45^0)$
2. $2\cos(\omega t)$
3. $10\sin(\omega t)$
4. $3\cos(\omega t + 30^0)$

Q336

Use phasor analysis to determine the voltage across the terminals a and b in the circuit diagram of Figure 4.7.

Q337

Draw the approximate Bode plots of the following transfer function

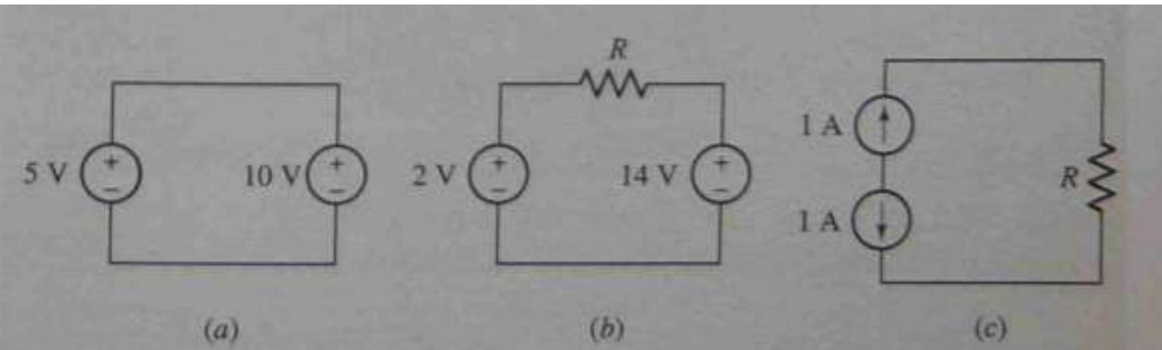
$$H(j\omega) = \frac{10(1 + j\omega 2)}{(j\omega 5)(1 + j\omega 10)^2}$$

Q338

The 560 Ω resistor shown in Fig. 2.24*b* is connected to a circuit which causes a current of 42.4 mA to flow through it. Calculate the voltage across the resistor and the power it is dissipating.

Q340

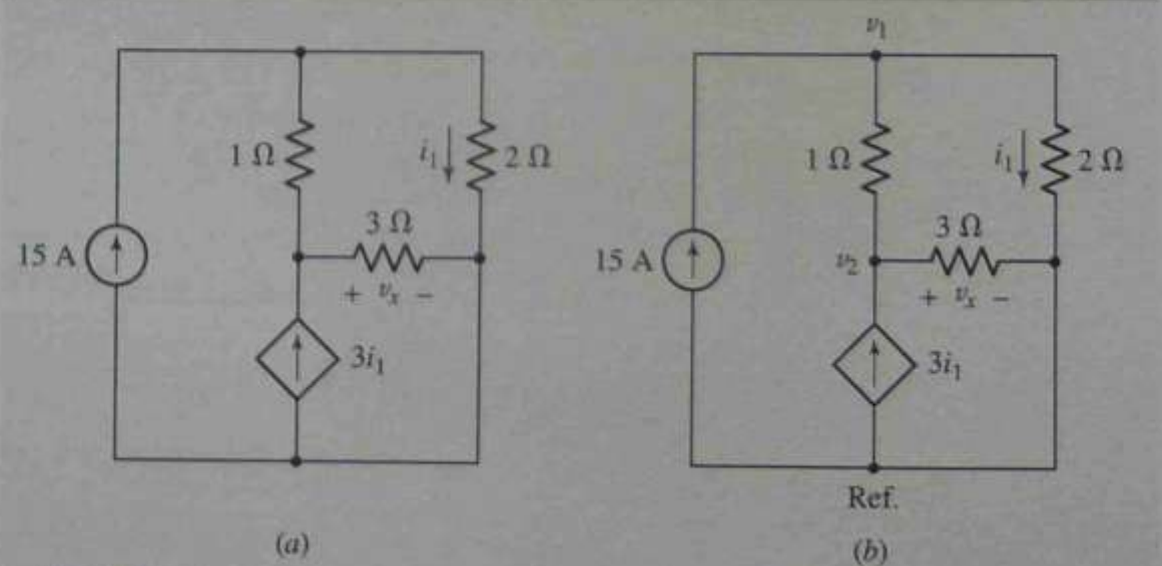
Determine which of the circuits of Fig. 3.24 are valid.



■ **FIGURE 3.24** (*a*) to (*c*) Examples of circuits with multiple sources, some of which violate Kirchhoff's laws.

Q341

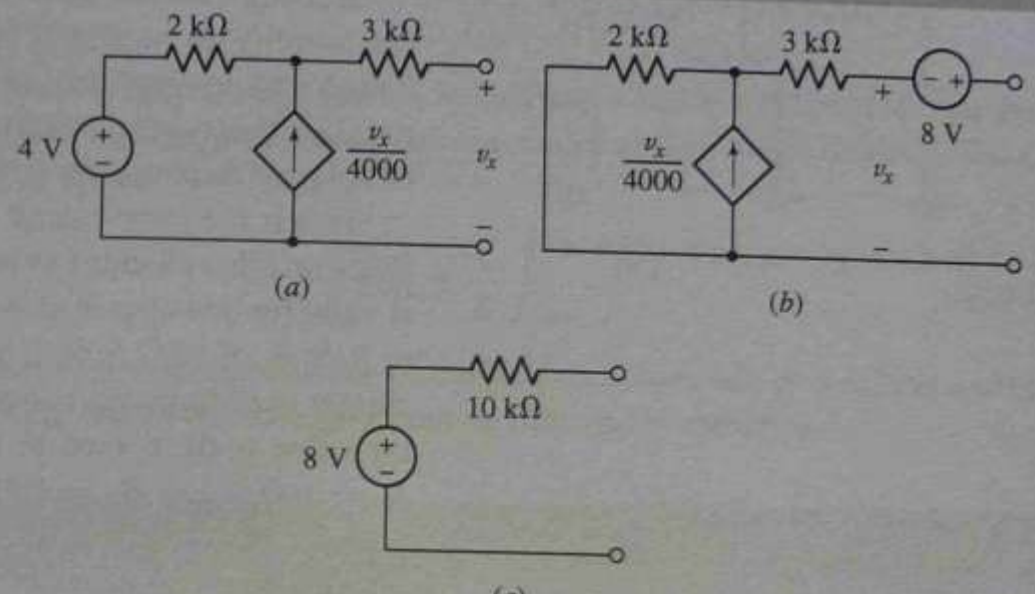
Determine the power supplied by the dependent source of Fig. 4.6*a*.



■ **FIGURE 4.6** (*a*) A four-node circuit containing a dependent current source. (*b*) Circuit labeled for nodal analysis.

Q342

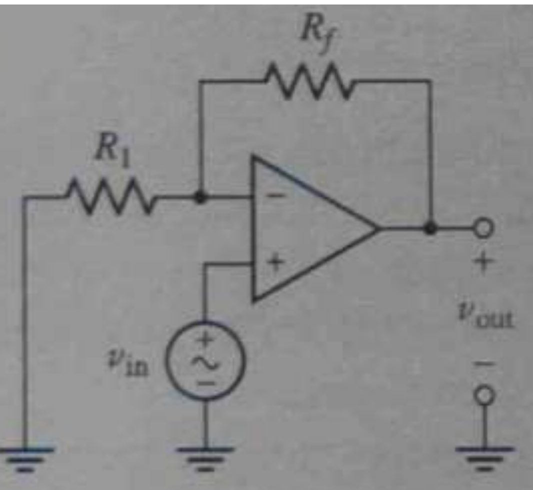
Determine the Thévenin equivalent of the circuit in Fig. 5.31*a*.



■ **FIGURE 5.31** (*a*) A given network whose Thévenin equivalent is desired. (*b*) A possible, but rather useless, form of the Thévenin equivalent. (*c*) The best form of the Thévenin equivalent for this linear resistive network.

Q343

Sketch the output waveform of the noninverting amplifier circuit in Fig. 6.6*a*. Use $v_{in} = 5 \sin 3t$ mV, $R_1 = 4.7$ kΩ, and $R_f = 47$ kΩ.



(*a*)

Q344

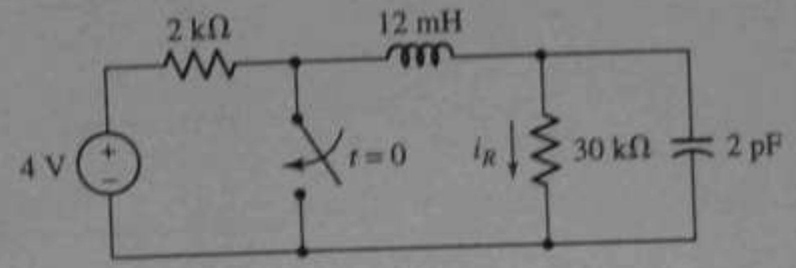
The voltage across a 2 H inductor is known to be 6 cos 5t V. Determine the resulting inductor current if $i(t = -\pi/2) = 1$ A.

Q345

For the circuit of Fig. 8.17a, find the voltage labeled v at $t = 200$ μs.

Q346

The circuit of Fig. 9.6a reduces to a simple parallel RLC circuit after $t = 0$. Determine an expression for the resistor current i_R valid for all time.



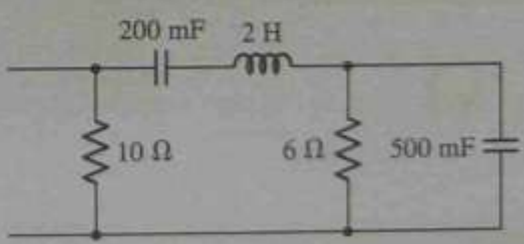
(a)

Q347

Determine $i_L(t)$ for the circuit of Fig. 9.17a, and plot the waveform.

Q349A

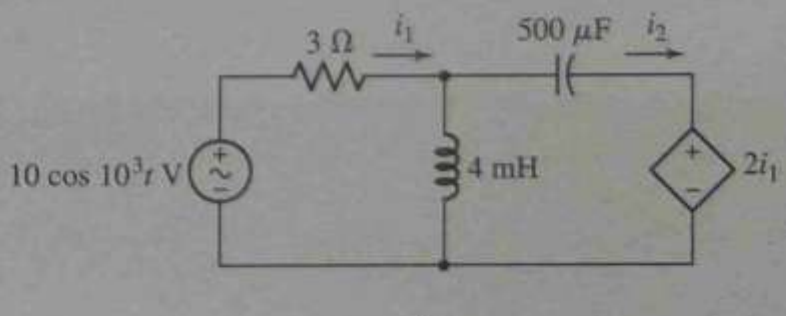
Determine the equivalent impedance of the network shown in Fig. 10.18a, given an operating frequency of 5 rad/s.



(a)

Q349B

Obtain expressions for the time-domain currents i_1 and i_2 in the circuit given as Fig. 10.24*a*.



(*a*)

Q350

Find the average power being delivered to an impedance $\mathbf{Z}_L = 8 - j11\ \Omega$ by a current $\mathbf{I} = 5\underline{/20^\circ}$ A.

Q351

Calculate values for the average power delivered to each of the two loads shown in Fig. 11.13, the apparent power supplied by the source, and the power factor of the combined loads.

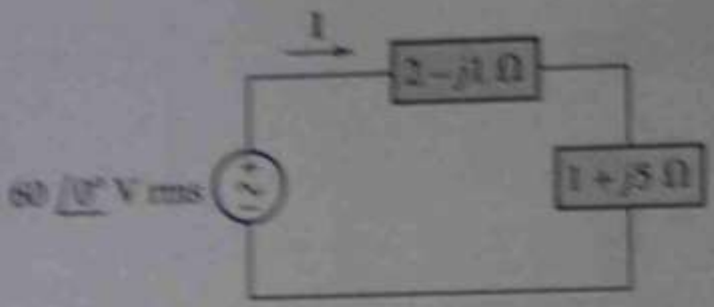


FIGURE 11.13 A circuit in which we seek the average power delivered to each element, the apparent power supplied by the source, and the power factor of the combined load.

Q352

For the circuit of Fig. 12.15, find both the phase and line currents, and the phase and line voltages throughout the circuit; then calculate the total power dissipated in the load.

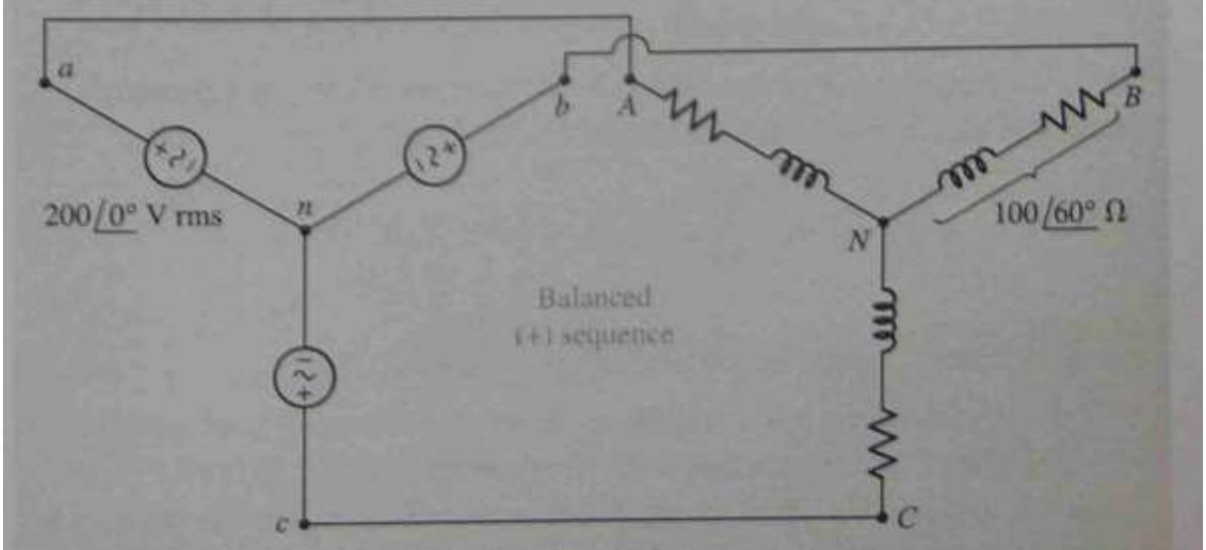


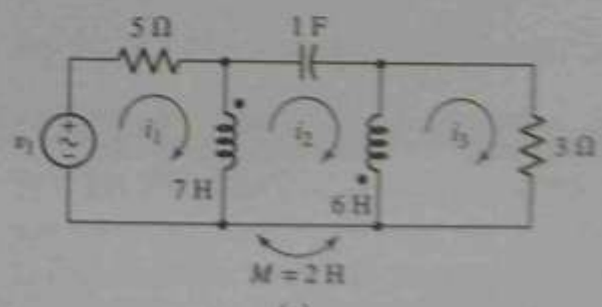
FIGURE 12.15 A balanced three-phase three-wire Y-Y connected system.

Q353

Determine the amplitude of the line current in a three-phase system with a line voltage of 300 V that supplies 1200 W to a Δ-connected load at a lagging PF of 0.8; then find the phase impedance.

Q354

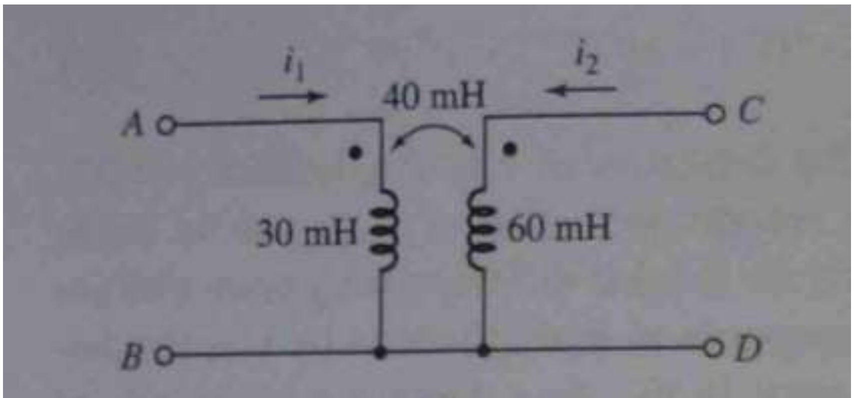
Write a complete set of phasor mesh equations for the circuit of Fig. 13.10*a*.



(*a*)

Q355

Find the T equivalent of the linear transformer shown in Fig. 13.19*a*.



(*a*)

Q356

Given the series *RL* circuit shown in Fig. 14.3, calculate the current through the 4 Ω resistor.

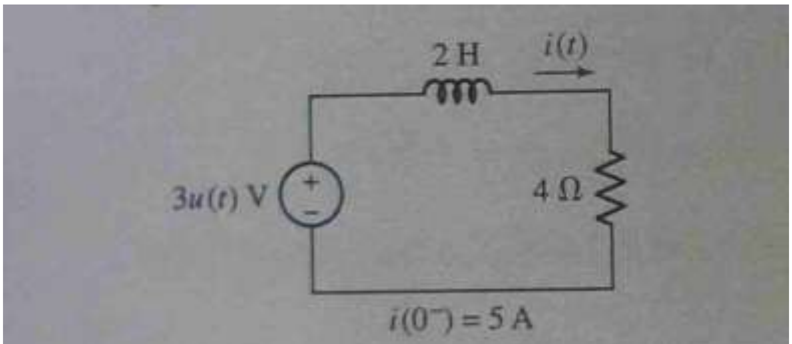


FIGURE 14.3 A circuit that is analyzed by transforming the differential equation $2\,di/dt + 4i = 3u(t)$ into $2[sI(s) - i(0^-)] + 4I(s) = 3/s$.

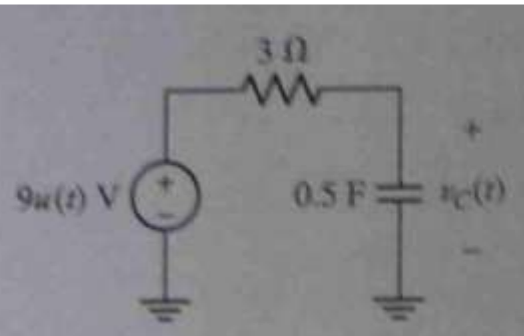
Q358

Compute the inverse transform of the function

$$V(s) = \frac{2}{s^3 + 12s^2 + 36s}$$

Q359

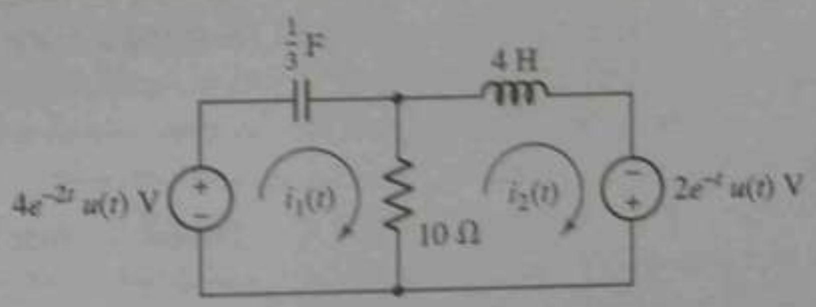
Determine $v_C(t)$ in the circuit of Fig. 15.6*a*, given an initial voltage $v_C(0^-) = -2$ V.



(*a*)

Q360

Determine the two mesh currents i_1 and i_2 in the circuit of Fig. 15.7*a*. There is no energy initially stored in the circuit.



(*a*)

Q362

The voltage $v_s = 100 \cos \omega t$ mV is applied to a series resonant circuit composed of a 10 Ω resistance, a 200 nF capacitance, and a 2 mH inductance. Use both exact and approximate methods to calculate the current amplitude if $\omega = 48$ krad/s.

Q363

Find the input impedance of the network shown in Fig. 17.5.

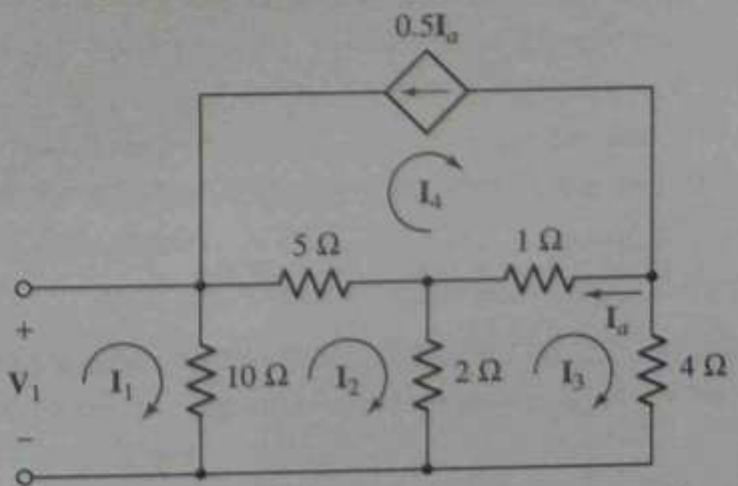


FIGURE 17.5 A one-port network containing a dependent source.

Q364

Find the four short-circuit admittance parameters for the resistive two-port shown in Fig. 17.9.

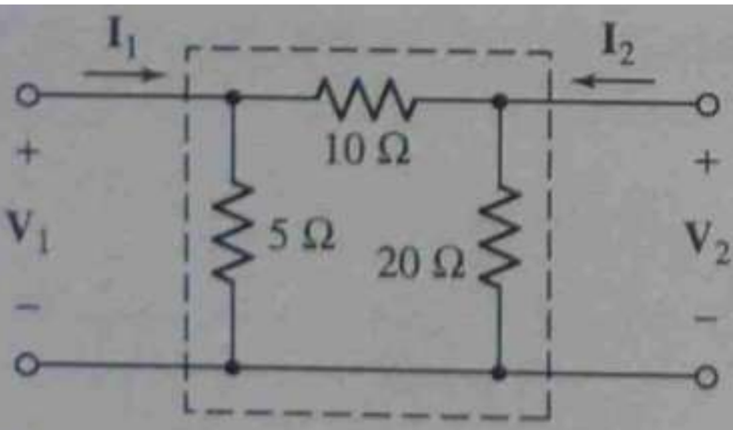
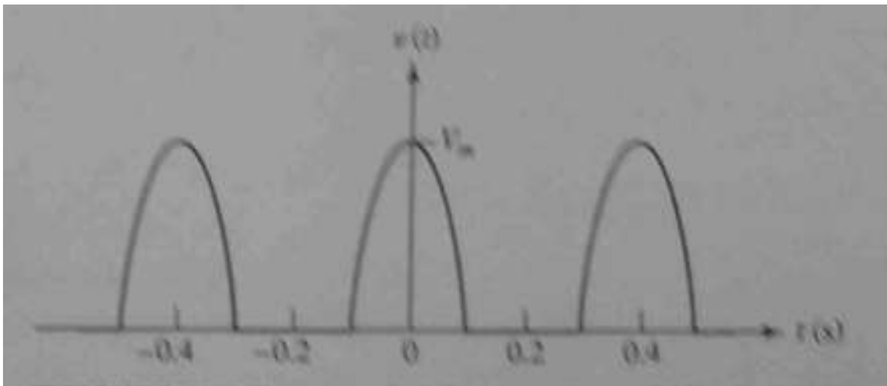


FIGURE 17.9 A resistive two-port.

Q365

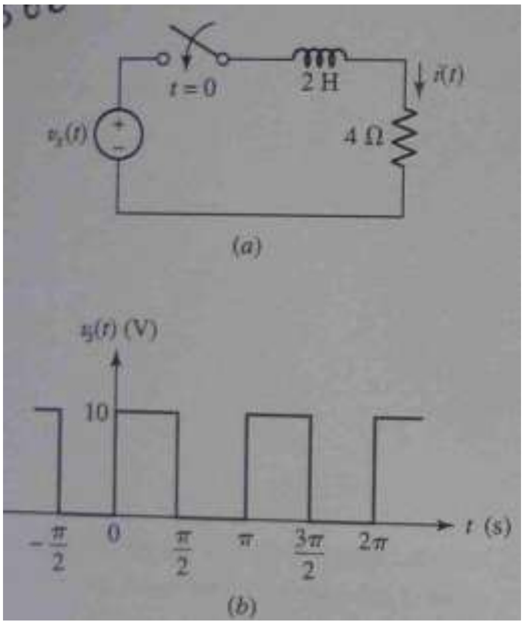
The "half-sinusoidal" waveform shown in Fig. 18.2 represents the voltage response obtained at the output of a half-wave rectifier circuit, a nonlinear circuit whose purpose is to convert a sinusoidal input voltage to a (pulsating) approximation to dc. Find the Fourier series representation of this waveform.



■ **FIGURE 18.2** The output of a half-wave rectifier to which a sinusoidal input is applied.

Q366

For the circuit of Fig. 18.8*a*, determine the periodic response *i*(*t*) corresponding to the forcing function shown in Fig. 18.8*b* if *i*(0) = 0.



Q367

A certain function $f(t)$ is a train of rectangular pulses of amplitude V_0 and duration τ, recurring periodically every T seconds, as shown in Fig. 18.11. Find the exponential Fourier series for $f(t)$.

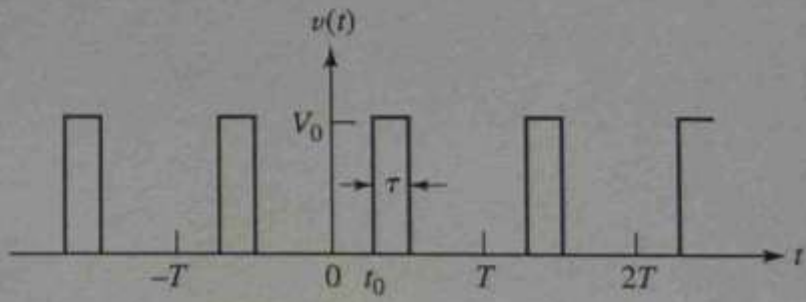


FIGURE 18.11 A periodic sequence of rectangular pulses.

Q378

2.1. A piece of infinitely permeable magnetic material completes the magnetic circuit in Fig. 2P.1 in such a way that it is free to move in the x- or y-direction. Under the assumption that the air gaps are short compared with their cross-sectional dimensions (i.e., that the fields are as shown), find $\lambda(x, y, i)$. For what range of x and y is this expression valid?

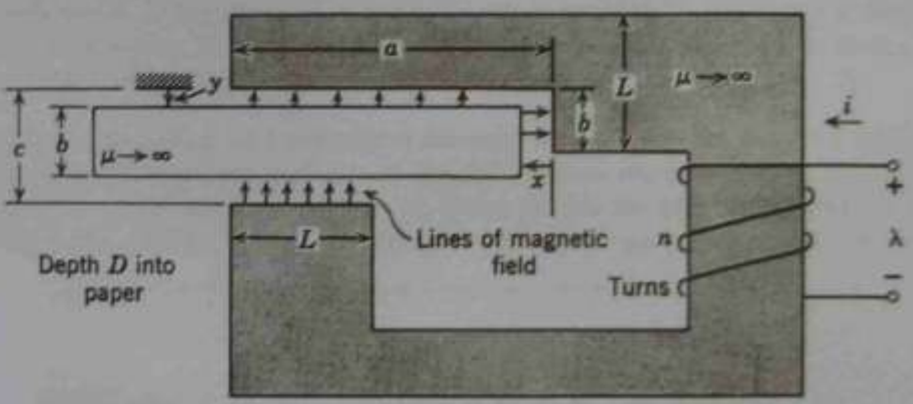


Fig. 2P.1

Q379

2.5. (a) Find the response $x(t)$ of the system shown in Fig. 2P.5a to a driving force $f(t)$ which is

(1) an impulse $\quad f(t) = I_0\mu_0(t),$

(2) a step $\quad f(t) = F_0\mu_{-1}(t).$

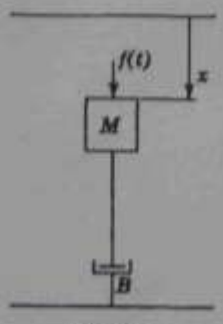


Fig. 2P.5a

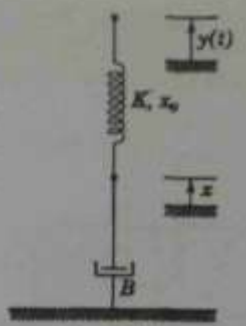


Fig. 2P.5b

Q380

2.7. In the system in Fig. 2P.7 the two springs have zero force when both x_1 and x_2 are zero. A mechanical force f is applied to node 2 in the direction shown. Write the equations governing the motion of the nodes 1 and 2. What are the natural frequencies involved?

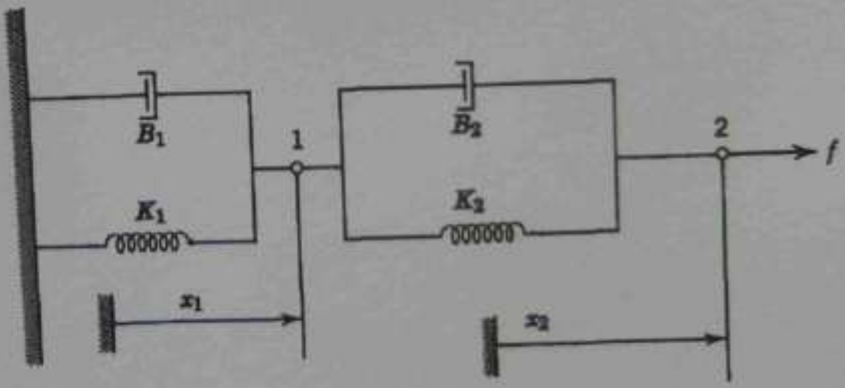


Fig. 2P.7

Q381

3.1. A simple plunger-type solenoid for the operation of relays, valves, etc., is represented in Fig. 3P.1. Assume that it is a conservative system and that its electrical equation of state is

$$\lambda = \frac{L_0 i}{1 + x/a}.$$

(a) Find the force that must be *applied to* the plunger to hold it in equilibrium at a displacement x and with a current i.

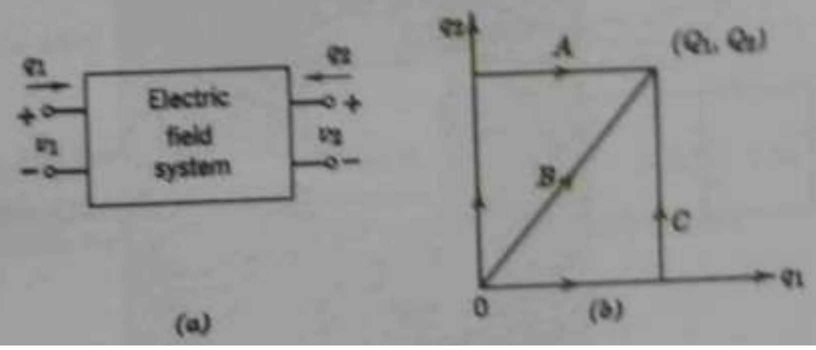
Q382

3.2. An electrically linear electric field system with two electrical terminal pairs is illustrated in Fig. 3P.2. The system has the electrical equations of state $v_1 = S_{11}q_1 + S_{12}q_2$ and $v_2 = S_{21}q_1 + S_{22}q_2$. (See Example 3.1.1 for a physical case of this type.)

(a) Calculate the energy input to the system over each of the three paths A, B, and C in the q_1-q_2 plane illustrated in Fig. 3P.2*b*.

(b) What is the relation between coefficients S_{12} and S_{21} to make these three values of energy the same?

(c) Derive the result of (b) by assuming that the system is conservative and applying reciprocity.



Q383

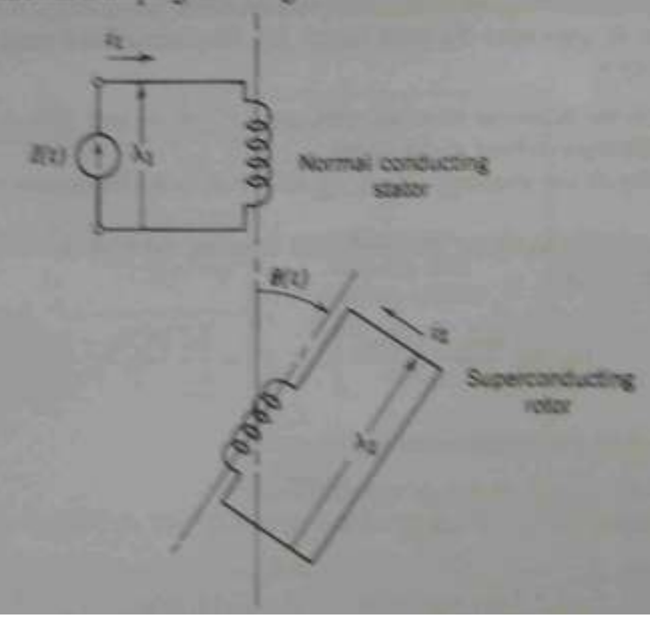
3.8. A schematic diagram of a rotating machine with a superconducting rotor (moment of inertia J) is shown in Fig. 3P.8. Tests have shown that $\lambda_1 = i_1L_1 + i_2L_m \cos\theta$ and $\lambda_2 = i_1L_m \cos\theta + i_2L_2$, where $\theta(t)$ is the angular deflection of the shaft to which coil (2) is attached. The machine is placed in operation as follows:

(a) With the (2) terminals open circuit and the shaft at $\theta = 0$, $I(t)$ is raised to I_0.

(b) Terminals (2) are shorted to conserve the flux λ_2 regardless of $\theta(t)$ or $i_1(t)$.

(c) $I(t)$ is now made a given driving function.

Write the equation of motion for the shaft. Your answer should be one equation involving only $\theta(t)$ as an unknown. Damping can be ignored.



Q384

3.10. The following equations of state describe the conservative, magnetic field coupling system of Fig. 3P.10 for the ranges of variables of interest ($i_1 > 0, i_2 > 0$). $\lambda_1 = L_0 i_1 + A i_1 i_2^2 x$ and $\lambda_2 = A i_1^2 i_2 x + L_0 i_2$, where L_0 and A are positive constants.

(a) Find the force applied by the coupling system on the external mechanical circuit as a function of i_1, i_2, and x.

(b) Write the *complete* set of differential equations for the system by using i_1, i_2, and x as dependent variables.

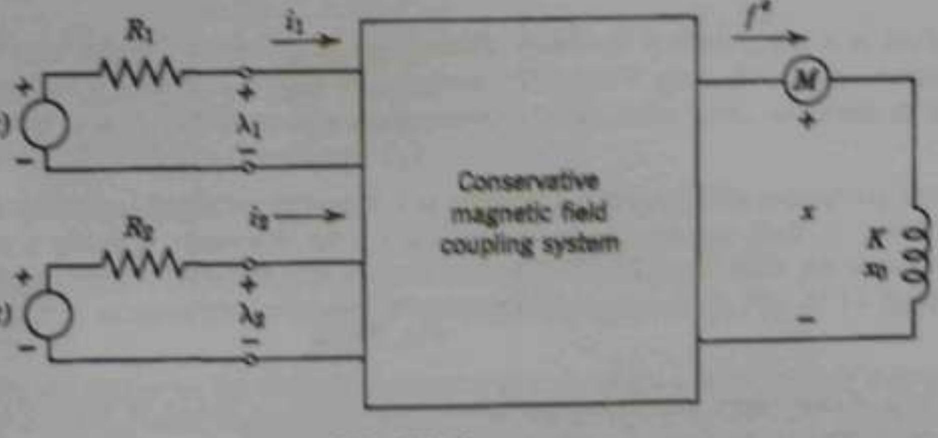


Fig. 3P.10

Q385

3.12. A magnetic field system has three electrical terminal pairs and two mechanical terminal pairs as shown in Fig. 3P.12. The system is electrically linear and may be described by the relations $\lambda_1 = L_{11}i_1 + L_{12}i_2 + L_{13}i_3$, $\lambda_2 = L_{21}i_1 + L_{22}i_2 + L_{23}i_3$, and $\lambda_3 = L_{31}i_1 + L_{32}i_2 + L_{33}i_3$. Each of the inductances L_{ij} ($i = 1, 2, 3; j = 1, 2, 3$) may be functions of the mechanical variables x_1 and x_2. *Prove* that if the system is conservative, $L_{12} = L_{21}$, $L_{13} = L_{31}$, and $L_{23} = L_{32}$. To do this recall that for a conservative system the energy (or coenergy) does not depend on the path of integration but only on the end point.

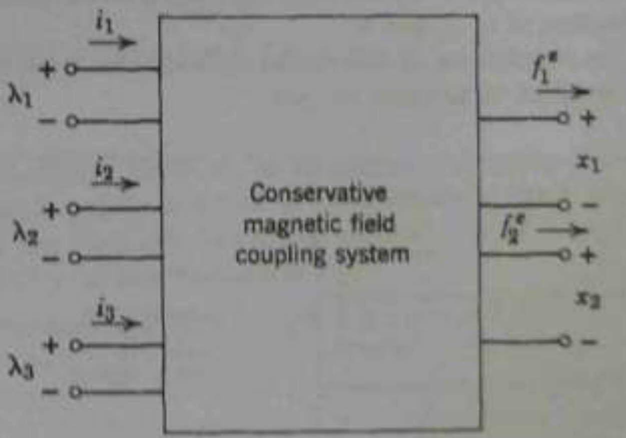


Fig. 3P.12

Q390

3.16. A plane electrode is free to move into the region between plane-parallel electrodes, as shown in Fig. 3P.16. The outer electrodes are at the same potential, whereas the inner electrode is at a potential determined by the constant voltage source V_0 in series with the output of an amplifier driven by a signal proportional to the displacement of the movable electrode itself. Hence the voltage of the inner electrode relative to that of the outer electrodes is $v = -V_0 + Ax$, where A is a given feedback gain. Find the force of electrical origin $f^e(x)$. (Note that this force is only a function of position, since the voltage is a known function of x.)

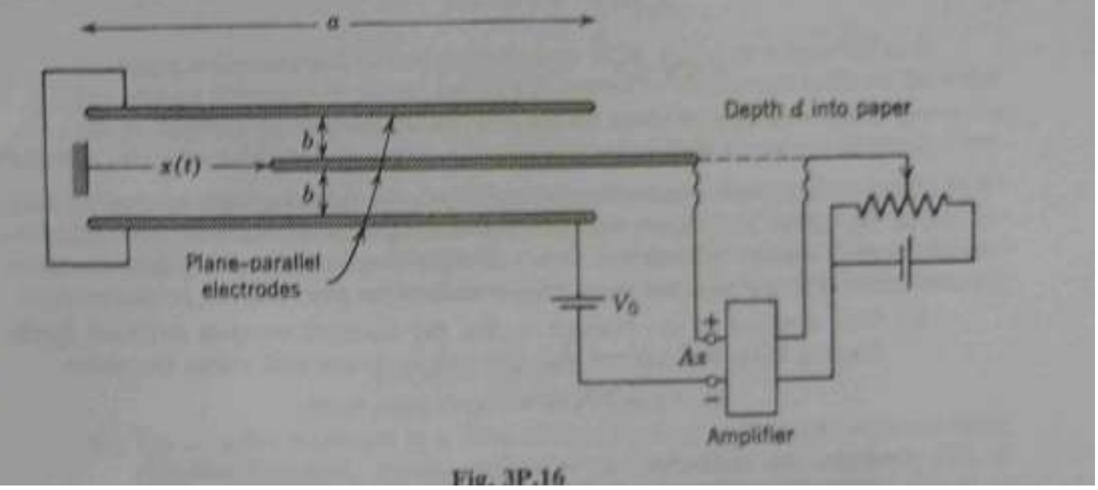


Fig. 3P.16

Q391

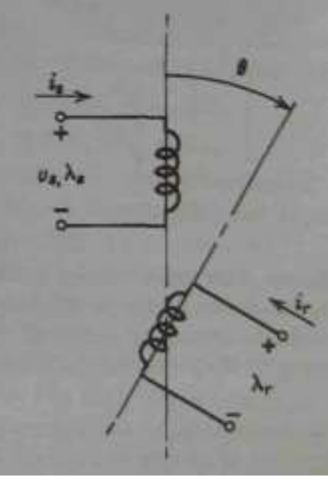
4.4. The machine represented schematically in Fig. 4P.4 has uniform winding distributions. As indicated by Problem 4.2, the electrical terminal relations are ideally

$$\lambda_s = L_s i_s + i_r \sum_{n\,\text{odd}} \frac{M_n}{n^4} \cos n\theta,$$

$$\lambda_r = L_r i_r + i_s \sum_{n\,\text{odd}} \frac{M_n}{n^4} \cos n\theta,$$

where L_s, L_r, and M_n are constants. We now constrain the machine as follows: $i_r = I =$ constant; $\theta = \omega t$, $\omega =$ constant, stator winding open-circuited $i_s = 0$.

(a) Find the instantaneous stator voltage $v_s(t)$.
(b) Find the ratio of the amplitude of the nth harmonic stator voltage to the amplitude of the fundamental component of stator voltage.
(c) Plot one complete cycle of $v_s(t)$ found in (a).



Q392

4.16. For nomenclature, refer to Fig. 4.1.17. The two-phase equivalent of a large, two-pole, polyphase, 60-Hz induction motor has the following parameters for operation at 60 Hz: $R_r = 0.100$ ohm, $\omega_s M = 4.50$ ohms, and $\omega_s(L_s - M) = \omega_s(L_r - M) = 0.300$ ohm. Neglect armature resistance. For operation at a constant amplitude of armature voltage $V_s = \sqrt{2}\,500$ V peak, calculate and plot torque, armature current, volt-ampere input, electrical power input, and mechanical power output as functions of mechanical speed for the range $0 < \omega_m < \omega_s = 120\pi$ rad/sec.

Q393

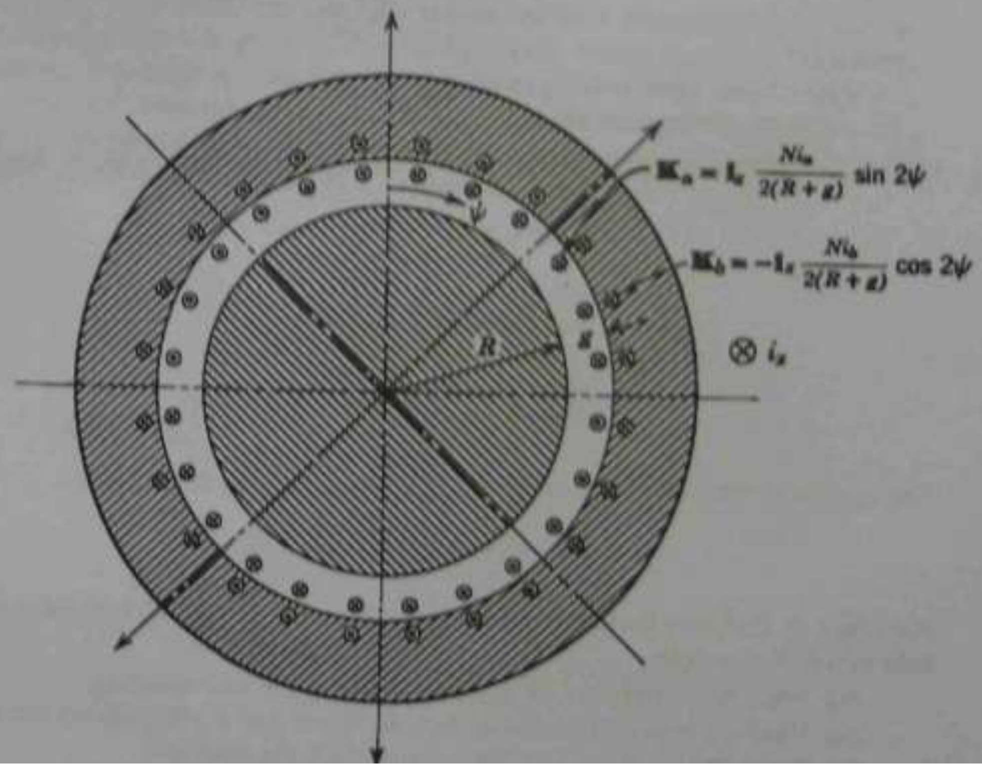
4.20. A four-pole smooth-air-gap machine has a two-phase set of stator windings, each with a total of N turns. The windings are distributed sinusoidally and currents in them produce surface current densities as indicated in Fig. 4P.20. When $g \ll R$, the radial flux density produced in the air gap by each winding is (see Problems 4.1 and 4.10)

$$B_{ra} = \frac{\mu_0 N i_a}{2g} \cos 2\psi,$$

$$B_{rb} = \frac{\mu_0 N i_b}{2g} \sin 2\psi.$$

(a) For the two-phase excitation, $i_a = I_a \cos \omega t$, $i_b = I_b \sin \omega t$, which is unbalanced in amplitude, find the total radial flux density.

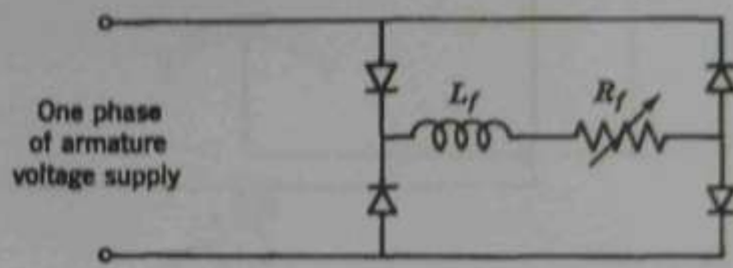
(b) Express the answer to (a) as a sum of two constant-amplitude traveling waves.



Q394

4.27. A two-phase, 60-Hz, salient-pole, 2-pole, synchronous motor has the following ratings and constants:

Rated output power,	1000 hp
Rated armature volts,	$\sqrt{2}$ 1000 V peak
Rated power factor,	unity
Direct axis reactance,	$\omega(L_0 + L_2) = 3.0$ ohms
Quadrature axis reactance,	$\omega(L_0 - L_2) = 2.0$ ohms
Speed voltage coefficient,	$\omega M = 150$ V/A



Q401

Example 6-1

A dc machine turning at 875 r/min carries an armature winding whose total weight is 40 kg. The current density is 5 A/mm^2 and the operating temperature is 80°C. The total iron losses in the armature amount to 1100 W.

Q402

Example 6-2

A dc compound motor having a rating of 10 kW, 1150 r/min, 230 V, 50 A, has the following losses at *full load*:

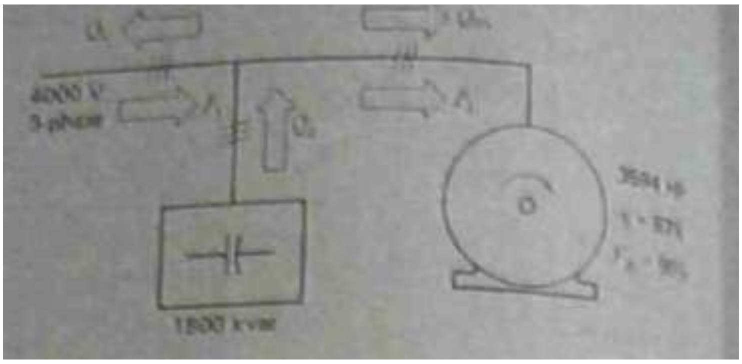
	bearing friction loss	=	40 W
	brush friction loss	=	50 W
	windage loss	=	200 W
(1)	total mechanical losses	=	290 W
(2)	iron losses	=	420 W
(3)	copper loss in the shunt field	=	120 W
	copper losses at full load:		
	a. in the armature	=	500 W
	b. in the series field	=	25 W
	c. in the commutating winding	=	70 W
(4)	total copper loss in the armature circuit at full load	=	595 W

Calculate the losses and efficiency at no-load and at 25, 50, 75, 100, and 150 percent of the nominal rating of the machine. Draw a graph showing efficiency as a function of mechanical load (neglect the losses due to brush contact drop).

Q403

Example 8-11

A 5000 hp wye-connected motor is connected to a 4000 V (line-to-line) 3-phase, 60 Hz line (Fig 8.21).



Q405

Example 20-6

Using the information revealed by the 60 Hz torque-speed and current-speed curves of Fig. 20.42, calculate the voltage and frequency required so that the machine will run at 5200 r/min while developing a torque of 100 N·m. What is the corresponding stator current?

Q410

Example 5-2

The following details are given on a 225 kW (≈ 300 hp), 250 V, 1200 r/min dc motor (see Figs. 5.4 and 5.5).

armature coils	243
turns per coil	1
type of winding	lap
armature slots	81
commutator segments	243
field poles	6
diameter of armature	559 mm
axial length of armature	235 mm

Q412

Q412

Example 10-11

A 100 kVA transformer is connected in parallel with an existing 250 kVA transformer to supply a load of 330 kVA. The transformers are rated 7200 V/240 V, but the 100 kVA unit has an impedance of 4 percent while the 250 kVA transformer has an impedance of 6 percent (Fig. 10.36a).

Calculate

a. The nominal primary current of each transformer

b. The impedance of the load referred to the primary side

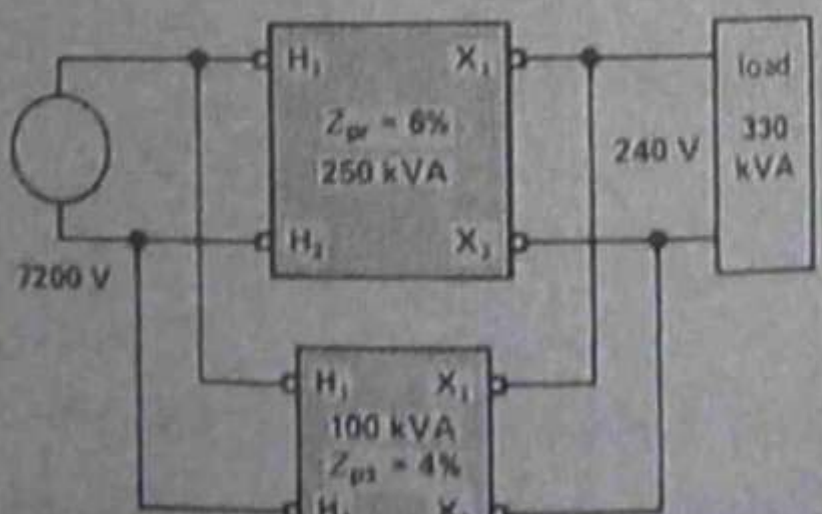
c. The impedance of each transformer referred to the primary side

d. The actual primary current in each transformer

Solution

a. Nominal primary current of the 250 kVA transformer is

$$I_{n1} = 250\,000/7200 = 34.7 \text{ A}$$



Q413

Example 12-1

Three single-phase transformers are connected in delta-delta to step down a line voltage of 138 kV to 4160 V to supply power to a manufacturing plant. The plant draws 21 MW at a lagging power factor of 86 percent.

Calculate

a. The apparent power drawn by the plant
b. The apparent power furnished by the HV line

Q414

Example 12-2

Three single-phase step-up transformers rated at 40 MVA, 13.2 kV/80 kV are connected in delta-wye on a 13.2 kV transmission line (Fig. 12.5). If they feed a 90 MVA load, calculate the following.

a. The secondary line voltage
b. The currents in the transformer windings
c. The incoming and outgoing transmission line currents

Q416

Example 12-4

The voltage of a 3-phase, 230 kV line has to be stepped up to 345 kV to supply a load of 200 MVA. Three single-phase transformers connected as autotransformers are to be used. Calculate the basic power and voltage rating of each transformer, assuming they are connected as shown in Fig 12.11b.

Q418

Example 13-2

A 0.5 hp, 6-pole induction motor is excited by a 3-phase, 60 Hz source. If the full-load speed is 1140 r/min, calculate the slip.

Q422

Example 16-3

A 30 MVA, 15 kV 60 Hz ac generator has a synchronous reactance of 1.2 pu and a resistance of 0.02 pu.

Calculate

a. The base voltage, base power and base impedance of the generator
b. The actual value of the synchronous reactance
c. The actual winding resistance, per phase
d. The total full-load copper losses

Q421

Example 16-2

A 3-phase synchronous generator produces an open-circuit line voltage of 6928 V when the dc exciting current is 50 A. The ac terminals are then short-circuited, and the three line currents are found to be 800 A.

a. Calculate the synchronous reactance per phase.
b. Calculate the terminal voltage if three 12 Ω resistors are connected in wye across the terminals.

Q424

Example 16-7

A 36 MVA, 21 kV, 1800 r/min, 3-phase generator connected to a power grid has a synchronous reactance of 9 Ω per phase. If the exciting voltage is 12 kV (line-to-neutral), and the system voltage is 17.3 kV (line-to-line), calculate the following:

a. The active power which the machine delivers when the torque angle δ is 30° (electrical)
b. The peak power that the generator can deliver before it falls out of step (loses synchronism)

Q425

Example 16-8 Q425

A 250 MVA, 25 kV, 3-phase steam-turbine generator has a synchronous reactance of 1.6 pu and a transient reactance X'_d of 0.23 pu. It delivers its rated output at a power factor of 100%. A short-circuit suddenly occurs on the line, close to the generating station.

Q427

Example 20-1

A 3-phase NEMA size 5 magnetic contactor rated at 270 A, 460 V possesses a 120 V, 60 Hz relay coil. The coil absorbs an apparent power of 2970 VA and 212 VA, respectively, in the open and closed contactor position. Calculate the following:

a. The inrush exciting current
b. The normal, sealed exciting current
c. The control power needed to actuate the relay coil compared to the power handled by the contactor

Q428

A 150 kW (200 hp), 460 V, 3-phase 3520 r/min, 60 Hz induction motor has a locked-rotor torque of 600 N·m and a locked-rotor current of 1400 A. Three resistors are connected in series with the line so as to reduce the voltage across the motor to 0.65 pu.

Calculate

a. The apparent power absorbed by the motor under full-voltage, locked-rotor conditions
b. The apparent power absorbed by the motor when the resistors are in the circuit

Q429

In Example 20-2, if the locked-rotor power factor of the motor alone is 0.35, calculate the value of the series resistors and the power they dissipate.

Q430

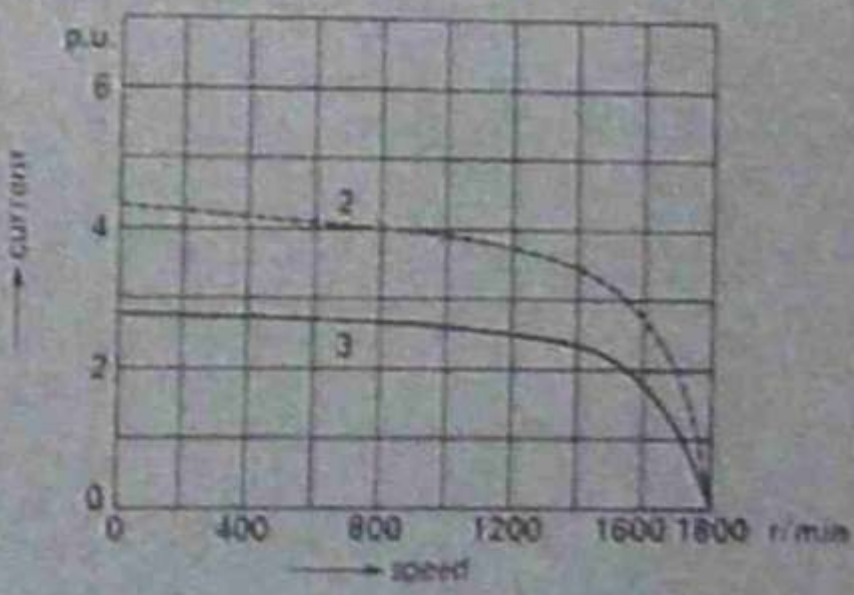
Example 20-4

A 200 hp (150 kW), 460 V, 3-phase, 3520 r/min, 60 Hz induction motor has a locked-rotor torque of 600 N·m and a locked-rotor current of 1400 A (same motor as in Example 20-2). Two autotransformers, connected in open delta, and having a 65 percent tap, are employed to provide reduced-voltage starting.

Calculate

a. The apparent power absorbed by the motor

b. The apparent power supplied by the 460 V line



Q448

Find the force on an electron (charge -1.602×10^{-19} C) which is 1 nm from a perfectly conducting plane. What is the electric field acting on the electron?

Q449

An air-spaced transmission line consists of two parallel cylindrical conductors each 2 mm in diameter with their centres 10 mm apart as shown in fig. 1.9. Calculate the maximum potential difference which can be applied to the conductors assuming that the electrical breakdown strength of air is $3\ \mathrm{MV \cdot m^{-1}}$.

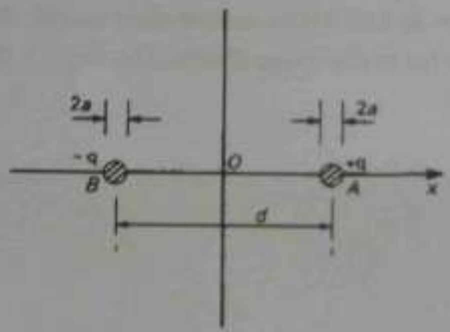


Fig. 1.9 A cross-sectional view of a parallel-wire transmission line.

Q450

Figure 1.13 shows a simplified form for the deflection plates for a low current electron beam. Given that the electron beam is launched from an electrode (the cathode) at a potential of -2000V and passes between the deflection plates as shown, estimate the angular deflection of the beam when the potentials of the plates are ±50 V.

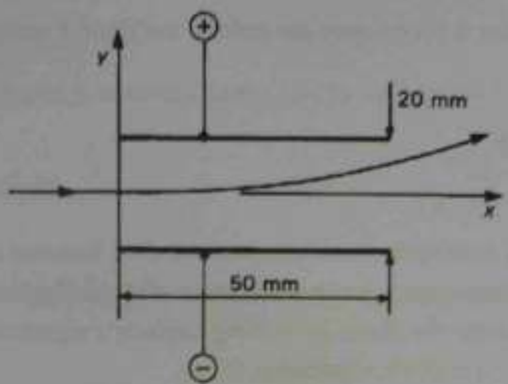


Fig. 1.13 The arrangement of a pair of electrostatic deflection plates for an electron beam.

Q451

Using the results of Example 2.1 calculate the maximum charge per unit area which can be induced in the semiconductor material. If there are 2.0×10^{18} atoms per square metre in the first layer of the silicon crystal, what proportion can be ionized by applying a voltage to the gate which is one sixth of the breakdown voltage?

Q452

A variable capacitor comprised a set of fixed plates, A, and a set of moving plates, B, as shown in Fig. 2.2. The capacitor is used to tune the frequency of a resonant circuit which varies inversely as the square root of the capacitance. Assuming that the effects of fringing fields can be neglected, find the shape which the moving plates must have if the frequency is to be proportional to the angle θ in the range 20-160° and 500-1500 kHz.

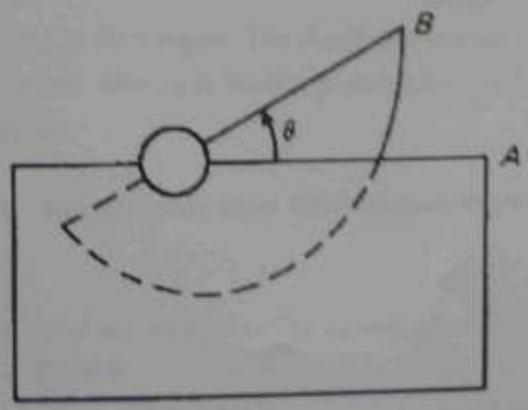


Fig. 2.2 Schematic diagram of a variable capacitor. A set of moving plates B rotates within a parallel set of fixed plates A.

Q453

A coaxial cable is to be made with two dielectric layers as shown in Fig. 2.7. The inner layer is made from a high-quality but expensive material which has dielectric strength E_1 and the outer layer from a cheaper material of dielectric strength E_2. Find an expression for the outer radius, b, of the inner layer if the cable is to be made as cheap as possible without reducing its maximum working voltage.

Q454

Figure 2.16 shows the distribution of fixed electric charges in the depletion region of a p-n junction in equilibrium. Assuming that the silicon has a permittivity ε, show that the potential difference between the two sides of the junction is given by:

$$V_0 = \frac{q}{2\varepsilon_s}\left(N_D d_n^2 + N_A d_p^2\right)$$

where q is the magnitude of each charge.

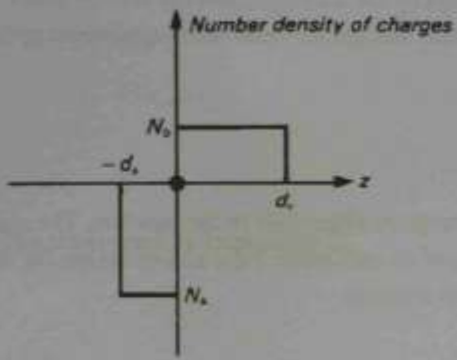


Fig. 2.16. The distribution of ionized donor and acceptor states in the depletion layer of an idealized p-n junction.

Q454B

A transmission line of characteristic impedance Z_0 terminated by a resistor $R = \frac{1}{2}Z_0$ as shown in fig.7.1 is connected to a matched source by a switch for a short time τ to produce a pulse on the line. If the propagation time along the line is T, where $T \gg \tau$, investigate the variation of voltage with time at each end of the line.

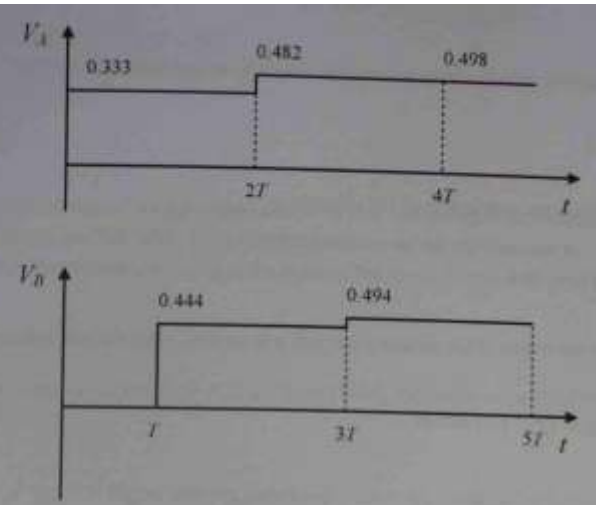


Fig. 7.5. Voltages across the line at *A* and *B* as a function of time for example 7.2

Q455

A solenoid is wound uniformly at 318 turns per metre on an insulating cylindrical former 100 mm in diameter and 2.5 m long. Calculate its inductance per unit length. Ten tapping points are made on the solenoid at regular intervals, and each is connected to earth through a 0.001 μF capacitor. The resulting network is a cascade of symmetrical tee sections as shown in Fig. 7.6. This line is used as the pulse-forming network for a high-power radar transmitter in the following manner: The line is charged to a potential difference of 20 kV with both ends open-circuited; a matched load is then connected across one end and the line is discharged through it. Assuming that the line can be treated as a uniform transmission line, calculate the amplitude and duration of the voltage pulse supplied to the load. Calculate also the current flowing in the load during the pulse, and the total pulse energy.

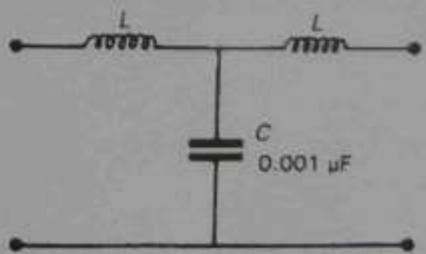


Fig. 7.6. One section of the pulse-forming network

Q457

The international guidelines for exposure of the general public to electromagnetic radiation in the range 400 to 2000 MHz state that it should not exceed a power density of $f/200$ W.m^{-2} where the frequency f is in MHz. Find the maximum electric and magnetic field strengths permissible at 2000 MHz.

(See: Guidelines for Limiting Exposure to Time-Varying Electric, Magnetic, and Electro-magnetic Fields (Up To 300 GHz), Table 7, International Commission on Non-Ionizing Radiation Protection, 1998; available at http://www.icnirp.de/documents/emfgdl.pdf)

Q458

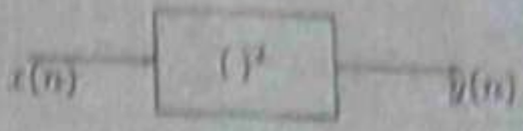
A long straight cylindrical wire carries a current I. Given that the wire has radius a and resistance R per unit length, calculate the power dissipated per unit length both directly and by integration of the Poynting vector.

Q512

Determine the linearity of the following system

Example 1.2.2 *Non linear, shift invariant system*

Consider a system described by the following input output relationship



or

$$y(n) = T\{x(n)\} = x^2(n)$$

Q513

Example 1.2.4 *To compute the relationship between the input and the output with a filter. $h(-k)$ is first plotted against k, $h(-k)$ is simply $h(k)$ reflected or "flipped" around $k = 0$, as shown in Figure 1.11. Replacing k by $k - n$, where n is a fixed integer, leads to a shift in the origin of the sequence $h(-k)$ to $k = n$. $y(n)$ is then obtained by the $\sum x(k)h(n-k)$. See Figure 1.12.*

Q514

Example 2.1.3 *Find the Fourier transform of the sequence*

$$x(n) = 0.5^{|n|}$$

We first observe that

$$x(n) = x_1(n) + x_2(n)$$

$$\text{where } x_1(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$\text{and } x_2(n) = \begin{cases} 0.5^{-n}, & n < 0 \\ 0, & n \geq 0 \end{cases}$$

Thus $X(f) = X_1(f) + X_2(f)$.

Q516

Example 2.5.3 *Solve the following LCCDE with IRC*

$$y(n) - 2y(n-1) - 3y(n-2) = x(n) + 4x(n-1),$$

where $x(n) = \delta(n)$.

Q515

Example 3.4.2 *Comparison of circular and linear convolution*

Let

$$x_1(n) = x_2(n) = \begin{cases} 1, & 0 \le n \le L-1, \\ 0, & \text{otherwise}. \end{cases}$$

Q 515

If $N = L$, then the N-point DFTs are given by

$$X_1(k) = X_2(k) = \sum_{n=0}^{N-1} e^{-j2\pi nk/N} = \begin{cases} N, & k = 0, \\ 0, & \text{otherwise} \end{cases}$$

Q517

Type I FIR filter. *Given a filter with impulse response*

$$h(n) = \begin{cases} 1, & 0 \le n \le 2 \\ 0, & \text{otherwise} \end{cases}$$